http://pubsonline.informs.org/journal/msom
MANUFACTURING & SERVICE OPERATIONS MANAGEMENT
Vol. 24, No. 3, May–June 2022, pp. 1511–1528
ISSN 1523-4614 (print), ISSN 1526-5498 (online)

Ride-Hailing Platforms: Competition and Autonomous Vehicles

Auyon Siddiq,[a] Terry A. Taylor[b,*]

[a] Anderson School of Management, University of California, Los Angeles, California 90095; [b] Haas School of Business, University of California, Berkeley, California 94720
*Corresponding author
Contact: auyon.siddiq@anderson.ucla.edu, https://orcid.org/0000-0003-2977-5558 (AS); taylor@haas.berkeley.edu, https://orcid.org/0000-0002-1488-8923 (TAT)

Received: July 26, 2019
Revised: September 29, 2020; April 15, 2021
Accepted: April 18, 2021
Published Online in Articles in Advance: January 10, 2022

https://doi.org/10.1287/msom.2021.1013

Abstract. ***Problem definition*:** Ride-hailing platforms, which are currently struggling with profitability, view autonomous vehicles (AVs) as important to their long-term profitability and prospects. Are competing platforms helped or harmed by platforms' obtaining access to AVs? Are the humans who participate on the platforms—driver-workers and rider-consumers (hereafter, agents)—collectively helped or harmed by the platforms' access to AVs? How do the conditions under which access to AVs reduces platform profits, agent welfare, and social welfare depend on the AV ownership structure (i.e., whether platforms or individuals own AVs)? ***Academic/practical relevance*:** AVs have the potential to transform the economics of ride-hailing, with welfare consequences for platforms, agents, and society. ***Methodology*:** We employ a game-theoretic model that captures platforms' price, wage, and AV fleet size decisions. ***Results*:** We characterize necessary and sufficient conditions under which platforms' access to AVs reduces platform profit, agent welfare, and social welfare. The structural effect of access to AVs on agent welfare is robust regardless of AV ownership; agent welfare decreases if and only if the AV cost is high. In contrast, the structural effect of access to AVs on platform profit depends on who owns AVs. The necessary and sufficient condition under which access to AVs decreases platform profit is high AV cost under platform-owned AVs and low AV cost under individually owned AVs. Similarly, the structural effect of access to AVs on social welfare depends on who owns AVs. Access to individually owned AVs increases social welfare; in contrast, access to platform-owned AVs decreases social welfare—if and only if the AV cost is high. ***Managerial implications*:** Our results provide guidance to platforms, labor and consumer advocates, and governmental entities regarding regulatory and public policy decisions affecting the ease with which platforms obtain access to AVs.

Supplemental Material: The online appendix is available at https://doi.org/10.1287/msom.2021.1013.

Keywords: game theory • service operations

1. Introduction

Ride-hailing platforms Lyft and Uber, which are currently struggling with profitability, view autonomous vehicles (AVs) as important to their long-term profitability and prospects. Wage payments to drivers constitute the largest expense for ride-hailing platforms. With the specific purpose of eliminating the variable cost of payments to drivers, Lyft and Uber have aggressively pursued the development of AVs, with each investing billions in their efforts (Siddiqui and Bensinger 2019). Lyft and Uber comprise 98% of the U.S. ride-hailing market (Bosa 2018). A central feature of these ride-hailing platforms is that they simultaneously compete over a common pool of supply—namely, independent driver-workers—and a common pool of demand—namely, rider-consumers. Each platform anticipates that after initially launching AVs, it will, for a period, serve customers with a mix of AVs and human-driven vehicles (Lyft 2019, Uber 2019).

Lyft and Uber each anticipate that its deployment of AVs will be crucial in improving its profitability and that its rival's deployment of AVs will threaten its profitability (Lyft 2019, Mims 2019, Uber 2019). When considering industry-wide access to AVs, it is unclear whether a platform's own benefit from obtaining access will be outweighed by the harm in facing a more formidable rival. *Do competing platforms benefit by both obtaining access to AVs?*

Platforms' deployment of AVs will affect the humans that participate on each side of the platform. It is natural that driver-workers will be hurt by being displaced by AVs and that rider-consumers will benefit through lower prices that result from the availability of a new supply source for platforms. What is less clear is the net impact on the human participants collectively (i.e., whether the benefit to consumers offsets the harm to workers). *Do human participants collectively benefit by platforms' access to AVs? Does society benefit by platforms' access to AVs?*

These questions are of interest to platforms, labor and consumer advocates, and governmental entities. The ease with which platforms obtain access to AVs will depend on regulatory and public policy decisions: the stringency of safety regulations governing ride-hailing AVs, infrastructure that affects the ease of platforms' use of AVs (e.g., infrastructure integrating ride-hailing AVs with public transit systems), and regulation that affects the ease of platforms' use of AVs (e.g., zoning for facilities that store, service, and charge ride-hailing AVs) (Duvall et al. 2019). The economic welfare consequences of platforms' access to AVs are one input to these decisions. To the extent that the harm to workers outweighs the benefit to consumers or the rest of society, labor advocates will be on stronger ground in pushing for barriers to ride-hailing AVs. The degree to which platforms push against such barriers and the strength of their arguments will depend on whether platforms and society benefit from access to AVs.

Addressing these questions requires some speculation regarding how AVs will be deployed and in particular, who will own the AVs used on each platform. There is evidence that Uber and Lyft will each own their AV fleets. Uber has said it intends to own and operate its own AVs (Isaac 2017, Uber 2019). Uber initiated its AV efforts in 2015 and in late 2017, agreed to purchase 24,000 AVs from Volvo, stating that "everything we're doing right now is about building autonomous vehicles at scale" (Boston 2017, Isaac 2017, p. B6). Lyft has said it will "most likely" lease AVs if it does not own them outright (Murphy 2016). Lyft envisions that its AV offering will be "asset intensive," which is consistent with Lyft owning AVs (Lyft 2019, p. 24). Lyft launched its in-house development of AVs in 2017, devoting one-10th of its engineers to the effort. The goal of Lyft's AV efforts in house and with industry partners (e.g., Waymo) is to bring "hundreds of thousands" of AVs to its platform (Bensinger 2017, p. B2).

Although Lyft's and Uber's in-house efforts are each aimed at developing technology for the platform's own AVs, each platform has expressed an openness to allowing AVs it does not wholly own on its network (Murphy 2016, Isaac 2017). One possibility is that a ride-hailing platform would partner with an outside entity that would put AVs on the platform's network. A second possibility is that a platform would allow AVs fully owned by external parties on its network.

Because it is difficult to capture all possible ownership scenarios in a single model, to build understanding we focus on two alternatives that represent opposite ends of the ownership spectrum. Under the *platform-owned AVs* ownership structure, each platform determines its AV fleet size and incurs an associated cost. This is consistent with the platform owning or leasing the AVs on its network. It is also consistent with the partnership model, to the extent that the partners make decisions with the objective of maximizing their combined profit. To the extent that independent AV fleet owners with market power seek to put their AVs on a platform's network, a different setup would be required. Because examining this scenario would require a significant level of speculation about how the various entities (including the fleet owners, which do not as yet exist) would interact, we defer its discussion to Section 5.

Under the *individually owned AVs* ownership structure, third parties lacking market power own the AVs. Although it is easiest to conceive of the owners as individuals, they could also be owners of small, independent AV fleets. Either would be consistent with the view of some analysts that question the viability of large AV fleets, owned either by platforms or by third parties (Motavalli 2020). A model in which a ride-hailing platform exclusively employs individually owned AVs has been proposed by Tesla CEO Elon Musk and could be adopted by ride-hailing incumbents that also use human drivers (Higgins 2019). (In our base model, for consistency across both ownership structures, we suppose each platform has access to a pool of AVs that exclusively serve that platform's customers; we relax this assumption for individually owned AVs in Section 4.3. We relax the assumption that AVs are owned by only one type of entity—either platforms or individuals—in Section 4.2.)

Although our work is primarily motivated by ride-hailing, parallel issues arise for delivery platforms. Delivery platforms, which are currently struggling with profitability, view AVs as important to their long-term profitability and prospects (Mims 2019). Competing platforms, such as DoorDash and Postmates, are developing and testing AVs for delivery (Luna 2020).

We characterize the impact of platforms' access to AVs on three performance measures: platform profit, the welfare of the human participants collectively (hereafter, agents), and social welfare (the sum of the previous two quantities). Table 1, which states the necessary and sufficient conditions under which access to AVs decreases each performance measure, summarizes our key findings. (We encourage the reader, at this point, to skip over the gray text in Table 1, as the results hold when that text is ignored.)

The structural effect of access to AVs on agent welfare is robust regardless of AV ownership; agent welfare decreases if and only if the AV cost is high. In contrast, the structural effect of access to AVs on platform profit depends on who owns AVs. The necessary and sufficient condition under which access to AVs decreases platform profit is high AV cost under platform-owned AVs and low AV cost under individually owned AVs. Similarly,

Table 1. Necessary and Sufficient Conditions for Access to AVs to Decrease Platform Profit, Agent Welfare, or Social Welfare—Under Platform-Owned AVs or Individually Owned AVs

<table>
<tr><th></th><th>Platform-owned AVs</th><th>Individually Owned AVs</th></tr>
<tr><td>Platform profit</td><td>AV cost is high
and relative price sensitivity of demand is greater than relative wage sensitivity of labor $\gamma/\beta > g_l/b_l$</td><td>AV cost is low
and relative price sensitivity of demand is high $\gamma/\beta > \underline{\eta}$, where $\underline{\eta} > g_l/b_l$</td></tr>
<tr><td>Agent welfare</td><td colspan="2">AV cost is high
and relative price sensitivity of demand is less than relative wage sensitivity of labor $\gamma/\beta < g_l/b_l$</td></tr>
<tr><td>Social welfare</td><td>AV cost is high
and relative price sensitivity of demand is greater than relative wage sensitivity of labor $\gamma/\beta > g_l/b_l$</td><td>None</td></tr>
</table>

the structural effect of access to AVs on social welfare depends on who owns AVs. Access to individually owned AVs increases social welfare; in contrast, access to platform-owned AVs decreases social welfare—if and only if the AV cost is high. We discuss the prescriptions that follow for platforms and advocates of human participants in Section 5.

This paper is primarily related to two streams of literature on competition: competition between ride-hailing platforms and the impact of changes in cost structure on competing firms. Ride-hailing platforms have been widely studied in the operations management literature. A large share of this work has focused on pricing, including dynamic pricing (Banerjee et al. 2016, Cachon et al. 2017, Bai et al. 2018, Hu et al. 2021), spatial pricing (Bimpikis et al. 2019, Besbes et al. 2021), and the impact of uncertainty (Taylor 2018). Other dimensions of ride-hailing platforms that have received attention are labor and staffing considerations (Afeche et al. 2018, Allon et al. 2019, Gurvich et al. 2019, Hu and Zhou 2019, Benjaafar et al. 2021) and matching mechanisms (Benjaafar et al. 2019, Ozkan and Ward 2020). Chen et al. (2020) examines research opportunities in ride-hailing and other contexts.

Within the ride-hailing literature, our work is most closely related to papers that investigate the impact of competition on platform profit and the welfare of workers and consumers. With respect to platform profit, Cohen and Zhang (2017) examines profit-sharing contracts between duopolist ride-hailing platforms and characterizes conditions under which such agreements benefit both platforms. Bai and Tang (2020) focuses on the factors that determine whether competing platforms earn strictly positive profit. Wu et al. (2020) considers how the timing of worker and consumer decisions affect the market share of each platform in equilibrium. Liu et al. (2019) examines the impact of different worker bonus schemes on platform profit. With respect to welfare, Bernstein et al. (2020) considers how equilibrium outcomes in a duopoly depend on whether drivers work for one or both platforms and show that both consumers and workers may be worse off when drivers work for both platforms. Nikzad (2018) and Benjaafar et al. (2020a) show consumers may be worse off under competition. Lin et al. (2018) shows that mergers between competing platforms can be beneficial for both consumers and workers. Our work differs from these papers in that we focus on the impact of access AVs on welfare and profit.

This paper also extends previous work on how changes in sourcing options or supply costs affect equilibrium outcomes. Using a general model of firm competition, Seade (1985) shows that industry-wide cost increases (e.g., taxes) can increase equilibrium profits. Salop and Scheffman (1987) shows that it can be advantageous for a firm to “overbuy” an input, so as to raise costs for a competitor. More recently, papers in the supply chain literature have examined settings where the change in cost structure can be either symmetric or asymmetric (i.e., can apply to one or both firms). In an asymmetric setting, Arya et al. (2008) shows that a firm may benefit from paying a premium to outsource production to a common supplier because of the resulting increase in its rival’s costs. Chen and Guo (2014) shows that for firms that compete over a single supplier, one firm’s access to a second supplier can increase a competitor’s profits because of a softening of supply-side competition. In a symmetric setting, Wu and Zhang (2014) shows that, in the context of outsourcing, higher supply costs for all firms can lift profits, again because of a softening of competition. Our work differs from these papers in that we focus on the impact of access to AVs under two distinct AV-ownership structures and consider consumer and worker welfare, as well as profit.

2. Model

Platforms compete simultaneously over consumers in a demand market (by setting prices) and workers in a labor market (by setting wages). Platforms can serve demand with worker-drivers or with AVs. We focus on two ownership structures for AVs. In the first setting, each platform obtains its own AV fleet; in the second, each platform recruits AVs owned by individuals. Next, we present a model for the consumer and labor markets, platform-owned AVs, and individually owned AVs. Then, we characterize platform profit, consumer surplus, and labor welfare.

2.1. Consumer and Labor Markets

Let p_i and $w_{l,i}$ denote the price offered to rider-consumers and wage offered to driver-workers by platform $i \in \{1,2\}$. Platform i's demand under prices $\mathbf{p} = (p_1, p_2)$ is

$$D_i(\mathbf{p}) = \alpha - \beta p_i + \gamma p_j, \tag{1}$$

where $\beta > \gamma \geq 0$ for $i \neq j$. Platform i's labor supply under wages $\mathbf{w}_l = (w_{l,1}, w_{l,2})$ is

$$L_i(\mathbf{w}_l) = b_l w_{l,i} - g_l w_{l,j}, \tag{2}$$

where $b_l > g_l \geq 0$. Note that γ is the crossprice sensitivity of demand and β is the own-price sensitivity of demand; accordingly, γ/β represents the *relative price sensitivity of demand*. Similarly, g_l is the crosswage sensitivity of labor supply, and b_l is the own-wage sensitivity of labor supply; accordingly, g_l/b_l represents the *relative wage sensitivity of labor*. Note that $g_l = 0$ and $\gamma = 0$ indicate an absence of competition in the labor and consumer markets, respectively. The assumption that labor supply is linear in wages has been used in the labor economics literature (e.g., Hamilton et al. 2000, Bhaskar et al. 2002) and parallels the commonly used assumption that demand is linear in prices.

2.2. Platform-Owned AVs

In the setting where platforms own AVs, fleet size decisions are made over a longer-term horizon than price and wage decisions. As such, we divide the time horizon into two periods. In the first period, platform $i \in \{1,2\}$ chooses the size of its AV fleet K_i, incurring cost $\theta c_k(K_i)$, where $\theta > 0$. We refer to θ as the platforms' *AV cost* and to $c_k(K_i)$ as the *AV cost function*. In the second period (spot market), each platform $i \in \{1,2\}$ observes the AV fleet of its rival platform K_j, $j \neq i$, before making price and wage decisions. In the base model, we suppose the AV cost function is linear $c_k(K_i) = K_i$; we relax this assumption in Section 4.1.

2.3. Individually Owned AVs

In the setting where individuals own AVs, platform $i \in \{1,2\}$ offers wage $w_{v,i}$ to individual AV owners in exchange for the deployment of their vehicle on the platform. In contrast to platform-owned AVs, sourcing individually owned AVs is a short-term decision. Analogous to labor supply, platform i's AV supply under wages $\mathbf{w}_v = (w_{v,1}, w_{v,2})$ is $V_i(\mathbf{w}_v) = b_v w_{v,i} - g_v w_{v,j}$, where $b_v > g_v \geq 0$.

2.4. Platform Profit

A unit of demand can be fulfilled by a unit of AV or labor. Accordingly, we restrict attention to the natural parameter range $(\mathbf{p}, \mathbf{w}_l, \mathbf{w}_v)$ wherein platform i balances total supply and demand: $D_i(\mathbf{p}) = K_i + L_i(\mathbf{w}_l) + V_i(\mathbf{w}_v)$. Under AV fleets $\mathbf{K} = (K_1, K_2)$, platform i chooses its price and wages $(p_i, w_{l,i}, w_{v,i})$ to maximize its second-period contribution

$$u_i(\mathbf{p}, \mathbf{w}_l, \mathbf{w}_v) = p_i D_i(\mathbf{p}) - w_{l,i} L_i(\mathbf{w}_l) - w_{v,i} V_i(\mathbf{w}_v). \tag{3}$$

Let $\mathbf{p}^*(\mathbf{K}) = (p_1^*(\mathbf{K}), p_2^*(\mathbf{K}))$, $\mathbf{w}_l^*(\mathbf{K}) = (w_{l,1}^*(\mathbf{K}), w_{l,2}^*(\mathbf{K}))$, and $\mathbf{w}_v^*(\mathbf{K}) = (w_{v,1}^*(\mathbf{K}), w_{v,2}^*(\mathbf{K}))$ denote equilibrium prices and wages under AV fleets $\mathbf{K}$. Platform i's second-period contribution under AV fleets $\mathbf{K}$ and equilibrium prices and wages $(\mathbf{p}^*(\mathbf{K}), \mathbf{w}_l^*(\mathbf{K}), \mathbf{w}_v^*(\mathbf{K}))$ is

$$r_i(\mathbf{K}) = u_i(\mathbf{p}^*(\mathbf{K}), \mathbf{w}_l^*(\mathbf{K}), \mathbf{w}_v^*(\mathbf{K})).$$

Platform i chooses its AV fleet K_i to maximize its (first-period) profit

$$\Pi_i(\mathbf{K}) = r_i(\mathbf{K}) - \theta c_k(K_i). \tag{4}$$

In the base model, we consider two ownership structures for AVs. Under *platform-owned AVs*, each platform acquires its own AV fleet, and individuals do not own AVs, which corresponds to the special case $b_v = g_v = 0$ and $\theta < \infty$. Under *individually owned AVs*, each platform recruits AVs owned by individuals, and platforms do not own AVs, which corresponds to the special case $b_v > g_v \geq 0$ and $\theta = \infty$. We relax the assumption that AVs are owned by platforms or individuals—but not both—in Section 4.2.

2.5. Consumer Surplus and Labor Welfare

Dixit (1986) shows that the demand in Equation (1) emerges under the following consumer utility model. A representative consumer facing prices (p_1, p_2) pays $p_1 D_1 + p_2 D_2$ for consuming (D_1, D_2) units. The consumer has quadratic utility from consumption $\tau D_1 + \tau D_2 - (\chi D_1^2 + 2\mu D_1 D_2 + \chi D_2^2)/2$, where $\tau > 0$ and $\chi > \mu > 0$. This utility function exhibits two features: decreasing marginal utility from consumption and utility from variety. The latter is natural if the consumer has different types of service needs (e.g., trips originating in different geographic areas) and perceives the platforms to be differentiated in their ability to meet these needs. The consumer chooses (D_1, D_2) to maximize her net utility. With a suitable mapping between (τ, χ, μ) and (α, β, γ), demand is given by Equation (1), and consumer surplus under symmetric equilibrium prices $\mathbf{p}^*$ is $CS = D_i(\mathbf{p}^*)^2/(\beta - \gamma)$; see Online Appendix E for the derivations.

A parallel model of worker utility results in the labor supply in Equation (2). A representative worker facing wages $(w_{l,1}, w_{l,2})$ receives payment $w_{l,1} L_1 + w_{l,2} L_2$ for providing (L_1, L_2) units of labor supply. The worker experiences quadratic disutility from providing labor $(x L_1^2 + 2m L_1 L_2 + x L_2^2)/2$, where $x > m > 0$. This disutility function exhibits two features: increasing marginal disutility from providing labor and utility for variety. The latter is natural if the worker's utility varies by the type of service it provides (e.g., trips originating in different geographic areas) and perceives the platforms to be differentiated in their ability to offer these service opportunities. The worker chooses (L_1, L_2) to maximize her net utility. With a suitable mapping between $(x,\ m)$ and (b_l, g_l), labor supply is given by Equation (2), and labor welfare under symmetric equilibrium wages $\mathbf{w}_l^*$ is $LW = L_i(\mathbf{w}_l^*)^2/(b_l - g_l)$; see Online Appendix E for the derivations. We refer to the sum of consumer surplus and labor welfare as agent welfare and the sum of agent welfare and both platforms' profits as social welfare.

3. Results

3.1. Platform-Owned AVs

This section examines the setting in which platforms, rather than individuals, own AVs. Because individuals do not own AVs, there is no market for such AVs: $b_v = g_v = 0$. Lemma 1 characterizes the existence and uniqueness of equilibria. All proofs for this section are in the Appendix, and all proofs for subsequent sections are in the Online Appendix.

Lemma 1. *Suppose $b_v = g_v = 0$. Under any platform-owned AV fleets $\mathbf{K}$, the equilibrium prices and wages $(\mathbf{p}^*(\mathbf{K}), \mathbf{w}_l^*(\mathbf{K}))$ are unique. Further, there exists $\tilde{g}_l > 0$ such that if $g_l < \tilde{g}_l$, then exactly one symmetric equilibrium AV fleet size, $K_1^* = K_2^* = K^*$, exists.*

Define $\theta_m = \lim_{K_1 \downarrow 0} \lim_{K_2 \downarrow 0} (\partial/\partial K_1) r_1(\mathbf{K})$. It is straightforward to show that the symmetric equilibrium AV fleet $K^* > 0$ if and only if the AV cost $\theta < \theta_m$. We say that platforms have *access to platform-owned AVs* when the AV cost $\theta \in (0, \theta_m)$. In the remainder of this section, we restrict attention to symmetric equilibria in AV fleet size, and for analytical tractability, we assume that $g_l < \tilde{g}_l$. To assess the restrictiveness of this assumption, we conducted a numerical study. Let Set A denote the approximately 300,000 combinations of parameters $\beta \in \{0.2, 0.4, \ldots, 1\}$, $\gamma = \vartheta\beta$, where $\vartheta \in \{0, 0.1, \ldots, 0.9, 0.99\}$, $b_l \in \{0.2, 0.4, \ldots, 1\}$, $g_l = \varsigma b_l$, where $\varsigma \in \{0, 0.1, \ldots, 0.9, 0.99\}$, and $\theta \in \{0.05, 0.10, \ldots, 5\}$. For each combination of parameters, we observed that exactly one symmetric equilibrium AV fleet size exists and that the results are consistent with the propositions in this section.

By symmetry, equilibrium platform profit under access to platform-owned AVs is $\Pi^P = \Pi_i(\mathbf{K}^*)$, where $\mathbf{K}^* = (K^*, K^*)$ is the symmetric equilibrium AV fleets, and equilibrium platform profit under no access to AVs is $\Pi^0 = \Pi_i(0,0) = r_i(0,0)$ for $i \in \{1,2\}$. Proposition 1 characterizes the impact of access to platform-owned AVs on equilibrium platform profit. Proposition 1 reveals that whether such access decreases platform profit depends in part on whether the relative price sensitivity of demand γ/β is greater than the relative wage sensitivity of labor g_l/b_l. For concreteness, if platform 1 decreases its price by one dollar, the relative price sensitivity of demand is the fraction of a dollar by which platform 2 must decrease her price to restore her demand to its level prior to platform 1's price reduction. Similarly, the relative wage sensitivity of labor is the fraction of a dollar by which platform 2 must increase her wage to restore her labor supply to its level prior to platform 1's increasing its wage by one dollar. Accordingly, these ratios are a measure of the intensity of competition in the consumer market and the labor market. In this sense, the relative price sensitivity of demand is greater than the relative wage sensitivity of labor when the intensity of competition is greater in the consumer market than the labor market.

Proposition 1. *There exists $\bar{\theta} \geq 0$ such that access to platform-owned AVs decreases platform profit $\Pi^P < \Pi^0$ if and only if the AV cost is high $\theta \in (\bar{\theta}, \theta_m)$. Further, $\bar{\theta} < \theta_m$ if and only if the relative price sensitivity of demand is greater than the relative wage sensitivity of labor*

$$\gamma/\beta > g_l/b_l. \tag{5}$$

Access to platform-owned AVs decreases platform profit if and only if the AV cost θ is high and the relative price sensitivity of demand is greater than the relative wage sensitivity of labor. The platforms obtain access to AVs when the AV cost decreases such that it is no longer prohibitively costly $\theta < \theta_m$. Hence, to understand the conditions under which access to platform-owned AVs decreases platform profit, it is useful to consider the effect of a reduction in the AV cost θ on platform i's profit:

$$\frac{d\Pi_i}{d\theta} = \Bigg[\underbrace{\frac{\partial Revenue_i(\mathbf{K}^*)}{\partial K_j}}_{\substack{<0 \\ \text{consumer market effect}}} - \underbrace{\frac{\partial LaborCost_i(\mathbf{K}^*)}{\partial K_j}}_{\substack{<0 \\ \text{labor market effect}}}\Bigg] \times \underbrace{\frac{dK_j^*}{d\theta}}_{<0} - \underbrace{c_k(K_i^*)}_{\substack{>0 \\ \text{AV sourcing cost effect}}}, \tag{6}$$

where under AV fleets $\mathbf{K}$, platform i's equilibrium revenue is $Revenue_i(\mathbf{K}) = p_i^*(\mathbf{K}) D_i(\mathbf{p}^*(\mathbf{K}))$ and equilibrium labor cost is $LaborCost_i(\mathbf{K}) = w_{i,l}^*(\mathbf{K}) L_i(\mathbf{w}_l^*(\mathbf{K}))$.

Reducing the AV cost has a direct beneficial *AV sourcing cost effect*; reducing θ reduces platform i's AV sourcing cost $\theta c_k(K_i^*)$. In addition, reducing the AV cost has two indirect (and opposing) effects that come through its impact on platform j's AV fleet size: a harmful *consumer market effect* and a beneficial *labor market effect*. Platform j responds to a reduction in the AV cost by expanding its AV fleet. This commits platform

j to compete more aggressively on price in the consumer market (the consumer market effect), hurting platform i. An increase in platform j's AV fleet reduces platform j's marginal value of labor, so platform j competes less aggressively on wage in the labor market (the labor market effect), benefiting platform i. Which effect dominates depends on a simple comparison of the relative price sensitivity of demand γ/β (which drives the magnitude of the consumer market effect) with the relative wage sensitivity of labor g_l/b_l (which drives the magnitude of the labor market effect).

To understand the impact of access to AVs on platform profit when the AV cost is high, consider the effect of decreasing the AV cost from the prohibitively costly threshold $\theta = \theta_m$. Because the AV cost is high, platform i's equilibrium AV fleet K_i^* is very small, and the AV sourcing cost effect is negligible. Hence, the harmful consumer market effect dominates the beneficial labor market effect, such that access to AVs decreases platform i's profit, if and only if the relative price sensitivity of demand is greater than the relative wage sensitivity of labor, Inequality (5).

If the relative price sensitivity of demand is less than the relative wage sensitivity of labor (Inequality (5) is violated), then the beneficial labor market effect dominates the harmful consumer market effect. Hence, both the indirect and direct effects of access to AVs are beneficial; access to AVs increases platform i's profit.

If the AV cost is small $\theta < \bar{\theta}$, then platform i's equilibrium AV fleet K_i^* is large, and the beneficial AV sourcing cost effect dominates; access to AVs increases platform i's profit.

It can be shown analytically that the AV cost threshold $\bar{\theta} > 0$ if $\gamma/\beta < g_l/b_l + \rho$ for some $\rho \in (0,1)$. In a numerical study of the parameters in Set A, we observed results consistent with the previous sentence, where "if" is replaced by "if and only if." This suggests that if the relative price sensitivity of demand is sufficiently larger than the relative wage sensitivity of labor $\gamma/\beta \geq g_l/b_l + \rho$, then access to platform-owned AVs decreases platform profit regardless of the AV cost. Intuitively, the harmful consumer market effect is so strong that it dominates the beneficial combined labor cost and AV sourcing cost effects.

Next, we consider the impact of AVs on the welfare of the human participants on the platform, namely consumers and workers. Naturally, access to platform-owned AVs decreases labor welfare (because AVs displace workers) and increases consumer surplus (because AVs provide platforms with an additional supply source). What is less clear is the net impact on the human participants collectively. That is, does the harm to workers offset the benefit to consumers?

We refer to the welfare of the human participants (that is, the sum of consumer surplus and labor welfare) as *agent welfare* $AW(\mathbf{K}) = CS(\mathbf{K}) + LW(\mathbf{K})$, where consumer surplus $CS(\mathbf{K}) = D_i(\mathbf{p}^*(\mathbf{K}))^2/(\beta - \gamma)$ and labor welfare $LW(\mathbf{K}) = L_i(\mathbf{w}_l^*(\mathbf{K}))^2/(b_l - g_l)$. Equilibrium agent welfare under access to platform-owned AVs is $AW^P = AW(\mathbf{K}^*)$; equilibrium agent welfare under no access to AVs is $AW^0 = AW(0,0)$. Proposition 2 characterizes the impact of access to platform-owned AVs on equilibrium agent welfare.

Proposition 2. *There exists $\tilde{\theta} \geq 0$ such that access to platform-owned AVs decreases agent welfare $AW^P < AW^0$ if and only if the AV cost is high $\theta \in (\tilde{\theta}, \theta_m)$. Further, $\tilde{\theta} < \theta_m$ if and only if the relative price sensitivity of demand is less than the relative wage sensitivity of labor*

$$\gamma/\beta < g_l/b_l. \tag{7}$$

Access to platform-owned AVs decreases agent welfare if and only if the AV cost θ is high and the relative price sensitivity of demand is less than the relative wage sensitivity of labor. The platforms obtain access to AVs when the AV cost decreases such that it is no longer prohibitively costly $\theta < \theta_m$. Hence, to understand the conditions under which access to platform-owned AVs decreases agent utility, it is useful consider the effect of a reduction in the AV cost θ on agent welfare:

$$\frac{dAW^P}{d\theta} = \Bigg[\underbrace{-D_T^*(\mathbf{K}^*)\frac{\partial p^*(\mathbf{K}^*)}{\partial K}}_{\substack{\frac{dCS}{dK} > 0 \\ \text{consumer surplus effect}}} + \underbrace{L_T^*(\mathbf{K}^*)\frac{\partial w^*(\mathbf{K}^*)}{\partial K}}_{\substack{\frac{dLW}{dK} < 0 \\ \text{labor welfare effect}}}\Bigg]\underbrace{\frac{dK^*}{d\theta}}_{<0}, \tag{8}$$

where $D_T^*(\mathbf{K}^*) = D_1^*(\mathbf{K}^*) + D_2^*(\mathbf{K}^*)$ is total equilibrium demand and $L_T^*(\mathbf{K}^*) = L_1^*(\mathbf{p}^*(\mathbf{K}^*), \mathbf{w}^*(\mathbf{K}^*)) + L_2^*(\mathbf{p}^*(\mathbf{K}^*), \mathbf{w}^*(\mathbf{K}^*))$ is total equilibrium labor supply. The platforms respond to a reduction in the AV cost by expanding their AV fleets, which has two opposing effects on agent utility: a beneficial *consumer surplus effect* and a harmful *labor welfare effect.* The expansion in AV fleets prompts the platforms to compete more aggressively on price in the consumer market $(\partial p^*(\mathbf{K}^*)/\partial K < 0)$, increasing consumer surplus. The expansion in AV fleets prompts the platforms to compete less aggressively on wage in the labor market $(\partial w^*(\mathbf{K}^*)/\partial K < 0)$, decreasing labor welfare.

To understand the impact of access to AVs on agent welfare when the AV cost is high, consider the effect of decreasing the AV cost from the prohibitively costly threshold $\theta = \theta_m$. Because the AV cost is high, the platforms' equilibrium AV fleets K^* are very small, so that the numbers of consumers receiving service and workers providing service are comparable: $D_T^*(\mathbf{K}^*) \approx L_T^*(\mathbf{K}^*)$. In the limit, as the AV cost approaches the level at which AVs are prohibitively costly $\theta \to \theta_m$, the net of the consumer surplus and labor welfare effects (i.e., the quantity in square brackets in Equation (8)) is strictly negative if and only if the platform's equilibrium margin increases in the fleet size

$$\frac{\partial[p^*(\mathbf{K}^*) - w^*(\mathbf{K}^*)]}{\partial K} > 0, \qquad (9)$$

which occurs if and only if the relative price sensitivity of demand is less than the relative wage sensitivity of labor, Inequality (7). When the relative price sensitivity of demand is less than the relative wage sensitivity of labor, the expansion of AV fleets prompts each platform to reduce its wage more aggressively than its price. Hence, each worker is hurt more than each consumer is helped. Because the numbers of consumers and workers are comparable, $D_T^*(\mathbf{K}^*) \approx L_T^*(\mathbf{K}^*)$, the harmful labor welfare effect outweighs the beneficial consumer surplus effect, with the net result that agent welfare decreases.

If the AV cost is small $\theta < \bar{\theta}$, then the platforms' equilibrium AV fleets K^* are large, and consequently, the number of consumers is significantly larger than the number of workers $D_T^*(\mathbf{K}^*) \gg L_T^*(\mathbf{K}^*)$. Because many consumers benefit from the price reduction and few workers are hurt by the wage reduction, the beneficial consumer surplus effect outweighs the harmful labor welfare effect, and the net result is that agent welfare increases.

Similarly, if the relative price sensitivity of demand is greater than the relative wage sensitivity of labor (Inequality (7) is reversed), then Inequality (9) is reversed; each consumer benefits more than each worker is hurt. Because the number of affected consumers is greater than the number of affected workers, the beneficial consumer surplus effect outweighs the harmful labor welfare effect, and the net result is that agent welfare increases.

Propositions 1 and 2 characterize the impact of access to platform-owned AVs on each of the two groups—platforms and human participants—separately. Taking these propositions together answers the question of how access to platform-owned AVs *jointly* affects these two groups. The answer is formalized in the following corollary, which shows that at least one group benefits. Let $\breve{\theta} = \min\{\bar{\theta}, \tilde{\theta}\}$. Note that it may be that $\breve{\theta} = 0$; further, $\breve{\theta} < \theta_m$ if and only if $\gamma/\beta \neq g_l/b_l$.

Corollary 1. *If AV cost is low $\theta \in (0, \breve{\theta})$, then access to platform-owned AVs increases platform profit and agent welfare. If the AV cost is high $\theta \in (\breve{\theta}, \theta_m)$, then access to platform-owned AVs either increases platform profit and decreases agent welfare or decreases platform profit and increases agent welfare. The former occurs if $\gamma/\beta < g_l/b_l$, and the latter occurs if $\gamma/\beta > g_l/b_l$.*

If the AV cost is low, both groups benefit from platforms' access to platform-owned AVs. If the AV cost is high, then one group benefits, and the other group is hurt; which group benefits is determined by a simple comparison between relative price sensitivity of demand and the relative wage sensitivity of labor.

We refer to the sum of agent welfare and the platforms' profits as *social welfare* $SW(\mathbf{K}) = AW(\mathbf{K}) + \Pi_1(\mathbf{K}) + \Pi_2(\mathbf{K})$. Equilibrium social welfare under access to platform-owned AVs is $SW^P = SW(\mathbf{K}^*)$; equilibrium social welfare under no access to AVs is $SW^0 = SW(0,0)$.

An immediately implication of Corollary 1 is that access to platform-owned AVs increases social welfare if the AV cost is low $\theta \in (0, \breve{\theta})$. When the AV cost is high, $\theta \in (\breve{\theta}, \theta_m)$, one group is hurt, and the other group benefits, which prompts the following question. Will the harm to the first group outweigh the benefit to the second group? The next proposition gives a sharp answer.

Proposition 3. *There exists $\hat{\theta} \geq 0$ such that access to platform-owned AVs decreases social welfare $SW^P < SW^0$ if and only if the AV cost is high $\theta \in (\hat{\theta}, \theta_m)$. Further, $\hat{\theta} < \theta_m$ if and only if the relative price sensitivity of demand is greater than the relative wage sensitivity of labor, Inequality* (5).

Access to platform-owned AVs decreases social welfare if and only if the AV cost θ is high and the relative price sensitivity of demand is greater than the relative wage sensitivity of labor.

The set of participants that is most obviously harmed by AVs is workers, as a portion of them is displaced by AVs. Accordingly, it might be natural to conjecture that if access to AVs was to reduce social welfare, it would do so because the harm to workers offsets the benefit to consumers and platforms. Proposition 3 reveals that this never occurs. Rather, a reduction in social welfare, when it occurs, is driven by the harm to platforms offsetting the benefit to human participants. To see this, observe that a necessary condition for platform-owned AVs to decrease social welfare is that the relative price sensitivity of demand is greater than the relative wage sensitivity of labor, Inequality (5). Under this condition, the benefit to consumers outweighs the harm to workers such that agent welfare increases (by Proposition 2).

We conclude by noting that Propositions 1–3 reveal that the impact of access to platform-owned AVs on each the three groups has a common structure; access to AVs decreases platform profit, agent welfare, and social welfare if and only if the AV cost is high. The next section reveals that this common structure no longer holds when individuals, rather than platforms, own AVs.

3.2. Individually Owned AVs

This section examines the setting in which individuals, rather than platforms, own AVs. The setting in which platforms do not own AVs is captured in our model by the cost of platform-owned AVs being prohibitive $\theta = \infty$. Lemma 2 characterizes the existence and uniqueness of the equilibrium.

Lemma 2. *Suppose $b_v > g_v \geq 0$ and the AV cost $\theta = \infty$. There exists a unique equilibrium in prices and wages, and it is symmetric: $p_1^* = p_2^* = p^*$, $w_{l,1}^* = w_{l,2}^* = w_l^*$, and $w_{v,1}^* = w_{v,2}^* = w_v^*$.*

We say that platforms have *access to individually owned AVs* when $b_v > 0$. (This parallels our definition of access to platform-owned AVs in that each platform sources individually owned AVs, $V_i(\mathbf{w}_v^*) > 0$, if and only if $b_v > 0$.) In the study in Section 3.1 of platform-owned AVs, each platform has access to a pool of AVs that exclusively serve that platform's customers. For consistency and to isolate the effect of AV ownership, in this section's study of individually owned AVs, we consider the parallel setting in which each platform has access to a pool of AVs that exclusively serve that platform's customers. That is, there is no competition in the individually owned AV market $g_v = 0$.

For an individually owned AV to serve a platform's customers, the AV must possess technology that allows it to interface with the platform and its customers. Conceivably, this technology would be developed by the platform and made available to AV owners (either at the time when the AV is manufactured or subsequently), under conditions imposed by the platform. A platform may find it attractive to impose restrictions to limit competition. The setting we consider in this section, $g_v = 0$, corresponds to the case where each platform imposes the condition on individual owners that an AV can only use the platform's proprietary technology on the condition that the AV exclusively serves that platform's customers (Tesla CEO Elon Musk has proposed such a model for exclusive use of individually owned AVs (Higgins 2019)). However, for completeness, in Section 4.3, we consider the setting where there is competition in the AV market $g_v > 0$.

Let $\underline{\eta} = [(2b_l - g_l)\beta - 2(\sqrt{b_l(b_l+\beta)} - b_l)(b_l - g_l)]/[(2b_l - g_l)\ \beta - (\sqrt{b_l(b_l+\beta)} - b_l)(b_l - g_l)]$ and $\bar{\eta} = 2[2b_l(b_l - g_l)(2b_l + g_l) + (2b_l - g_l)^2\beta]/[2b_l(b_l - g_l)(6b_l - g_l) + 2(2b_l - g_l)^2\beta]$. Note that $\max(g_l/b_l, 2/3) < \underline{\eta} < \min(\bar{\eta}, 1)$, and $\bar{\eta} < 1$ if and only if $g_l/b_l < 2/3$. Let $\phi = 1/b_v$; we refer to ϕ as the *AV cost*, in the setting with individually owned AVs. We define equilibrium platform profit Π^I, agent welfare AW^I, and social welfare SW^I under access to individually owned AVs analogously to that under platform-owned AVs; for completeness, formal definitions are in Online Appendix C.

Proposition 4. *There exists $\bar{\phi} \geq 0$ such that access to individually owned AVs decreases platform profit $\Pi^I < \Pi^0$ if and only if the AV cost is low $\phi < \bar{\phi}$. Further, if the relative price sensitivity of demand is low $\gamma/\beta \leq \underline{\eta}$, then $\bar{\phi} = 0$; if $\gamma/\beta \in (\underline{\eta}, \bar{\eta})$, then $0 < \bar{\phi} < \infty$; and if $\gamma/\beta \geq \bar{\eta}$, then $\bar{\phi} = \infty$.*

Access to individually owned AVs decreases equilibrium profits if and only if the AV cost is low $\phi < \bar{\phi}$ and the relative price sensitivity of demand is high $\gamma/\beta > \underline{\eta}$. The platforms obtain access to AVs when the AV cost decreases such that it is no longer prohibitively costly $\phi < \infty$. Hence, to understand the conditions under which access to individually owned AVs decreases platform profit, it is useful to consider the effect of a reduction in the AV cost ϕ on platform i's profit. To cleanly delineate the mechanisms by which a reduction in the AV cost affects platform i's profit, it is useful to consider the case where labor is prohibitively costly $b_l = 0$. Because there is no competition in the autonomous vehicle market $g_v = 0$, platform i's AV sourcing cost is $\phi c_v(V_i)$, where $c_v(V_i) = V_i^2$. The effect of reducing the AV cost ϕ on platform i's profit is

$$\frac{d\Pi_i}{d\phi} = \underbrace{\frac{\partial p_j^*}{\partial \phi}\frac{\gamma}{\beta} D_i(\mathbf{p}^*)}_{\substack{>0 \\ \text{competitor price effect}}} - \underbrace{c_v(V_i(\mathbf{w}_v^*))}_{\substack{>0 \\ \text{AV sourcing cost effect}}}.$$

Reducing the AV cost ϕ has a direct beneficial *AV sourcing cost effect*; reducing ϕ reduces platform i's AV sourcing cost $\phi c_v(V_i(\mathbf{w}_v^*))$. In addition, reducing the AV cost has an indirect harmful *competitor price effect*; platform j responds to a reduction in the AV cost by reducing its price, which hurts platform i by reducing its demand. Intuitively, the magnitude of this impact is increasing in the relative price sensitivity of demand γ/β. If the relative price sensitivity of demand is low, $\gamma/\beta \leq \underline{\eta}$, then the competitor price effect is small, and the beneficial AV sourcing cost effect dominates. In contrast, if the relative price sensitivity of demand is high, $\gamma/\beta \geq \bar{\eta}$, then the harmful competitor price effect dominates. If the relative price sensitivity of demand is moderate, $\gamma/\beta \in (\underline{\eta}, \bar{\eta})$, then the harmful competitor price effect dominates if and only if platform j's price is quite sensitive to the AV cost. The sensitivity of platform j's price to the AV cost decreases in the AV cost. (The intuition is that as the AV cost ϕ decreases, the equilibrium AV supply V^* increases, which implies that $c_v'(V^*)$ increases (because the AV cost function $c_v(\cdot)$ is strictly convex). Hence, platform j's marginal cost of supply $\phi c_v'(V^*)$, and hence, platform j's price, becomes more sensitive to the AV cost ϕ.) Consequently, the harmful competitor price effect dominates if and only if the AV cost is low. The observation that the harmful competitor price effect dominates when the AV cost is low is not driven by our assumption that the AV sourcing cost is quadratic; the result holds for any strictly convex $c_v(\cdot)$ (see Online Appendix C). We are not the first to observe that convexity in the sourcing cost can drive competitors to be harmed by a reduction in the sourcing cost; see Fuess and Loewenstein (1991).

This logic continues to hold when labor is not prohibitively costly $b_l > 0$. The primary effect of the platforms obtaining access to high-cost ($\phi \geq \bar{\phi}$) individually owned AVs is to reduce each platform's sourcing cost to the benefit of both platforms. The primary effect of the platforms obtaining access to low-cost ($\phi < \bar{\phi}$) individually owned AVs is to trigger aggressive price competition to the detriment of both platforms.

Strikingly, the structure of this result is reversed when platforms rather than individuals own AVs, as can be seen by comparing Propositions 1 and 4. Access to individually owned AVs harms platforms if and only if the AV cost is low, whereas access to platform-owned AVs harms platforms if and only if the AV cost is high. Across the two ownership structures, the effect of access to AVs is to reduce the platforms' marginal cost of supply at the point in time when the platforms compete in prices and wages (period 2). The critical difference between the ownership structures is that under platform-owned AVs, each platform commits ex ante (by incurring a cost) to reduce its marginal cost ex post, whereas under individually owned AVs, there is no such commitment. This difference in commitment drives the difference in structural conditions under which platforms are harmed by access to AVs. (For evidence that this difference in structural conditions is not driven by the assumed form of the platform-owned AV cost function, see Proposition 7 in Section 4.1. Although we have not explicitly modeled the utility of individual owners, in a representative owner model paralleling that of Section 2's representative worker model, with increasing marginal disutility from providing supply, the resulting AV cost function $c_v(\cdot)$ is convex. Such increasing marginal disutility is natural in that as the owner provides more supply, the AV is less available for her personal use, and the owner's personal use naturally exhibits decreasing marginal utility.)

Because the structural conditions under which platforms are harmed by access to AVs depend on the ownership structure of AVs, one might conjecture that the structural conditions under which agents are harmed by access to AVs would also depend on the ownership structure. Proposition 5 reveals that this conjecture is false.

Proposition 5. *There exists $\tilde{\phi} \geq 0$ such that access to individually owned AVs decreases agent welfare $AW^I < AW^0$ if and only if the AV cost is high $\phi > \tilde{\phi}$. Further, $\tilde{\phi} < \infty$ if and only if the relative price sensitivity of demand is less than the relative wage sensitivity of labor, Inequality* (7).

Just as in the case with platform-owned AVs, under individually owned AVs, access to AVs decreases agent welfare if and only if the AV cost is high and the relative price sensitivity of demand is less than the relative wage sensitivity of labor. The intuition under individually owned AVs parallels that under platform-owned AVs. Across both ownership structures, the effect of access to AVs is to displace workers to their detriment and to the benefit of consumers. Although the nature of competition is different under the two ownership structures because of the commitment involved in platform ownership, the nature of how consumers and workers are affected by AVs is not affected by the ownership structure.

Propositions 4 and 5 reveal that who benefits from platform access to individually owned AVs depends on the AV cost. When the AV cost is low, agents benefit, and the platforms are harmed. When the AV cost is high, the platforms benefit, and agents are harmed. This prompts the following question. Across all parameter regimes, will the harm caused by access to AVs outweigh the benefit? It might be natural to conjecture that, at least in some parameter regime, the answer is "yes." The next proposition reveals that when individuals own AVs, the answer is always "no."

Proposition 6. *Platform access to individually owned AVs increases social welfare $SW^I > SW^0$.*

Thus, the insight from the study in Proposition 3 of platform-owned AVs that access to AVs can decrease social welfare is reversed when individuals own AVs.

For consistency with our treatment of social welfare under platform-owned AVs, we have defined social welfare to be the sum of the utility of three groups: platforms, consumers, and workers. Individual ownership of AVs introduces a fourth group: individual owners. Naturally, individuals would not choose to own AVs if doing so reduced their utility. (For simplicity, we have not modeled individual owners' utility and AV acquisition decisions.) Hence, taking into account utility of AV-owning individuals would presumably strengthen the conclusion of Proposition 6.

Table 1 in Section 1 summarizes our key analytical results: Propositions 1–6. The structural effect of access to AVs on agent welfare does not depend on who owns AVs. In contrast, the structural effect of access to AVs on platform profit and social welfare does depend on AV ownership. Nonetheless, a common theme that cuts across both ownership structures is that if the relative price sensitivity of demand is less than the relative wage sensitivity of labor, then access to AVs increases platform profit and social welfare. Even if the AV cost is high, such that access to AVs harms agents, this harm is outweighed by the benefit to platforms. A second theme that cuts across both ownership structures is that if the AV cost is low, then access to AVs increases social welfare. These themes and results are illustrated in Figure 1, which depicts the parameter regions in which access to AVs increases or decreases platform profit, agent welfare, and social welfare.

4. Extensions

4.1. Nonlinear Platform AV Cost

This section extends the study in Section 3.1 of platform-owned AVs by allowing the platform's cost of its AV fleet to be nonlinear in the fleet size. (Under no access to AVs, platforms do not incur AV costs; consequently, platform profit is $\Pi^0 = r_i(0,0)$ for $i \in \{1,2\}$.) Section 3.1 shows that access to platform-owned AVs decreases platform profit, agent welfare, and social welfare if the AV cost θ is high. This section shows that this result is robust to the assumption that the AV cost function is linear $c_k(K_i) = K_i$. In particular, similar results to Propositions 1–3 hold when the AV cost function has a general form, provided that two conditions hold. There exists a single symmetric equilibrium AV fleet size K^*, and the equilibrium AV fleet size K^* decreases in the AV cost θ. The latter restriction is mild, as it reflects the natural relationship between marginal cost and investment. The former is a common requirement in symmetric settings for analysis to proceed. These two conditions are formalized in Assumption 1.

Assumption 1. *There exists exactly one symmetric equilibrium AV fleet size, $K_i^* = K_j^* = K^*$. Further, if $K^* > 0$, then $(d/d\theta)K^* < 0$.*

Note that Assumption 1 allows for $c_k(0) > 0$, which can be interpreted as the fixed cost incurred by the platform in developing AV technology and production capability. (A platform might incur minimal fixed costs if it was to lease or purchase AVs from a third party that bore the fixed technology development and production costs.)

Next, we present a result that shows that Assumption 1 is not especially onerous. Let $\bar{\bar{K}}_i = \max\{K_i : D_i(\mathbf{p}^*(\mathbf{K})) \geq K_i\}$. Note $\bar{\bar{K}}_i$ depends on K_j where $j \neq i$; a closed form expression is given immediately before Lemma A.4 in the appendix. We generalize the definition $\theta_m = \lim_{K_1 \downarrow 0} \lim_{K_2 \downarrow 0}[(\partial/\partial K_1) r_1(\mathbf{K})/(\partial/\partial K_1) c_k(K_1)]$ to reflect the generalized AV cost function $c_k(\cdot)$; note that $K^* > 0$ if and only if $\theta < \theta_m$.

Lemma 3. *Suppose the AV cost function $c_k(K)$ is strictly increasing and twice differentiable, with $(\partial/\partial K)c_k(K)|_{K=0} > 0$. Then, Assumption 1 holds if* (i) *$c_k(K)$ is weakly convex or* (ii) *$c_k(K)$ is concave and for any $K_j \geq 0$ and $K_i \in (0, \bar{\bar{K}}_i)$, $c_k(K)$ satisfies*

$$(\partial^2/\partial K_i^2)c_k(K_i)/[(\partial/\partial K_i)c_k(K_i)|_{K_i=0}] > [(\partial^2/\partial K_i^2)r_i(\mathbf{K}) - (\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K})]/[(\partial/\partial K_i)r_i(\mathbf{K})|_{(K_i,K_j)=(0,0)}]. \quad (10)$$

Inequality (10) can be interpreted as requiring the AV cost function to not be "too concave."

Proposition 7 establishes that notable structural results regarding the impact of access to platform-owned AVs continue to hold when the assumption that the AV cost function is linear is replaced with Assumption 1.

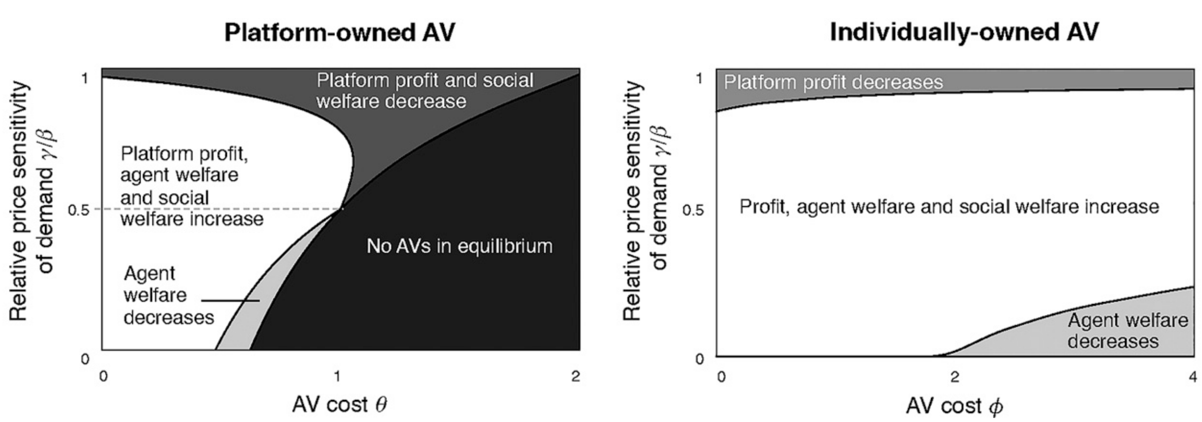
Proposition 7. *Suppose Assumption 1 holds. There exist $\bar{\theta} \geq 0$, $\tilde{\theta} \geq 0$, and $\hat{\theta} \geq 0$ such that access to platform-owned AVs*

i. *decreases platform profit $\Pi^P < \Pi^0$ if the AV cost is high $\theta \in (\bar{\theta}, \theta_m)$;*

ii. *decreases agent welfare $AW^P < AW^0$ if and only if the AV cost is high $\theta \in (\tilde{\theta}, \theta_m)$; and*

iii. *decreases social welfare $SW^P < SW^0$ if the AV cost is high $\theta \in (\hat{\theta}, \theta_m)$.*

Figure 1. Impact of Access to AVs on Platform Profit, Agent Welfare, and Social Welfare



Notes. Each performance measure increases unless stated otherwise (for example, in the region marked "Agent welfare decreases," platform profit and social welfare increase). The left panel depicts the impact of access to platform-owned AVs, and the right panel depicts the impact of access to individually owned AVs. Parameters are $\alpha = \beta = b_l = 1$ and $g_l = 0.5$. Hence, the relative wage sensitivity of labor $g_l/b_l = 0.5$.

Further, both $\bar{\theta} < \theta_m$ *and* $\hat{\theta} < \theta_m$ *if* $\gamma/\beta > g_l/b_l$, *and* $\tilde{\theta} < \theta_m$ *if and only if* $\gamma/\beta < g_l/b_l$.

Regarding the impact of access to platform-owned AVs on agent welfare, Proposition 7(ii) shows that Proposition 2 extends without modification when the assumption that the AV cost function is linear is relaxed. Regarding platform profit, Proposition 7(i) shows that Proposition 1's sufficient condition for access to AVs to reduce platform profit continues to hold. Consequently, the divergence in results across ownership structures persists; if the AV cost is high (more precisely, the platform-owned AV cost $\theta \in (\bar{\theta}, \theta_m)$ and the individually owned AV cost $\phi > \bar{\phi}$), then access to platform-owned AVs decreases platform profit, but access to individually owned AVs increases platform profit. Similarly, regarding social welfare, Proposition 7(iii) shows that Proposition 3's sufficient condition for access to AVs to reduce social welfare continues to hold. Consequently, the divergence in results across ownership structures persists; access to high-cost platform-owned AVs decreases social welfare, whereas access to individually owned AVs increases social welfare.

4.2. Platform-Owned and Individually Owned AVs

This section extends Section 3.1's study of platform-owned AVs and Section 3.2's study of individually owned AVs by considering the setting in which AVs are owned by platforms *and* individuals. More precisely, we consider the impact of access to platform-owned and individually owned AVs on equilibrium platform profit, agent welfare, and social welfare. For the first two of these performance measures, Sections 3.1 and 3.2 provide conditions under which access to AVs owned by one type of entity (platforms or individuals) decreases that performance measure. This section shows that those conditions are sufficient for access to AVs owned by both types of entities to decrease that performance measure. Further, this section shows that the conditions for platform-owned AVs to decrease social welfare, when suitably adapted, are sufficient for access to AVs owned by both types of entities to decrease social welfare.

It is straightforward to generalize the argument in Section 3.1 to establish that for each individually owned AV cost $\phi < \infty$, there exists a unique, symmetric equilibrium AV fleet K^* and a threshold $\theta_m(\phi)$ such that $K^* > 0$ if and only if the platform-owned AV cost $\theta < \theta_m(\phi)$. We say that platforms have *access to platform-owned and individually owned AVs* when the individually owned AV cost $\phi < \infty$ and the platform-owned AV cost $\theta \in [0, \theta_m(\phi))$. Let Π^M, AW^M, and SW^M denote the equilibrium platform profit, agent welfare, and social welfare under access to platform-owned and individually owned AVs.

Proposition 8. *There exist* $\bar{\phi} \geq 0$, $\tilde{\phi} \geq 0$, *and* $\hat{\phi} \geq 0$ *such that*

i. *for each* $\phi < \bar{\phi}$, *there exists* $\bar{\theta} < \theta_m(\phi)$ *such that access to platform-owned and individually owned AVs decreases platform profit* $\Pi^M < \Pi^0$ *if and only if the platform-owned AV cost is high* $\theta \in (\bar{\theta}, \theta_m(\phi))$;

ii. *for each* $\phi > \tilde{\phi}$, *there exists* $\bar{\theta} < \theta_m(\phi)$ *such that access to platform-owned and individually owned AVs decreases agent welfare* $AW^M < AW^0$ *if and only if the platform-owned AV cost is high* $\theta \in (\bar{\theta}, \theta_m(\phi))$; *and*

iii. *for each* $\phi > \hat{\phi}$, *there exist* $\underline{\theta} < \theta_m(\phi)$ *and* $\bar{\theta} > \underline{\theta}$ *such that access to platform-owned and individually owned AVs decreases social welfare* $SW^M < SW^0$ *if and only if the platform-owned AV cost is moderate* $\theta \in (\underline{\theta}, \bar{\theta})$.

Further, $\bar{\phi} > 0$ *if* $\gamma/\beta > \underline{\eta}$, $\tilde{\phi} < \infty$ *if* $\gamma/\beta < g_l/b_l$, *and* $\hat{\phi} < \infty$ *if* $\gamma/\beta > g_l/b_l$.

Proposition 1 shows that access to platform-owned AVs decreases platform profit if the platform-owned AV cost θ is high, and Proposition 4 shows that access to individually owned AVs decreases platform profit if the individually owned AV cost ϕ is low. Proposition 8(i) shows that access to platform-owned and individually owned AVs decreases platform profit if both of the aforementioned cost conditions hold.

Proposition 2 shows that access to platform-owned AVs decreases agent welfare if the platform-owned AV cost is high, and Proposition 5 shows that access to individually owned AVs decreases agent welfare if the individually owned AV cost is high. Proposition 8(ii) shows that access to platform-owned and individually owned AVs decreases agent welfare if both of the aforementioned cost conditions hold.

Proposition 3 shows that access to platform-owned AVs decreases social welfare if and only if the platform-owned AV cost is high. Proposition 8(iii) shows that the necessary and sufficient condition for access to platform-owned and individually owned AVs to decrease social welfare is similar, provided that the individually owned AV cost is high. The condition differs in that when the platform-owned AV cost is very high $\theta > \bar{\theta}$, access to platform-owned and individually owned AVs increases social welfare. The intuition behind this divergence stems from Proposition 6, which shows that access to individually owned AVs increases social welfare. It follows that when the platform-owned AV cost is very high, the presence of individually owned AVs (and their positive impact on social welfare) dominates.

4.3. Competition over Individually Owned AVs

For consistency, Sections 3.1 and 3.2 both consider a common structure for the AV supply market; each platform has access to a pool of AVs that exclusively serves that platform's customers. Hence, comparing the results in the two sections illuminates how the *structure of AV ownership* affects the impact of access to AVs on platforms and agents.

This section extends Section 3.2's study of individually owned AVs by considering an alternative market structure: competition over AVs. Such competition occurs when individually owned AVs possess the technology to interface with each platform and its customers. This section shows that the structure of how access to AVs affects agent welfare is unaffected by this alternative market structure. In contrast, the structure of how access to AVs affects platform profit, and social welfare is sensitive to the structure of the AV supply market.

Competition in the individually owned AV market corresponds to $g_v > 0$. Because the AV cost $\phi = 1/b_v$, the restriction that $b_v > g_v$ implies that $\phi < \phi_m$, where $\phi_m = 1/g_v$. In other words, parallel to the setting with platform-owned AVs, the platforms have access to individually owned AVs if and only if the AV cost $\phi \in (0, \phi_m)$. (In Section 3.2's setting with no competition over AVs ($g_v = 0$), $\phi_m = \infty$.)

Proposition 9. *Suppose there is competition in the individually owned AV market $g_v > 0$. There exist $\bar{\phi} \geq 0$, $\tilde{\phi} \geq 0$, and $\hat{\phi} \geq 0$ such that access to individually owned AVs*

i. *decreases platform profit $\Pi^I < \Pi^0$ if the AV cost is high $\phi \in (\bar{\phi}, \phi_m)$;*

ii. *decreases agent welfare $AW^I < AW^0$ if and only if the AV cost is high $\phi \in (\tilde{\phi}, \phi_m)$; and*

iii. *decreases social welfare $SW^I < SW^0$ if the AV cost is high $\phi \in (\hat{\phi}, \phi_m)$.*

Further, both $\bar{\phi} < \phi_m$ and $\hat{\phi} < \phi_m$ if $\gamma/\beta > g_l/b_l$, and $\tilde{\phi} < \phi_m$ if and only if $\gamma/\beta < g_l/b_l$.

Regarding the impact of access to AVs on agent welfare, Proposition 9(ii) shows that Proposition 5 extends without modification when the assumption that there is no competition in the individually owned AV market is relaxed. Across both AV market structures, the effect of access to AVs is to displace workers to their detriment and to the benefit of consumers. Although the nature of competition over supply is different under the two market structures, the nature of how consumers and workers are affected by AVs is not affected by the AV market structure.

In contrast, Proposition 9, (i) and (iii) contrasts with Propositions 4 and 6; the structural conditions under which access to individually owned AVs decrease platform profit and social welfare are sensitive to the presence of competition in the individually owned AV market. Competition in the individually owned AV market drives up the marginal cost of supply, directly and adversely affecting the platforms. This expands the parameter regime where platform profit (and hence, social welfare) is pushed down by access to AVs.

5. Discussion

This paper characterizes the conditions under which ride-hailing platforms' access to AVs benefits or harms the most affected constituencies: platforms and the humans who participate on the platform—rider-consumers and driver-workers. These conditions depend almost exclusively on three key quantities: the AV cost, the relative price sensitivity of demand, and the relative wage sensitivity of labor (see Table 1). These results suggest prescriptions for platforms and advocates concerned for humans who participate on the platforms, prescriptions that will become more actionable after aspects of AV technology (e.g., its cost) come into clearer view.

Suppose the relative price sensitivity of demand is less than relative wage sensitivity of labor. Then, access to AVs (1) increases platform profit and social welfare and (2) increases agent welfare if and only if the AV cost is low. The first result suggests platforms should push regulators to make decisions that ease platforms' access to AVs and that they could argue that such access benefits society as whole. The implication of the second result for advocates concerned for humans that participate on the platform is more subtle. If the AV cost is low, these advocates should encourage regulators to make decisions that ease access to AVs. If the AV cost is high, these advocates could lobby regulators to block platforms' access to AVs. If the advocates deem such efforts unlikely to succeed, they could take the opposite tack, lobbying regulators to take actions that would reduce the cost of AVs. These prescriptions do not depend on the AV ownership structure.

This last conclusion is reversed if the relative price sensitivity of demand is notably greater than relative wage sensitivity of labor. In that case, (1) access to platform-owned AVs increases platform profit and social welfare if and only if the AV cost is low, and (2) access to individually owned AVs increases platform profit if and only if the AV cost is high and increases social welfare. Thus, if the AV cost is high, platforms should act in opposite ways depending on who owns AVs. If it appears that platforms would own AVs, they should push regulators to block platforms' access to AVs, and they could argue that doing so benefits society as a whole. If it appears that individuals would own AVs, platforms should do the opposite, pushing regulators to make decisions that ease platforms' access to AVs, again arguing that this benefits society. The AV cost being low reverses these prescriptions for platforms. If the AV cost is low, platforms may benefit by lobbying regulators to take actions that would increase the cost of individually owned AVs. What drives the divergence in results across the two ownership structures is that platform ownership requires the platform to make a costly commit ex ante to reduce its marginal cost ex post, whereas under individually owned AVs, there is no such commitment.

It is natural to expect that as technological innovation advances, the AV cost will decrease over time. Our static model suggests that the attitudes of platforms and advocates concerned for human participants, accordingly, may change over time. For example, if the relative price sensitivity of demand is less than relative wage sensitivity of labor, then advocates for human participants that oppose platforms' access to AVs when the initial cost of AVs is high would shift to favoring AVs when the AV cost drops sufficiently. Platform's attitudes toward access to platform-owned AVs would undergo a parallel evolution as the AV cost decreases over time. In contrast, if the relative price sensitivity of demand is notably greater than relative wage sensitivity of labor, then platforms that favor access to individually owned AVs when the initial cost of AVs is high would shift to opposing AVs when the AV cost drops sufficiently.

We have assumed that consumers are indifferent to whether their transportation is provided by a human driver or an AV. To the extent that consumers develop strong preferences between these transportation modes (e.g., because of perceived health or safety risks), it may be fruitful to incorporate these preferences.

We have assumed that the platforms are powerful in that in deploying AVs, either the platform decides the size of its AV fleet (under platform ownership) or engages with AV owners that lack market power (under individual ownership). However, a platform might negotiate with owners of large AV fleets to put their AVs on the platform's network. In contrast to the relatively simple arms-length financial transactions between a ride-hailing platform and an individual AV owner, the structure of the financial arrangement between a platform and an owner of a large fleet might be quite complex, specifying when and how many AVs the fleet owner would make available, how the platform would allocate consumer requests to the fleet owner's AVs versus other vehicles, fixed payments, revenue-dependent payments, etc. As AV technology develops and the manner in which AVs integrate with and/or compete against ride-hailing platforms comes into sharper focus, future research opportunities should abound.

Appendix A. Proof of Lemma 1

Before proving Lemma 1, we establish several supporting results. Lemmas A.1 and A.2 provide the best response and equilibrium prices and wages for both platforms under any AV fleets $\mathbf{K} = (K_1, K_2)$, respectively; Lemmas A.3–A.5 are technical results; and Lemmas A.6 establishes that only one symmetric equilibrium exists in the more general case where platform i's cost of AV fleet K_i is $\theta c_k(K_i)$ for $i \in \{1,2\}$, where $c_k(\cdot)$ is weakly convex and strictly increasing. The proof of Lemma 1 follows immediately after Lemma A.6.

We begin by considering the platforms' price and wage decisions, for a given platform of AV fleets $\mathbf{K}$. Note that $b_v = g_v = 0$ and $w_{v,i} = 0$ for $i \in \{1,2\}$ when platforms own AV; for brevity, we drop the last argument from $u_i(\mathbf{p}, \mathbf{w}_l, \mathbf{w}_v)$ and the subscript l from b_l, g_l, $w_{l,1}$, and $w_{l,2}$. We say that a platform i *sources labor* if $L_i(\mathbf{w}) > 0$. If platform i's price is low $p_i < (\alpha + \gamma p_j - K_i)/\beta$, then its demand exceeds its AV fleet $D_i(\mathbf{p}) > K_i$, which implies the platform sources labor to satisfy the demand unmet by its AV fleet $L_i(\mathbf{w}) = D_i(\mathbf{p}) - K_i > 0$; this implies the platform's wage $w_i = [D_i(\mathbf{p}) - K_i + g w_j]/b$. In this case, platform i's second-period contribution is

$$u^l(\mathbf{p}, w_j) = p_i D_i(\mathbf{p}) - [(D_i(\mathbf{p}) - K_i + g w_j)/b][D_i(\mathbf{p}) - K_i].$$

If platform 1's price is high $p_i \ge (\alpha + \gamma p_j - K_i)/\beta$, then $D_i(\mathbf{p}) \le K_i$, and the platform does not source labor $L_i(\mathbf{w}) = 0$. In this case, platform i's second-period contribution is

$$u^s(\mathbf{p}) = p_i D_i(\mathbf{p}).$$

Thus, platform i's second-period contribution is

$$u_i(\mathbf{p}, w_j) = \begin{cases} u^l(\mathbf{p}, w_j) & \text{if } p_i < (\alpha + \gamma p_j - K_i)/\beta \\ u^s(\mathbf{p}) & \text{if } p_i \ge (\alpha + \gamma p_j - K_i)/\beta, \end{cases} \quad \text{(A.1)}$$

where the argument w_i is eliminated. Next, let $\tilde{p}_i^l(K_i) = [(\alpha + \gamma p_j)(2\beta + b) - 2\beta K_i + \beta g w_j]/[2\beta(\beta + b)]$, $\tilde{p}_i^e(K_i) = (\alpha + \gamma p_j - K_i)/\beta$, $\tilde{p}_i^s(K_i) = (\alpha + \gamma p_j)/(2\beta)$, $\tilde{w}_i^l(K_i) = [(\alpha + \gamma p_j - 2K_i)b + (\beta + 2b) g w_j]/[2(\beta + b)b]$, $\tilde{w}_i^e(K_i) = \tilde{w}_i^s(K_i) = g w_j/b$. The superscript l is mnemonic for sourcing labor, s for slack AV capacity, and e for equating AV fleet with demand.

Lemma A.1. *Under AV fleet K_i, platform i's best response price and wage to platform j's price and wage (p_j, w_j) is*

$$(\tilde{p}_i(K_i), \tilde{w}_i(K_i)) = \begin{cases} (\tilde{p}_i^l(K_i), \tilde{w}_i^l(K_i)) & \text{if } K_i < (\alpha + \gamma p_j - g\beta w_j/b)/2, \\ (\tilde{p}_i^e(K_i), \tilde{w}_i^e(K_i)) & \text{if } K_i \in [(\alpha + \gamma p_j - g\beta w_j/b)/2, (\alpha + \gamma p_j)/2], \\ (\tilde{p}_i^s(K_i), \tilde{w}_i^s(K_i)) & \text{if } K_i > (\alpha + \gamma p_j)/2. \end{cases}$$

Further, $K_i < D_i(\tilde{p}_i(K_i), p_j)$ if and only if $K_i < (\alpha + \gamma p_j - g\beta w_j/b)/2$; $K_i = D_i(\tilde{p}_i(K_i), p_j)$ if and only if $K_i \in [(\alpha + \gamma p_j - g\beta w_j/b)/2, (\alpha + \gamma p_j)/2]$; and $K_i > D_i(\tilde{p}_i(K_i), p_j)$ if and only if $K_i > (\alpha + \gamma p_j)/2$.

Proof of Lemma A.1. It is straightforward to show that platform i's second-period contribution $u_i(\mathbf{p}, w_j)$, as given in (A.1), is strictly concave in p_j. Further, $\lim_{p_i \uparrow \tilde{p}_i^e(K_i)} (\partial/\partial p_i) u_i(\mathbf{p}, w_j) > \lim_{p_i \downarrow \tilde{p}_i^e(K_i)} (\partial/\partial p_i) u_i(\mathbf{p}, w_j)$. If $K_i < (\alpha + \gamma p_j - g\beta w_j/b)/2$, then $\lim_{p_i \uparrow \tilde{p}_i^e(K_i)} (\partial/\partial p_i) u_i(\mathbf{p}, w_j) < 0$, and platform i's best response price is the unique solution to the first-order condition $(\partial/\partial p_i)u^l(\mathbf{p}, w_j) = 0$, namely $p_i = \tilde{p}_i^l(K_i)$; further, $K_i < D_i(\tilde{p}_i^l(K_i), p_j)$. If $K_i \in [(\alpha + \gamma p_j - g\beta w_j/b)/2, (\alpha + \gamma p_j)/2]$, then $\lim_{p_i \downarrow \tilde{p}_i^e(K_i)}(\partial/\partial p_i)u_i(\mathbf{p}, w_j) < 0 < \lim_{p_i \uparrow \tilde{p}_i^e(K_i)}(\partial/\partial p_i)u_i(\mathbf{p}, w_j)$ and platform i's best response price $p_i = \tilde{p}_i^e(K_i)$; further, $K_i = D_i(\tilde{p}_i^e(K_i), p_j)$. If $K_i > (\alpha + \gamma p_j)/2$, then $\lim_{p_i \downarrow \tilde{p}_i^e(K_i)}(\partial/\partial p_i)u_i(\mathbf{p}, w_j) > 0$, and platform i's best response price is the unique solution to the first-order condition $(\partial/\partial p_i)u^s(\mathbf{p}, w_j) = 0$, namely $p_i = \tilde{p}_i^s(K_i)$; further, $K_i > D_i(\tilde{p}_i^s(K_i), p_j)$. If $K_i < (\alpha + \gamma p_j - g\beta w_j/b)/2$, then platform i's best response price is sufficiently small that the platform sources labor $L_i(\mathbf{w}) > 0$; thus, platform i's best response wage is $w_i = [D_i(\tilde{p}_i^l(K_i), p_j) - K_i + g w_j]/b = \tilde{w}_i^l(K_i)$. If $K_i \geq (\alpha + \gamma p_j - g\beta w_j/b)/2$, then platform i's best response price is sufficiently large that the platform does not source labor $L_i(\mathbf{w}) = 0$; thus, platform i's best response wage $w_i = g w_j/b = \tilde{w}_i^s(K_i) = \tilde{w}_i^e(K_i)$. □

Let $(p_1^{uv}(K_1), w_1^{uv}(K_1), p_2^{uv}(K_2), w_2^{uv}(K_2))$ denote the unique solution to $p_1^{uv}(K_1) = \tilde{p}_1^u(K_1)$, $\tilde{w}_1^{uv}(K_1) = \tilde{w}_1^u(K_1)$, $p_2^{uv}(K_2) = \tilde{p}_2^v(K_2)$ and $\tilde{w}_2^{uv}(K_2) = \tilde{w}_2^v(K_2)$, where $\{u, v\} \in \{e, l, s\}^2$. Further, let

$$\psi_L^l = \frac{(2\beta b + \gamma g)(\gamma b - \beta g)}{[\beta(\beta+\gamma)(2b+g) + b(2\beta+\gamma)(b+g)](b-g)}$$

$$\psi_H^l = \frac{(4\beta^2-\gamma^2)b(b^2-g^2) + \beta(2\beta^2-\gamma^2)(2b^2-g^2) - \beta^2\gamma b g}{\beta[\beta(\beta+\gamma)(2b+g) + b(2\beta+\gamma)(b+g)](b-g)}$$

$$\psi_L^s = \frac{\beta\gamma(2b^2-g^2)}{\beta(\beta+\gamma)(2b^2-g^2) + (2\beta+\gamma)b(b^2-g^2)}$$

$$\psi_H^s = \frac{(4\beta^2-\gamma^2)b(b^2-g^2) + \beta(2\beta^2-\gamma^2)(2b^2-g^2)}{\beta[\beta(\beta+\gamma)(2b^2-g^2) + b(2\beta+\gamma)(b^2-g^2)]}.$$

In Lemma A.2(i), we assume, without loss of generality, that $K_1 \leq K_2$. This restriction implies that $(2 - \gamma/\beta)K_1 \leq \psi_L^s K_1 + \psi_H^s K_2 \leq \psi_L^l K_1 + \psi_H^l K_2$, where the inequalities are strict if and only if $K_1 < K_2$. For use in Lemmas A.2 and A.4 and the proof of Lemma A.6, define $\underline{K}_i = (\alpha - \psi_H^l K_j)/\psi_L^l$ if $g/b > \gamma/\beta$ and $\underline{K}_i = -\infty$ if $g/b \leq \gamma/\beta$.

Lemma A.2. (i) *Assume $K_1 \leq K_2$. Under AV fleets $\mathbf{K}$, the equilibrium prices and wages are unique and given by*

$$(\mathbf{p}^*(\mathbf{K}), \mathbf{w}^*(\mathbf{K})) = \begin{cases} (p_1^{ss}(K_1), p_2^{ss}(K_2), w_1^{ss}(K_1), w_2^{ss}(K_2)) & \text{if } \alpha < (2-\gamma/\beta)K_1, \\ (p_1^{es}(K_1), p_2^{es}(K_2), w_1^{es}(K_1), w_2^{es}(K_2)) & \text{if } \alpha = (2-\gamma/\beta)K_1 \text{ and } K_1 < K_2, \\ (p_1^{ee}(K_1), p_2^{ee}(K_2), w_1^{ee}(K_1), w_2^{ee}(K_2)) & \text{if } \alpha = (2-\gamma/\beta)K_1 = (2-\gamma/\beta)K_2, \\ (p_1^{ls}(K_1), p_2^{ls}(K_2), w_1^{ls}(K_1), w_2^{ls}(K_2)) & \text{if } \alpha \in ((2-\gamma/\beta)K_1, \psi_L^s K_1 + \psi_H^s K_2), \\ (p_1^{le}(K_1), p_2^{le}(K_2), w_1^{le}(K_1), w_2^{le}(K_2)) & \text{if } \alpha \in [\psi_L^s K_1 + \psi_H^s K_2, \psi_L^l K_1 + \psi_H^l K_2], \\ (p_1^{ll}(K_1), p_2^{ll}(K_2), w_1^{ll}(K_1), w_2^{ll}(K_2)) & \text{if } \alpha > \psi_L^l K_1 + \psi_H^l K_2. \end{cases}$$

(ii) *If $K_2 < \alpha/(2-\gamma/\beta)$, then*

$$(\mathbf{p}^*(\mathbf{K}), \mathbf{w}^*(\mathbf{K})) = \begin{cases} (p_1^{sl}(K_1), p_2^{sl}(K_2), w_1^{sl}(K_1), w_2^{sl}(K_2)) & \text{if } K_1 > (\alpha - \psi_L^s K_2)/\psi_H^s \\ (p_1^{el}(K_1), p_2^{el}(K_2), w_1^{el}(K_1), w_2^{el}(K_2)) & \text{if } (\alpha - \psi_L^l K_2)/\psi_H^l \leq K_1 \leq (\alpha - \psi_L^s K_2)/\psi_H^s, \\ (p_1^{ll}(K_1), p_2^{ll}(K_2), w_1^{ll}(K_1), w_2^{ll}(K_2)) & \text{if } \underline{K}_1 < K_1 < (\alpha - \psi_L^l K_2)/\psi_H^l, \\ (p_1^{le}(K_1), p_2^{le}(K_2), w_1^{le}(K_1), w_2^{le}(K_2)) & \text{if } K_1 \leq \underline{K}_1. \end{cases}$$

If $K_2 = \alpha/(2-\gamma/\beta)$, then

$$(\mathbf{p}^*(\mathbf{K}), \mathbf{w}^*(\mathbf{K})) = \begin{cases} (p_1^{se}(K_1), p_2^{se}(K_2), w_1^{se}(K_1), w_2^{se}(K_2)) & \text{if } K_1 > \alpha/(2-\gamma/\beta), \\ (p_1^{ee}(K_1), p_2^{ee}(K_2), w_1^{ee}(K_1), w_2^{ee}(K_2)) & \text{if } K_1 = \alpha/(2-\gamma/\beta), \\ (p_1^{lv}(K_1), p_2^{lv}(K_2), w_1^{lv}(K_1), w_2^{lv}(K_2)) & \text{if } K_1 < \alpha/(2-\gamma/\beta), \end{cases}$$

where $v = e$ if $g/b \geq \gamma/\beta$ and $v = l$ if $g/b < \gamma/\beta$.

Proof of Lemma A.2. (i) Lemma A.1 implies that under AV fleets $\mathbf{K}$, prices and wages $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^{ss}(\mathbf{K}), \mathbf{w}^{ss}(\mathbf{K}))$ are an equilibrium if and only if $K_i > D_i(\mathbf{p}^{ss}(\mathbf{K}))$ for $i \in \{1,2\}$; $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^{es}(\mathbf{K}), \mathbf{w}^{es}(\mathbf{K}))$ is an equilibrium if and only if $K_1 = D_1(\mathbf{p}^{es}(\mathbf{K}))$ and $K_2 > D_2(\mathbf{p}^{es}(\mathbf{K}))$; $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^{ee}(\mathbf{K}), \mathbf{w}^{ee}(\mathbf{K}))$ is an equilibrium if and only if $K_i = D_i(\mathbf{p}^{ee}(\mathbf{K}))$ for $i \in \{1,2\}$; $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^{ls}(\mathbf{K}), \mathbf{w}^{ls}(\mathbf{K}))$ is an equilibrium if and only if $K_1 < D_1(\mathbf{p}^{ls}(\mathbf{K}))$ and $K_2 > D_2(\mathbf{p}^{ls}(\mathbf{K}))$; $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^{le}(\mathbf{K}), \mathbf{w}^{le}(\mathbf{K}))$ is an equilibrium if and only if $K_1 < D_1(\mathbf{p}^{le}(\mathbf{K}))$ and $K_2 = D_2(\mathbf{p}^{le}(\mathbf{K}))$; and $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^{ll}(\mathbf{K}), \mathbf{w}^{ll}(\mathbf{K}))$ is an equilibrium if and only if $K_i < D_i(\mathbf{p}^{ll}(\mathbf{K}))$ for $i \in \{1,2\}$. Further, it is straightforward to verify $K_i > D_i(\mathbf{p}^{ss}(\mathbf{K}))$, $i \in \{1,2\}$ if and only if $\alpha < (2-\gamma/\beta)K_1$; $K_1 = D_1(\mathbf{p}^{es}(\mathbf{K}))$ and $K_2 > D_2(\mathbf{p}^{es}(\mathbf{K}))$ if and only if $\alpha = (2-\gamma/\beta)K_1$ and $K_1 < K_2$; $K_i = D_i(\mathbf{p}^{ee}(\mathbf{K}))$ for $i \in \{1,2\}$ if and only if $\alpha = (2-\gamma/\beta)K_1 = (2-\gamma/\beta)K_2$; $K_1 < D_1(\mathbf{p}^{ls}(\mathbf{K}))$ and $K_2 > D_2(\mathbf{p}^{ls}(\mathbf{K}))$ if and only if $\alpha \in ((2-\gamma/\beta)K_1, \psi_L^s K_1 + \psi_H^s K_2)$; $K_1 < D_1(\mathbf{p}^{le}(\mathbf{K}))$ and $K_2 = D_2(\mathbf{p}^{le}(\mathbf{K}))$ if and only if $\alpha \in [\psi_L^s K_1 + \psi_H^s K_2, \psi_L^l K_1 + \psi_H^l K_2]$; and $K_i < D_i(\mathbf{p}^{ll}(\mathbf{K}))$, $i \in \{1,2\}$ if and only if $\alpha > \psi_L^l K_1 + \psi_H^l K_2$. (ii) By interchanging indices in part (i), it is straightforward to write $(\mathbf{p}^*(\mathbf{K}), \mathbf{w}^*(\mathbf{K}))$ in closed form for the case where $K_1 \geq K_2$; we refer to this, along with part (i), as the *extended part* (i). If $K_2 = \alpha/(2-\gamma/\beta)$, then $\alpha = (\psi_L^u + \psi_H^u)K_2$ for $u \in \{l, s\}$; the result follows from the extended part (i). For the remainder of the proof, suppose $K_2 < \alpha/(2-\gamma/\beta)$. This implies $K_2 < (\alpha - \psi_u^s K_2)/\psi_u^s$ for $u \in \{L, H\}$. If $g/b > \gamma/\beta$, then $\psi_L^l < 0$ and $(\alpha - \psi_H^l K_2)/\psi_L^l < K_2 < (\alpha - \psi_L^l K_2)/\psi_H^l$; the result follows from the extended part (i). If $g/b \leq \gamma/\beta$, then $\psi_L^l \geq 0$. Therefore, if $K_1 \leq K_2$, then $\psi_L^l K_1 + \psi_H^l K_2 \leq (\psi_L^l + \psi_H^l)\, K_2 < \alpha$, where the last inequality holds because $K_2 < \alpha/(2-\gamma/\beta)$. The result follows from the extended part (i). □

Lemma A.3. *If $D_i(\mathbf{p}^*(\mathbf{K})) \geq K_i$ and $r_i(\mathbf{K})$ is differentiable in K_i at $\mathbf{K}$, then $(\partial^2/\partial K_i^2)r_i(\mathbf{K}) < (\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K}) \leq 0$. If $D_i(\mathbf{p}^*(\mathbf{K})) < K_i$, then $(\partial^2/\partial K_i^2)r_i(\mathbf{K}) = (\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K}) = 0$.*

Proof of Lemma A.3. Let i = 1 without loss of generality. We prove the statements in order. First, if $D_i(\mathbf{p}^*(\mathbf{K})) \geq K_i$, then the price and wage equilibrium must be one of six types: *es*, *ee*, *el*, *ls*, *le*, or *ll*. Using the expressions for the equilibrium prices and wages $(\mathbf{p}^*(\mathbf{K}), \mathbf{w}^*(\mathbf{K}))$ in Lemma A.2, $r_i(\mathbf{K})$ can be written in closed form for each of these equilibrium types. For equilibrium types *es*, *ee*, *el*, *ls*, and *le*, it can be verified algebraically that $(\partial^2/\partial K_i^2)r_i(\mathbf{K}) < (\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K})$ and $(\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K}) \leq 0$. For the *ll*-type equilibrium, it is straightforward to verify algebraically that $(\partial^2/\partial K_i^2)r_i(\mathbf{K}) < (\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K})$, $\lim_{\gamma \to 0}(\partial^2/\partial K_i \partial K_j) r_i(\mathbf{K}) < 0$, and $(\partial^3/\partial K_i \partial K_j \partial \gamma)r_i(\mathbf{K}) < 0$ for all $\gamma \in [0, \beta)$; the latter two imply that $(\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K}) \leq 0$. Second, if $D_i(\mathbf{p}^*(\mathbf{K})) < K_i$, then the price and wage equilibrium must be one of three types: *ss*, *se*, or *sl*. For each of these equilibrium types, $(\partial/\partial K_i)r_i(\mathbf{K}) = 0$, which implies $(\partial^2/\partial K_i^2) r_i(\mathbf{K}) = (\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K}) = 0$. □

Let $\bar{\bar{K}}_i = (\alpha - \psi_L^s K_j)/\psi_H^s$ if $K_j < \alpha/(2-\gamma/\beta)$ and $\bar{\bar{K}}_i = \alpha/(2-\gamma/\beta)$ if $K_j = \alpha/(2-\gamma/\beta)$. Note $\bar{\bar{K}}_i \in (0, \infty)$. In Lemmas A.4–A.6, we consider a more general formulation where platform i's cost of AV fleet K_i is $\theta c_k(K_i)$ for $i \in \{1,2\}$; in Section 2, $c_k(K_i) = K_i$.

Lemma A.4. *Suppose the AV cost function $c_k(K)$ is weakly convex and strictly increasing, and $K_j \in [0, \alpha/(2-\gamma/\beta)]$. Then, there exists $\tilde{g} > 0$ such that the following statements hold for $g \in [0, \tilde{g})$. Platform i's profit $\Pi_i(\mathbf{K})$ is continuous and strictly quasiconcave in K_i on $K_i \in [0, \infty)$; platform i's best response AV fleet to platform j's AV fleet, $\tilde{K}_i(K_j)$, is unique; $\tilde{K}_i(K_j) \in [0, \bar{\bar{K}}_i]$; and $\tilde{K}_i(K_j) \neq \underline{K}_i$. Further, if $\gamma = 0$, then $\tilde{g} = b$.*

Proof of Lemma A.4. Because $c_k(K_i)$ is weakly convex and strictly increasing, to establish that $\Pi_i(\mathbf{K})$ is continuous and strictly quasiconcave in K_i, it is sufficient to show that $r_i(\mathbf{K})$ is strictly quasiconcave in K_i on $K_i \in (0, \bar{\bar{K}}_i)$, invariant to K_i on $K_i \in [\bar{\bar{K}}_i, \infty)$ and continuous in K_i on $K_i \in [0, \infty)$. It is straightforward to verify the latter two properties algebraically using the expressions for $(\mathbf{p}^*, \mathbf{w}^*)$ given in Lemma A.2(i). It remains to show that there exists $\tilde{g} > 0$ such that $r_i(\mathbf{K})$ is strictly quasiconcave in K_i on $K_i \in [0, \bar{\bar{K}}_i)$. Let $i = 1$ without loss of generality. First, suppose $K_j = \alpha/(2-\gamma/\beta)$. Using the expressions for $(\mathbf{p}^*, \mathbf{w}^*)$ given in Lemma A.2(ii), it is straightforward to show that $r_i(\mathbf{K})$ is differentiable in K_i for $K_i \in [0, \bar{\bar{K}}_i)$. Hence, $(\partial^2/\partial K_i^2) r_i(\mathbf{K}) < 0$ for $K_i \in [0, \bar{\bar{K}}_i)$ (by Lemma A.3). Second, suppose $K_j < \alpha/(2-\gamma/\beta)$. By parallel argument, $(\partial^2/\partial K_i^2) r_i(\mathbf{K}) < 0$ for $K_i \in [0, \max\{0, \underline{K}_i\}) \cup (\max\{0, \underline{K}_i\}, (\alpha - \psi_L^l K_j)/\psi_H^l) \cup (\alpha - \psi_L^l K_j)/\ ^{Hl}_{\psi}, \bar{\bar{K}}_i)$. Using the expressions for $(\mathbf{p}^*, \mathbf{w}^*)$ given in Lemma A.2(ii), it is straightforward to show the following: $\lim_{g\to 0} \lim_{K_i \uparrow (\alpha - \psi_L^l K_j)/\psi_H^l} (\partial/\partial K_i) r_i(\mathbf{K}) > \lim_{g\to 0} \lim_{K_i \downarrow (\alpha - \psi_L^l K_j)/\psi_H^l} (\partial/\partial K_i) r_i(\mathbf{K})$; if $g/b > \gamma/\beta$, then $\lim_{g\to 0} \lim_{K_i \uparrow \underline{K}_i} (\partial/\partial K_i) r_i(\mathbf{K}) < 0$ and $\lim_{g\to 0} \lim_{K_i \downarrow \underline{K}_i} (\partial/\partial K_i) r_i(\mathbf{K}) < 0$. The former implies that there exists $\tilde{g}^x > 0$ such that $r_i(\mathbf{K})$ is strictly quasiconcave in K_i for $K_i \in (\max(0, \underline{K}_i), \bar{\bar{K}}_i)$ for $g \in [0, \tilde{g}^x)$. The latter implies that if $g/b > \gamma/\beta$, then there exists $\tilde{g}^y > 0$ such that $\lim_{K_i \uparrow \underline{K}_i} (\partial/\partial K_i) r_i(\mathbf{K}) < 0$ and $\lim_{K_i \downarrow \underline{K}_i} (\partial/\partial K_i) r_i(\mathbf{K}) < 0$ for $g \in [0, \tilde{g}^y)$. This implies that $r_i(\mathbf{K})$ is strictly quasiconcave in K_i for $K_i \in (0, \alpha - \psi_L^l K_j)/\psi_H^l)$ and $\tilde{K}_i(K_j) \neq \underline{K}_i$ for $g \in [0, \tilde{g}^y)$. Thus, $r_i(\mathbf{K})$ is strictly quasiconcave in K_i on $K_i \in (0, \bar{\bar{K}}_i)$ for $g \in [0, \tilde{g})$, where $\tilde{g} = \min\{\tilde{g}^x, \tilde{g}^y\}$. In the special case where $\gamma = 0$, it is straightforward to show by parallel argument to that shown here that $\tilde{g}^x = \tilde{g}^y = \tilde{g} = b$. Uniqueness of the best response $\tilde{K}_i(K_j)$ follows from strict quasiconcavity of $\Pi_i(\mathbf{K})$ in K_i on $K_i \in [0, \infty)$. Because $r_i(\mathbf{K})$ is invariant to K_i on $K_i \in [\bar{\bar{K}}_i, \infty)$, $\Pi_i(\mathbf{K})$ is strictly decreasing in K_i on $K_i \in [\bar{\bar{K}}_i, \infty)$. This implies $\tilde{K}_i(K_j) \in [0, \bar{\bar{K}}_i]$. □

Lemma A.5. *Suppose the AV cost function $c_k(K)$ is strictly increasing. If $K_1^* = K_2^* = K^*$ is a symmetric equilibrium, then $K^* < \alpha/(2-\gamma/\beta)$ and $(\mathbf{p}^*, \mathbf{w}^*) = (\mathbf{p}^{ll}(K^*, K^*), \mathbf{w}^{ll}(K^*, K^*))$.*

Proof of Lemma A.5. Let $\mathring{K}_i(K_j) = \max\{K : \tilde{K}_i(K_j) = K\}$, where $\tilde{K}_i(K_j)$ denotes platform i's best response AV fleet to platform j's AV fleet; in words, $\mathring{K}_i(K_j)$ denotes platform i's largest best response. To establish that a symmetric equilibrium cannot have $K^* \geq \alpha/(2-\gamma/\beta)$, it is sufficient to show that $\mathring{K}_i(K_j) < K_j$ when $K_j \geq \alpha/(2-\gamma/\beta)$. Suppose $K_j \geq \alpha/(2-\gamma/\beta)$. Using the expressions for $(\mathbf{p}^*, \mathbf{w}^*)$ given in Lemma A.2(i), it is straightforward to verify that $r_i(\mathbf{K})$ is invariant to K_i on $K_i \in [\alpha/(2-\gamma/\beta), \infty)$. Therefore, because $c_k(K_i)$ is strictly increasing, $\Pi_i(\mathbf{K})$ is strictly decreasing in K_i on $K_i \in [\alpha/(2-\gamma/\beta), \infty)$. Therefore, $\mathring{K}_i(K_j) \leq \alpha/(2-\gamma/\beta)$. Hence, if $K_j > \alpha/(2-\gamma/\beta)$, $\mathring{K}_i(K_j) < K_j$. Suppose instead that $K_j = \alpha/(2-\gamma/\beta)$. Using the expressions for $(\mathbf{p}^*, \mathbf{w}^*)$ given in Lemma A.2(ii), it is straightforward to verify that $\lim_{K_i \uparrow \alpha/(2-\gamma/\beta)} (\partial/\partial K_i) r_i(\mathbf{K}) \leq 0$. Because $\theta > 0$ and $c_k(K_i)$ is strictly increasing, $\lim_{K_i \uparrow \alpha/(2-\gamma/\beta)} (\partial/\partial K_i) \Pi_i(\mathbf{K}) < 0$. Consequently, it cannot be that $\tilde{K}_i(K_j) = \alpha/(2-\gamma/\beta)$ is a best response for platform i. Hence, $\mathring{K}_i(K_j) < K_j$ when $K_j \geq \alpha/(2-\gamma/\beta)$. Because $\psi_L^l + \psi_H^l = 2 - \gamma/\beta$, $K_i^* < \alpha/(2-\gamma/\beta)$ for $i \in \{1,2\}$ implies $\alpha > \psi_L^l K_1^* + \psi_H^l K_2^*$, which by Lemma A.2(i), implies $(\mathbf{p}^*, \mathbf{w}^*) = (\mathbf{p}^{ll}(K^*, K^*), \mathbf{w}^{ll}(K^*, K^*))$. □

Lemma A.6. *Suppose the AV cost function $c_k(K)$ is weakly convex and strictly increasing. There exists $\tilde{g} > 0$ such that if $g < \tilde{g}$, then only one symmetric equilibrium, $K_1^* = K_2^* = K^*$, exists.*

Proof of Lemma A.6. We refer to $K_i \in [0, \alpha/(2-\gamma/\beta)]$ for $i \in \{1,2\}$ as the *truncated strategy space* and $K_i \in [0, \infty)$ for $i \in \{1,2\}$ as the *full strategy space.* The proof proceeds in three steps. First, we show that there exists only one equilibrium on the truncated strategy space and that it is symmetric. We denote this equilibrium by $K_1^t = K_2^t = K^t$. Second, we show that K^t is also an equilibrium on the full strategy space. Third, we show that K^t is the only symmetric equilibrium on the full strategy space.

Step 1. Because the game is symmetric, the truncated strategy space $K_i \in [0, \alpha/(2-\gamma/\beta)]$ is compact and convex for $i \in \{1,2\}$, and the profit functions $\Pi_i(\mathbf{K})$ are continuous and quasiconcave in K_i on $K_i \in [0, \alpha/(2-\gamma/\beta)]$ for $i \in \{1,2\}$ (from Lemma A.4), there exists at least one symmetric equilibrium, K^t, on the truncated strategy space (Cachon and Netessine 2004). Next, we show that K^t is the only equilibrium on the truncated strategy space. By Lemma A.4, $\tilde{K}_i(K_j)$ is unique for $K_j \in [0, \alpha/(2-\gamma/\beta)]$. It follows from Berge's maximum theorem that the best response $\tilde{K}_i(K_j)$ is continuous in K_j on $K_j \in [0, \alpha/(2-\gamma/\beta)]$ for $i \in \{1,2\}$. Because an equilibrium exists on the truncated strategy space, to prove uniqueness, it suffices to show that the magnitudes of the slopes of the best response functions are strictly less than one everywhere on the truncated strategy space (Cachon and Netessine 2004). Because the platforms are symmetric and because $\tilde{K}_i(K_j)$ is continuous in K_j on $K_j \in [0, \alpha/(2-\gamma/\beta)]$, it is sufficient to show that $|(d/dK_j)\tilde{K}_i(K_j)| < 1$ for $K_j \in (0, \alpha/(2-\gamma/\beta))$. First, consider the case where $(\partial^2/\partial K_i \partial K_j)\Pi_i(\mathbf{K})$ exists at $\mathbf{K} = (\tilde{K}_i(K_j), K_j)$. Note that if $(\partial/\partial K_i)\Pi_i(\mathbf{K})|_{K_i = \alpha/(2-\gamma/\beta)} > 0$, then by quasiconcavity of $\Pi_i(\mathbf{K})$ in K_i, the best response on the truncated strategy space is $\tilde{K}_i(K_j) = \alpha/(2-\gamma/\beta)$. In this case, $(d/dK_j)\tilde{K}_i(K_j) = 0$, and thus, $|(d/dK_j)\tilde{K}_i(K_j)| < 1$ holds immediately. If $(\partial/\partial K_i)\Pi_i(\mathbf{K})|_{K_i = \alpha/(2-\gamma/\beta)} \leq 0$, then the best response $\tilde{K}_i(K_j)$ on $K_i \in [0, \alpha/(2-\gamma/\beta)]$ is given by the solution to $(\partial/\partial K_i)\Pi_i(\mathbf{K}) = 0$. Because $\tilde{K}_i(K_j)$ is continuous and $\tilde{K}_i(K_j)$ is the solution to $(\partial/\partial K_i)\Pi_i(\mathbf{K}) = 0$, we may apply the implicit function theorem to obtain $|(d/dK_j)\tilde{K}_i(K_j)| = |[(\partial^2/\partial K_i \partial K_j) r_i(\mathbf{K})]/[(\partial^2/\partial K_i^2) r_i(\mathbf{K}) - \theta(\partial^2/\partial K_i^2) c_k(K_i)]_{K_i = \tilde{K}_i(K_j)}|$. Next, because $\theta > 0$, note that $D_i(\mathbf{p}^*(\tilde{K}_i(K_j), K_j)) < \tilde{K}_i(K_j)$ cannot hold at a best response $\tilde{K}_i(K_j)$. It follows that $D_i(\mathbf{p}^*(\tilde{K}_i(K_j), K_j)) \geq \tilde{K}_i(K_j)$. Therefore,

$|(d/dK_j)\tilde{K}_i(K_j)| \leq |[(\partial^2/\partial K_i \partial K_j) r_i(\mathbf{K})]/[(\partial^2/\partial K_i^2) r_i(\mathbf{K})]_{K_i=\tilde{K}_i(K_j)}| < 1$, where the first inequality follows because $c_k(\cdot)$ is weakly convex, and the second inequality follows from Lemma A.3 because the existence of $(\partial^2/\partial K_i \partial K_j)\Pi_i(\mathbf{K})$ at $\mathbf{K} = (\tilde{K}_i(K_j), K_j)$ implies $r_i(\mathbf{K})$ is differentiable in K_i at $\mathbf{K} = (\tilde{K}_i(K_j), K_j)$ and because $D_i(\mathbf{p}^*(\tilde{K}_i(K_j), K_j)) \geq \tilde{K}_i(K_j)$. Therefore, if $(\partial^2/\partial K_i \partial K_j)\Pi_i(\mathbf{K})$ exists at $\mathbf{K} = (\tilde{K}_i(K_j), K_j)$, $|(d/dK_j)\tilde{K}_i(K_j)| < 1$. Second, consider the case where $(\partial^2/\partial K_i \partial K_j)\Pi_i(\mathbf{K})$ does not exist at $\mathbf{K} = (\tilde{K}_i(K_j), K_j)$. By Lemma A.2(ii), this can only occur if $\tilde{K}_i(K_j) = (\alpha - \psi_L^u K_j)/\psi_H^u$ for $u \in \{s, l\}$ or if $\tilde{K}_i(K_j) = \underline{K}_i$. By Lemma A.4, $\tilde{K}_i(K_j) \neq \underline{K}_i$. If $\tilde{K}_i(K_j) = (\alpha - \psi_L^u K_j)/\psi_H^u$, then $|(d/dK_j)\tilde{K}_i(K_j)| = \psi_L^u/\psi_H^u < 1$ for $u \in \{s, l\}$, where the inequality follows by straightforward algebra. It follows that $K_1^t = K_2^t = K^t$ is the unique equilibrium on the truncated strategy space.

Step 2. By definition, $K^t = \arg\max_{K_i \in [0, \alpha/(2-\gamma/\beta))} \Pi_i(\mathbf{K})|_{K_j = K^t}$ for $i \in \{1, 2\}$. By Lemma A.4, $\Pi_i(\mathbf{K})$ is quasiconcave in K_i on $K_i \in [0, \infty)$ for $K_j \in [0, \alpha/(2-\gamma/\beta)]$ and $i \in \{1, 2\}$. Because $K^t \in [0, \alpha/(2-\gamma/\beta)]$, it follows that $K^t = \arg\max_{K_i \in [0,\infty)} \Pi_i(\mathbf{K})|_{K_j = K^t}$ for $i \in \{1, 2\}$, which implies $\tilde{K}_i(K^t) = K^t$ for $i \in \{1, 2\}$. Therefore, $K_1^t = K_2^t = K^t$ is also an equilibrium on the full strategy space.

Step 3. Suppose that in addition to K^t, there exists a second symmetric equilibrium on the full strategy space, $K_1^a = K_2^a = K^a$. By Lemma A.5, it must be that $K^a \in [0, \alpha/(2-\gamma/\beta))$. However, this contradicts the result in the first step of this proof that K^t is the unique equilibrium on the truncated strategy space $K_i \in [0, \alpha/(2-\gamma/\beta)]$ for $i \in \{1, 2\}$. We conclude that $K^* = K^t$ is the only symmetric equilibrium on the full strategy space. □

Proof of Lemma 1. Lemma A.2 establishes uniqueness of the equilibrium price and wage for any $\mathbf{K} = (K_1, K_2)$. By Lemma A.6, for any weakly convex and strictly increasing $c_k(K)$, there exists $\tilde{g} > 0$ such that if $g < \tilde{g}$, then exactly one symmetric equilibrium exists. The result follows because the AV cost function $c_k(K) = K$. □

Appendix B. Proofs for Platform-Owned AVs

Lemma B.1 deals with the general formulation where platform i's cost of AV fleet K_i is $\theta c_k(K_i)$ for $i \in \{1, 2\}$; in Section 2, $c_k(K_i) = K_i$. We generalize the definition $\theta_m = \lim_{K_1 \downarrow 0} \lim_{K_2 \downarrow 0} [(\partial/\partial K_1) r_1(\mathbf{K})/(\partial/\partial K_1) c_k(K_1)]$ to reflect the generalized AV cost function $c_k(\cdot)$; note that $K^* > 0$ if and only if $\theta < \theta_m$. The proofs of Lemma B.2 and Propositions 1 and 3 are stated in terms of $c_k(K)$, where $c_k(K) = K$.

Lemma B.1. *Suppose the AV cost function $c_k(K)$ is weakly convex and strictly increasing. If $K^* > 0$, then $(d/d\theta)K^* < 0$. Further, if AV cost function is linear, $c_k(K) = K$, then $(d^2/d\theta^2)\, K^* = 0$.*

Proof of Lemma B.1. First, suppose $c_k(K)$ is weakly convex and strictly increasing; we will show that $(d/d\theta)K^* < 0$. Let $\mathcal{A}(K_i, K_j) = (\partial^2/\partial K_j \partial\theta)\Pi_j(\partial^2/\partial K_i \partial K_j)\Pi_i - (\partial^2/\partial K_i \partial\theta)\Pi_i(\partial^2/\partial K_j^2)\Pi_j$ and $\mathcal{B}(K_i, K_j) = (\partial^2/\partial K_i^2)\Pi_i(\partial^2/\partial K_j^2)\Pi_j - (\partial^2/\partial K_i \partial K_j)\Pi_j(\partial^2/\partial K_j \partial K_i)\Pi_i$. It follows immediately from the analysis in Dixit (1986) that $(d/d\theta)K_i^* = \mathcal{A}(K_i^*, K_j^*)/\mathcal{B}(K_i^*, K_j^*)$. It suffices to show that $\mathcal{B}(K_i^*, K_j^*) > 0$ and $\mathcal{A}(K_i^*, K_j^*) < 0$. It follows from Lemma A.3 that $\mathcal{B}(K_i^*, K_j^*) > 0$. Next, we show $\mathcal{A}(K_i^*, K_j^*) < 0$. By symmetry, $(\partial^2/\partial K_i \partial\theta)\Pi_i = (\partial^2/\partial K_j \partial\theta)\Pi_j$, $(\partial^2/\partial K_i^2)\Pi_i = (\partial^2/\partial K_j^2)\Pi_j$, and $(\partial^2/\partial K_i \partial K_j)\Pi_i = (\partial^2/\partial K_i \partial K_j)\Pi_j$. Then, through straightforward substitution and factoring, $\mathcal{A}(K_i^*, K_j^*) = (\partial/\partial K_j \partial\theta)[(\partial^2/\partial K_i \partial K_j)\Pi_i - (\partial^2/\partial K_j^2)\,\Pi_j]$ and $\mathcal{B}(K_i^*, K_j^*) = [(\partial^2/\partial K_j^2)\Pi_j + (\partial^2/\partial K_i \partial K_j)\,\Pi_i] \cdot [(\partial^2/\partial K_j^2)\Pi_j - (\partial^2/\partial K_i \partial K_j)\Pi_i]$. Therefore, $(d/d\theta)K^* = \mathcal{A}(K^*, K^*)/\mathcal{B}(K^*, K^*) = -[(\partial^2/\partial K_j \partial\theta)\,\Pi_j]/[(\partial^2/\partial K_j^2)\Pi_j + (\partial^2/\partial K_i \partial K_j)\Pi_i]$. Note $(\partial^2/\partial K_j \partial\theta)\Pi_j = -(\partial/\partial K_j) c_k(K_j) < 0$ because $c_k(K_j)$ is strictly increasing, and $(\partial^2/\partial K_j^2)\Pi_j + (\partial^2/\partial K_i \partial K_j)\,\Pi_i < 0$ by Lemma A.3. Therefore, $(d/d\theta)K^* < 0$. Second, suppose $c_k(K) = K$; we will show $(d^2/d\theta^2)K^* = 0$. Note $c_k(K) = K$ implies $(\partial^2/\partial K^2)c_k(K) = 0$. Therefore, $\mathcal{A}(K_i^*, K_j^*)$ and $\mathcal{B}(K_i^*, K_j^*)$ are both invariant to θ. It follows that $(d/d\theta)K^*$ is invariant to θ, and thus, $(d^2/d\theta^2)K^* = 0$. □

For use in Lemma B.2 and the proofs of Propositions 1 and 3, let $\pi(\theta) = \Pi_1(K^*(\theta), K^*(\theta))$ and $sw(\theta) = SW(K^*(\theta), K^*(\theta))$ be equilibrium platform profit and social welfare, respectively, under AV cost θ.

Lemma B.2. (i) *Equilibrium profit $\pi(\theta)$ is strictly convex in θ on $\theta \in (0, \theta_m)$.* (ii) *Equilibrium social welfare $sw(\theta)$ is strictly convex in θ on $\theta \in (0, \theta_m)$.*

Proof of Lemma B.2. (i) We denote the AV cost function as $c_k(K)$, where $c_k(K) = K$. Note $(d^2/d\theta^2)\pi(\theta) = (d^2/d\theta^2)K^*(\theta)(\partial/\partial K)\Pi_1(K, K) + (d/d\theta)K^*(\theta)[(d/d\theta)K^*(\theta)(\partial^2/\partial K^2)\;\Pi_1(K, K) + 2(\partial^2/\partial K \partial\theta)\Pi_1(K, K)] + (\partial^2/\partial\theta^2)\Pi_1(K, K)$. Note $(\partial^2/\partial\theta^2)\Pi_1(K, K) = 0$ and $(\partial^2/\partial K \partial\theta)\Pi_1(K, K) = -(\partial/\partial K)c_k(K)$. Further, because $c_k(K) = K$, $(d^2/d\theta^2)K^*(\theta) = 0$ (by Lemma B.1). By simplifying, $(d^2/d\theta^2)\pi(\theta) = (d/d\theta)K^*(\theta)[(d/d\theta)K^*(\theta)(\partial^2/\partial K^2)\Pi_1(K, K) - 2(\partial/\partial K)c_k(K)]$. Because $(d/d\theta)K^*(\theta) < 0$ (by Lemma B.1), it suffices to show $(d/d\theta)K^*(\theta)(\partial^2/\partial K^2)\Pi_1(K, K) - 2(\partial/\partial K)c_k(K) < 0$. Note $(d/d\theta)K^*(\theta) = (\partial/\partial K)c_k(K)/[(\partial^2/\partial K_1^2)\Pi_1(\mathbf{K}) + (\partial^2/\partial K_1 \partial K_2)\Pi_1(\mathbf{K})]$ from the proof of Lemma B.1. Substituting this expression for $(d/d\theta)K^*(\theta)$ into the preceding inequality and simplifying, it remains to show that $(\partial^2/\partial K^2)\Pi_1(K, K)/[(\partial^2/\partial K_1^2)\Pi_1(\mathbf{K}) + (\partial^2/\partial K_1 \partial K_2)\Pi_1(\mathbf{K})] < 2$. Note

$$
\begin{aligned}
&(\partial^2/\partial K^2)\Pi_1(K, K)/\{(\partial^2/\partial K_1^2)\Pi_1(\mathbf{K}) + (\partial^2/\partial K_1 \partial K_2)\Pi_1(\mathbf{K})\} \\
&= (\partial^2/\partial K^2)[r_1(K, K) - \theta c_k(K)]/\{(\partial^2/\partial K_1^2)[r_1(\mathbf{K}) - \theta c_k(K_1)] \\
&\quad + (\partial^2/\partial K_1 \partial K_2) r_1(\mathbf{K})\} \\
&= (\partial^2/\partial K^2) r_1(K, K)/\{(\partial^2/\partial K_1^2) r_1(\mathbf{K}) + (\partial^2/\partial K_1 \partial K_2) r_1(\mathbf{K})\} \\
&< 2,
\end{aligned}
$$

where the second equality follows from $(\partial^2/\partial K^2)c_k(K) = 0$, and the inequality can be verified algebraically.

(ii) Because $sw(\theta) = 2\pi(\theta) + AW(K^*(\theta), K^*(\theta))$ and $(d^2/d\theta^2)\pi(\theta) > 0$ (by part (i)), it suffices to show $(d^2/d\theta^2)AW(K^*(\theta), K^*(\theta)) > 0$ for $\theta \in (0, \theta_m)$. Note $(d^2/d\theta^2)AW(K^*(\theta), K^*(\theta)) = (d^2/d\theta^2)K^*(\theta)(\partial/\partial K)AW(K, K) + [(d/d\theta)K^*(\theta)]^2(\partial^2/\partial K^2)AW(K, K) = [(d/d\theta)K^*(\theta)]^2(\partial^2/\partial K^2)AW(K, K)$, where the second equality follows because $(d^2/d\theta^2)K^*(\theta) = 0$ (by Lemma B.1). It can be shown algebraically that $(\partial^2/\partial K^2)AW(K, K) > 0$. The result follows because $(d/d\theta)K^*(\theta) < 0$ for $\theta \in (0, \theta_m)$ (by Lemma B.1). □

Proof of Proposition 1. The proof proceeds in two steps. First, we show that $\lim_{\theta\uparrow\theta_m}(d/d\theta)\pi(\theta)>0$ if and only if $\gamma/\beta>g_l/b_l$. Second, we prove the main result.

Step 1. We denote the AV cost function as $c_k(K)$, where $c_k(K)=K$. Note $\lim_{\theta\uparrow\theta_m}(d/d\theta)\pi(\theta)=\lim_{\theta\uparrow\theta_m}[(\partial/\partial K)\Pi_1(K^*,K^*)(d/d\theta)K^*(\theta)+(\partial/\partial\theta)\pi(\theta)]$. Because $(d/d\theta)K^*(\theta)<0$ (by Lemma B.1) and $\lim_{\theta\uparrow\theta_m}(\partial/\partial\theta)\pi(\theta)=-c_k(K)|_{K=0}=0$, $\lim_{\theta\uparrow\theta_m}(d/d\theta)\pi(\theta)>0$ if and only if $\lim_{\theta\uparrow\theta_m}(\partial/\partial K)\Pi_1(K^*,K^*)<0$. Further, because $\lim_{\theta\uparrow\theta_m}K^*=0$, $\lim_{\theta\uparrow\theta_m}(\partial/\partial K)\Pi_1(K^*,K^*)=\lim_{K\downarrow 0}(\partial/\partial K)[r_1(K,K)-\theta_m c_k(K)]$. Substituting $\theta_m=\lim_{K_1\downarrow 0}\lim_{K_2\downarrow 0}[(\partial/\partial K_1)\Pi_1(\mathbf{K})/(\partial/\partial K_1)c_k(K_1)]$, it follows that $\lim_{\theta\uparrow\theta_m}(\partial/\partial K)\Pi_1(K^*,K^*)<0$ if and only if $\lim_{K\downarrow 0}(\partial/\partial K)\ r_1(K,K)-\lim_{K_1\downarrow 0}\lim_{K_2\downarrow 0}(\partial/\partial K_1)r_1(\mathbf{K})<0$. Next, it can be shown that $\lim_{K\downarrow 0}(\partial/\partial K)r_1(K,K)-\lim_{K_1\downarrow 0}\lim_{K_2\downarrow 0}(\partial/\partial K_1)r_1(\mathbf{K})=\xi(\beta,\gamma,b_l,g_l)/\zeta(\alpha,\beta,\gamma,b_l,g_l)$ for some functions $\xi(\beta,\gamma,b_l,g_l)$ and $\zeta(\alpha,\beta,\gamma,b_l,g_l)$, where $\zeta(\alpha,\beta,\gamma,b_l,g_l)>0$ for $\gamma<\beta$ and $g_l<b_l$, and $\xi(\beta,\gamma,b_l,g_l)<0$ if and only if $\gamma/\beta>g_l/b_l$. It follows that $\lim_{\theta\uparrow\theta_m}(d/d\theta)\pi(\theta)>0$ if and only if $\gamma/\beta>g_l/b_l$.

Step 2. Suppose $\gamma/\beta>g_l/b_l$. Note that $\Pi^P=\pi(\theta)$ where $\theta\in(0,\theta_m)$. Because $\pi(\theta)$ is continuous in θ, $\pi(\theta_m)=\Pi^0$, and $\lim_{\theta\uparrow\theta_m}(d/d\theta)\pi(\theta)>0$, there exists $\bar{\theta}<\theta_m$ such that $\pi(\theta)<\Pi^0$ for all $\theta\in(\bar{\theta},\theta_m)$. Further, because $\pi(\theta)$ is strictly convex on $\theta\in(0,\theta_m)$ (by Lemma B.2(i)), $\pi(\theta)>\Pi^0$ for all $\theta<\bar{\theta}$. Next, suppose $\gamma/\beta\le g_l/b_l$. It follows from the first step of the proof that $\lim_{\theta\uparrow\theta_m}(d/d\theta)\pi(\theta)\le 0$. Because $\lim_{\theta\uparrow\theta_m}(d/d\theta)\pi(\theta)\le 0$, $\pi(\theta_m)=\Pi^0$ and $\pi(\theta)$ is strictly convex in θ, it follows that $\pi(\theta)>\Pi^0$ for all $\theta\in(0,\theta_m)$. □

Proof of Proposition 2. The proof proceeds in two steps. First, we show that $\lim_{K\to 0}(\partial/\partial K)AW(K,K)<0$ if and only if $\gamma/\beta<g_l/b_l$. Second, we prove the main result.

Step 1. It can be shown algebraically that $\lim_{K\to 0}(\partial/\partial K)AW(K,K)=(\gamma b_l-\beta g_l)\xi(\beta,\gamma,g_l,b_l)$, where $\xi(\beta,\gamma,g_l,b_l)=2b_l(b_l-g_l)\alpha\beta/[(2b_l-g_l)\ \beta(\beta-\gamma)+(b_l-g_l)(2\beta-\gamma)]^2$. Note $\xi(\beta,\gamma,g_l,b_l)>0$. It follows that $\lim_{K\to 0}(\partial/\partial K)AW(K,K)<0$ if and only if $\gamma/\beta<g_l/b_l$.

Step 2. We consider two cases: $\gamma/\beta\ge g_l/b_l$ and $\gamma/\beta<g_l/b_l$. Case 1. $\gamma/\beta\ge g_l/b_l$. It suffices to show $AW(K,K)\ge AW(0,0)$ for all $K\ge 0$. From the first step, $\gamma/\beta\ge g_l/b_l$ implies $\lim_{K\to 0}(\partial/\partial K)AW(K,K)\ge 0$. Next, it can be verified algebraically that $(\partial^2/\partial K^2)AW(K,K)>0$ for all $K\ge 0$. Because $\lim_{K\to 0}(\partial/\partial K)AW(K,K)\ge 0$ and $(\partial^2/\partial K^2)AW(K,K)>0$ for all $K\ge 0$, $(\partial/\partial K)AW(K,K)\ge 0$ for all $K\ge 0$. This, together with the fact that $\lim_{K\to 0}AW(K,K)=AW(0,0)$, implies that $AW(K,K)\ge AW(0,0)$ for all $K\ge 0$. Case 2. $\gamma/\beta<g_l/b_l$. We first show that there exists $\bar{K}>0$ such that $AW(K,K)<AW(0,0)$ for all $K\in(0,\bar{K})$. From the first step, $\gamma/\beta<g_l/b_l$ implies $\lim_{K\to 0}(\partial/\partial K)AW(K,K)<0$. This, together with the facts that $(\partial^2/\partial K^2)AW(K,K)>0$ for all $K\ge 0$ and $\lim_{K\to 0}AW(K,K)=AW(0,0)$, implies that there exists $\bar{K}>0$ such that $AW(K,K)<AW(0,0)$ if and only if $K\in(0,\bar{K})$. Next, note that because $K^*(\theta_m)=0$, $K^*(0)>0$, and $K^*(\theta)$ is strictly decreasing in θ on $\theta\in(0,\theta_m)$ (by Lemma B.1), there exists $\tilde{\theta}\in(0,\theta_m)$ such that $K^*(\theta)\in(0,\bar{K})$ if and only if $\theta\in(\tilde{\theta},\theta_m)$. It follows that $AW(K^*,K^*)<AW(0,0)$ if and only if $\theta\in(\tilde{\theta},\theta_m)$. □

Proof of Proposition 3. The proof proceeds in two steps. First, we show that $\lim_{\theta\uparrow\theta_m}(d/d\theta)sw(\theta)>0$ if and only if $\gamma/\beta>g_l/b_l$. Second, we prove the main result.

Step 1. Note $\lim_{\theta\uparrow\theta_m}(d/d\theta)sw(\theta)=\lim_{\theta\uparrow\theta_m}[(\partial/\partial K)SW(K^*,K^*)\ (d/d\theta)K^*(\theta)+(\partial/\partial\theta)sw(\theta)]$. Because $(d/d\theta)K^*(\theta)<0$ (by Lemma B.1) and $\lim_{\theta\uparrow\theta_m}(\partial/\partial\theta)sw(\theta)|_{\theta=\theta_m}=-2c_k(K)|_{K=0}=0$, $\lim_{\theta\uparrow\theta_m}(d/d\theta)sw(\theta)>0$ if and only if $\lim_{\theta\uparrow\theta_m}(\partial/\partial K)SW(K^*,K^*)<0$. Further, because $\lim_{\theta\uparrow\theta_m}K^*=0$, $\lim_{\theta\uparrow\theta_m}(\partial/\partial K)SW(K^*,K^*)=\lim_{K\downarrow 0}(\partial/\partial K)\ [2r_1(K,K)\ -2\theta_m c_k(K)+AW(K,K)]$. Substituting $\theta_m=\lim_{K_1\downarrow 0}\lim_{K_2\downarrow 0}[(\partial/\partial K_1)\Pi_1(\mathbf{K})/(\partial/\partial K_1)c_k(K_1)]$, it follows that $\lim_{\theta\uparrow\theta_m}(\partial/\partial K)SW(K^*,K^*)<0$ if and only if $\lim_{K\downarrow 0}(\partial/\partial K)[r_1(K,K)+AW(K,K)/2]-\lim_{K_1\downarrow 0}\lim_{K_2\downarrow 0}(\partial/\partial K_1)r_1(\mathbf{K})<0$. Next, it can be shown that $\lim_{K\downarrow 0}(\partial/\partial K)[r_1(K,K)+AW(K,K)/2]-\lim_{K_1\downarrow 0}\lim_{K_2\downarrow 0}(\partial/\partial K_1)r_1(\mathbf{K})=(\gamma b_l-\beta g_l)\xi(\alpha,\beta,\gamma,b_l,g_l)$ for some function $\xi(\beta,\gamma,b_l,g_l)$. It can be verified that $\xi(\beta,\gamma,b_l,g_l)>0$ for $\gamma<\beta$ and $g_l<b_l$. It follows that $\lim_{\theta\uparrow\theta_m}(d/d\theta)sw(\theta)>0$ if and only if $\gamma/\beta>g_l/b_l$.

Step 2. Note that $SW^P=sw(\theta)$ where $\theta\in(0,\theta_m)$. To prove the main result, we show that there exists $\hat{\theta}\ge 0$ such that $sw(\theta)<SW^0$ if and only if $\theta\in(\hat{\theta},\theta_m)$, where $\hat{\theta}<\theta_m$ if and only if $\gamma/\beta>g_l/b_l$. First, suppose $\gamma/\beta>g_l/b_l$. Because $sw(\theta)$ is continuous in θ, $sw(\theta_m)=SW^0$, and $\lim_{\theta\uparrow\theta_m}(d/d\theta)sw(\theta)>0$ (by Step 1), there exists $\hat{\theta}<\theta_m$ such that $sw(\theta)<SW^0$ for all $\theta\in(\hat{\theta},\theta_m)$. Further, because $(d^2/d\theta^2)sw(\theta)>0$ for $\theta\in(0,\theta_m)$ (by Lemma B.2(ii)), $sw(\theta)>SW^0$ for all $\theta<\hat{\theta}$. Next, suppose $\gamma/\beta\le g_l/b_l$. It follows from Step 1 that $\lim_{\theta\uparrow\theta_m}(d/d\theta)sw(\theta)<0$. Because $\lim_{\theta\uparrow\theta_m}(d/d\theta)sw(\theta)<0$, $sw(\theta_m)=SW^0$ and $(d^2/d\theta^2)sw(\theta)>0$, it follows that $sw(\theta)>SW^0$ for all $\theta\in(0,\theta_m)$. Hence $\hat{\theta}\ge\theta_m$. □

References

Afeche P, Liu Z, Maglaras C (2018) Ride-hailing networks with strategic drivers: The impact of platform control capabilities on performance. Working paper, University of Toronto, Toronto, Canada.

Allon G, Cohen M, Sinchaisri P (2019) The impact of behavioral and economic drivers on gig economy workers. Working paper, University of Pennsylvania, Philadelphia.

Arya A, Mittendorf B, Sappington D (2008) The make-or-buy decision in the presence of a rival: Strategic outsourcing to a common supplier. *Management Sci.* 54(10):1747–1758.

Bai J, Tang CS (2020) Can two competing on-demand service platforms be both profitable? Working paper, State University of New York at Binghamton, Binghamton, NY.

Bai J, So KC, Tang CS, Chen XM, Wang H (2018) Coordinating supply and demand on an on-demand platform: Price, wage, and payout ratio. *Manufacturing Service Oper. Management* 21(3):556–570.

Banerjee S, Johari R, Riquelme C (2016) Pricing in ride-sharing platforms: A queueing-theoretic approach. Working paper, Cornell University, Ithaca, NY.

Benjaafar S, Xiao S, Yang X (2020a) Do workers and customers benefit from competition between on-demand service platforms? Working paper, University of Minnesota, Minneapolis.

Benjaafar S, Ding J-Y, Kong G, Taylor T (2021) Labor welfare in on-demand service platforms. *Manufacturing Service Oper. Management*, ePub ahead of print April 6, https://doi.org/10.1287/msom.2020.0964.

Benjaafar S, Kong G, Li X, Courcoubetis C (2019) Peer-to-peer product sharing: Implications for ownership, usage and social welfare in the sharing economy. *Management Sci.* 65(2):477–493.

Bensinger G (2017) Lyft jumps into driverless. *Wall Street Journal* (July 22), B2.

Bernstein F, DeCroix G, Keskin B (2020) Competition between two-sided platforms under demand and supply congestion effects. *Manufacturing Service Oper. Management*, ePub ahead of print October 1, https://doi.org/10.1287/msom.2020.0866.

Besbes O, Castro F, Lobel I (2021) Surge pricing and its spatial supply response. *Management Sci.* 67(3):1350–1367.

Bhaskar V, Manning A, To T (2002) Oligopsony and monopsonistic competition in labor markets. *J. Econom. Perspect.* 16(2):155–174.

Bimpikis K, Candogan O, Saban D (2019) Spatial pricing in ride-hailing networks. *Oper. Res.* 67(3):744–769.

Bosa D (2018) Lyft claims it now has more than one-third of the US ride-sharing market. *CNBC Online* (May 14), https://www.cnbc.com/2018/05/14/lyft-market-share-051418-bosa-sf.html.

Boston W (2017) Uber in pact for self-driving Volvos. *Wall Street Journal* (November 21), B1.

Cachon G, Netessine S (2004) Game theory in supply chain analysis. Simchi-Levi D, Wu SD, Shen ZM, eds. *Handbook of Quantitative Supply Chain Analysis: Modeling in the eBusiness Era* (Springer, New York), 13–66.

Cachon G, Daniels K, Lobel R (2017) The role of surge pricing on a service platform with self-scheduling capacity. *Manufacturing Service Oper. Management* 19(3):368–384.

Chen J, Guo Z (2014) Strategic sourcing in the presence of uncertain supply and retail competition. *Production Oper. Management* 23 (10):1748–1760.

Chen Y, Dai T, Korpeoglu C, Korpeoglu E, Sahin O, Tang C, Xiao S (2020) Innovative online platforms: Research opportunities. *Manufacturing Service Oper. Management* 22(3):430–445.

Cohen M, Zhang R (2017) Coopetition and profit sharing for ride-sharing platforms. Working paper, New York University, New York.

Dixit A (1986) Comparative statics for oligopoly. *Internat. Econom. Rev.* 27(1):107–122.

Duvall T, Hannon E, Katseff J, Safran J, Wallace T (2019) A new look at autonomous-vehicle infrastructure. *McKinsey Insights* (May 22) https://www.mckinsey.com/industries/capital-projects-and-infrastructure/our-insights/a-new-look-at-autonomous-vehicle-infrastructure.

Fuess SM Jr., Lowenstein MA (1991) On strategic cost increases in a duopoly. *Internat. J. Indust. Organ.* 9(3):389–395.

Gurvich I, Lariviere M, Moreno A (2019) Operations in the on-demand economy: Staffing services with self-scheduling capacity. Hu M, ed. *Sharing Economy: Making Supply Meet Demand* (Springer, New York), 249–278.

Hamilton J, Thisse JF, Zenou Y (2000) Wage competition with heterogeneous workers and firms. *J. Labor Econom.* 18(3):453–472.

Higgins T (2019) Tesla plans self-driving taxi fleet. *Wall Street Journal* (April 23), B3.

Hu B, Hu M, Zhu H (2021) Surge pricing and two-sided temporal responses in ride-hailing. *Manufacturing Service Oper. Management*, ePub ahead of print February 3, https://doi.org/10.1287/msom.2020.0960.

Hu M, Zhou Y (2019) Price, wage and fixed commission in on-demand matching. Working paper, University of Toronto, Toronto, Canada.

Isaac M (2017) Uber in deal with Volvo to buy self-driving cars for its future network. *New York Times* (November 21), B6.

Lin X, Lu T, Wang X (2018) Mergers between on-demand service platforms: The impact on consumer surplus and labor welfare. Working paper, South China University of Technology, Guangzhou, China.

Liu X, Cui Y, Chen L (2019) Bonus competition in the gig economy. Working paper, Cornell University, Ithaca, NY.

Luna N (2020) Report: After GrubHub snub, Uber looks to buy Postmates. *Nation's Restaurant News* (June 30), https://www.nrn.com/delivery-takeout-solutions/report-after-grubhub-snub-uber-looks-buy-postmates.

Lyft (2019) Form S-1. Accessed May 1, 2019, https://www.sec.gov/Archives/edgar/data/1759509/000119312519059849/d633517ds1.htm.

Mims C (2019) Exchange - technology - keywords: In a strong job market, the gig is up. *Wall Street Journal* (May 4), B5.

Motavalli J (2020) Will driverless cars also be riderless? *New York Times* (May 29), B7.

Murphy M (2016) Lyft's cofounder has his own vision for a self-driving taxi fleet. *Quartz* (October 27), https://qz.com/819335/lyft-cofounder-john-zimmer-has-his-own-vision-for-a-self-driving-taxi-fleet-to-take-on-uber-and-tesla-tsla/.

Nikzad A (2018) Thickness and competition in ride-sharing markets. Working paper, University of Southern California, Los Angeles, California.

Ozkan E, Ward A (2020) Dynamic matching for real-time ridesharing. *Stochastic Systems* 10(1):29–70.

Salop S, Scheffman DT (1987) Cost-raising strategies. *J. Indust. Econom.* 36(1):19–34.

Seade J (1985) Profitable cost increases and the shifting of taxation: Equilibrium responses of markets in oligopoly. Technical report, University of Warwick, Coventry, United Kingdom.

Siddiqui F, Bensinger G (2019) Uber, Lyft bet on the driverless dream. *Washington Post* (March 31), A1.

Taylor T (2018) On-demand service platforms. *Manufacturing Service Oper. Management* 20(4):704–720.

Uber (2019) Form S-1. Accessed May 1, 2019, https://www.sec.gov/Archives/edgar/data/1543151/000119312519103850/d647752ds1.htm.

Wu S, Xiao S, Benjaafar S (2020) Two-sided competition between on-demand service platforms. Working paper, University of Minnesota, Minneapolis.

Wu X, Zhang F (2014) Home or overseas? An analysis of sourcing strategies under competition. *Management Sci.* 60(5):1223–1240.

Ride-hailing Platforms: Competition and Autonomous Vehicles

Online Supplement

This online supplement contains Appendices C, D and E.

Appendix C: Proofs for Individually-Owned AVs

Proof of Lemma 2: We begin by characterizing platform i's best response prices and wages $(p_i, w_{l,i}, w_{v,i})$ to platform j prices and wages $(p_j, w_{l,j}, w_{v,j})$. Because platform i's best response price and wages satisfy $D_i(\mathbf{p}) = L_i(\mathbf{w}_l) + V_i(\mathbf{w}_v)$ and because $D_i(\mathbf{p}) \leq \alpha + \gamma p_j$, in characterizing platform i's best response, we can restrict attention to $w_{l,i}$ and $w_{v,i}$ such that $L_i(\mathbf{w}_l) + V_i(\mathbf{w}_v) \leq \alpha + \gamma p_j$. Let $\hat{p}_i(p_j, \mathbf{w}_l, \mathbf{w}_v)$ be the solution to $D_i(\hat{p}_i, p_j) = L_i(\mathbf{w}_l) + V_i(\mathbf{w}_v)$; that is, let $\hat{p}_i(p_j, \mathbf{w}_l, \mathbf{w}_v) = [\alpha + \gamma p_j - L_i(\mathbf{w}_l) - V_i(\mathbf{w}_v)]/\beta$. Because $D_i(\mathbf{p}) = L_i(\mathbf{w}_l) + V_i(\mathbf{w}_v)$, platform i's second-period contribution (equation (3)) can be rewritten as

$$u_i(p_j, \mathbf{w}_l, \mathbf{w}_v) = [\hat{p}_i(p_j, \mathbf{w}_l, \mathbf{w}_v) - w_{l,i}]L_i(\mathbf{w}_l) + [\hat{p}_i(p_j, \mathbf{w}_l, \mathbf{w}_v) - w_{v,i}]V_i(\mathbf{w}_v),$$

where the argument p_i is eliminated. Next, define $\tilde{p}_i = [(b_l + b_v)(\alpha + \gamma p_j) + \beta(2\alpha + 2\gamma p_j + g_l w_{l,j} + g_v w_{v,j})]/[2\beta(b_l + b_v + \beta)]$, $\tilde{w}_{l,i} = [g_l w_{l,j}(b_v + \beta) + b_l(\alpha + \gamma p_j + 2g_l w_{l,j} + g_v w_{v,j})]/[2b_l(b_l + b_v + \beta)]$, $\tilde{w}_{v,i} = [g_v w_{v,j}(b_l+\beta)+b_v(\alpha+\gamma p_j+g_l w_{l,j}+2g_v w_{v,j})]/[2b_v(b_l+b_v+\beta)]$. Let $(p_1^*, w_{l,1}^*, w_{v,1}^*, p_2^*, w_{l,2}^*, w_{v,2}^*)$ be the unique solution to $p_1 = \tilde{p}_1$, $w_{l,1} = \tilde{w}_{l,1}$, $w_{v,1} = \tilde{w}_{v,1}$, $p_2 = \tilde{p}_2$, $w_{l,2} = \tilde{w}_{l,2}$ and $w_{v,2} = \tilde{w}_{v,2}$.

Note platform i's best response wages are attained at a maximizer of $u_i(p_j, \mathbf{w}_l, \mathbf{w}_v)$. Note also that platform i's best response price must satisfy $p_i = \hat{p}_i(p_j, \mathbf{w}_l, \mathbf{w}_v)$, because $\hat{p}_i(p_j, \mathbf{w}_l, \mathbf{w}_v)$ solves $D_i(\mathbf{p}) = L_i(\mathbf{w}_l) + V_i(\mathbf{w}_v)$, which must hold at a best response price. It follows that platform i's best response prices and wages $(p_i, w_{l,i}, w_{v,i})$ to platform j prices and wages $(p_j, w_{l,j}, w_{v,j})$ must satisfy

$$p_i = \hat{p}_i(p_j, \mathbf{w}_l, \mathbf{w}_v),\ (\partial/\partial w_{l,i})u_i(p_j, \mathbf{w}_l, \mathbf{w}_v) = 0,\ (\partial/\partial w_{v,i})u_i(p_j, \mathbf{w}_l, \mathbf{w}_v) = 0.$$

It can be shown algebraically that $(\tilde{p}_i, \tilde{w}_{l,i}, \tilde{w}_{v,i})$ is the unique maximizer of $u_i(p_j, \mathbf{w}_l, \mathbf{w}_v)$; hence $(\tilde{p}_i, \tilde{w}_{l,i}, \tilde{w}_{v,i})$ is platform i's best response. The result follows because $(p_1^*, w_{l,1}^*, w_{v,1}^*, p_2^*, w_{l,2}^*, w_{v,2}^*)$ is by definition the intersection of the best responses functions; that is, $(p_1^*, w_{l,1}^*, w_{v,1}^*, p_2^*, w_{l,2}^*, w_{v,2}^*)$ is the unique solution to $p_1 = \tilde{p}_1$, $w_{l,1} = \tilde{w}_{l,1}$, $w_{v,1} = \tilde{w}_{v,1}$, $p_2 = \tilde{p}_2$, $w_{l,2} = \tilde{w}_{l,2}$ and $w_{v,2} = \tilde{w}_{v,2}$. Further, by symmetry, $p_1^* = p_2^*$, $w_{l,1}^* = w_{l,2}^*$ and $w_{v,1}^* = w_{v,2}^*$. □

Note that platform i's equilibrium profit is given by taking the equilibrium prices and wages $(p_1^*, w_{l,1}^*, w_{v,1}^*, p_2^*, w_{l,2}^*, w_{v,2}^*)$ into the profit function $\Pi_i(\mathbf{K})$ (given in equation (4)), where $\mathbf{K} = (0, 0)$ because $\theta = \infty$.

In the remainder of this appendix, unless explicitly stated otherwise, we suppose there is no competition in the AV market $g_v = 0$. Lemma 1 is useful in the proof of Proposition 4. For use in the proofs of Lemma 1 and Proposition 4, let Π^I denote equilibrium platform profit under own-wage sensitivity of AV supply $b_v > 0$; that is, Π^I represents equilibrium profit under access to AVs. Let $\underline{\Pi} = \lim_{b_v\to\infty} \Pi^I$. Let Π^0 denote equilibrium platform profit under $b_v = g_v = 0$; that is, Π^0 represents equilibrium profit under no access to AVs.

Lemma 1 *Suppose there is no competition in the AV market $g_v = 0$. (i) If $\gamma/\beta \leq 2/3$, then Π^I weakly increases in b_v on $b_v > 0$. (ii) If $\gamma/\beta \in (2/3, \bar{\eta})$, then there exists $\underline{b}_v > 0$ such that Π^I*

strictly increases in b_v on $b_v \in (0, \underline{b}_v)$ and strictly decreases in b_v on $b_v \in (\underline{b}_v, \infty)$. If $\gamma/\beta \in (2/3, \underline{\eta}]$, then $\Pi^I \geq \Pi^0$ for $b_v > 0$. If $\gamma/\beta \in (\underline{\eta}, \bar{\eta})$, then there exists $\bar{b}_v \in (\underline{b}_v, \infty)$ such that $\Pi^I < \Pi^0$ if and only if $b > \bar{b}_v$. (iii) If $\gamma/\beta \geq \bar{\eta}$, then Π^I strictly decreases in b_v on $b_v > 0$.

Proof of Lemma 1: Let $\psi(b_v) = 2\beta[2b_l(b_l - g_l)(2b_l + g_l) + (2b_l - g_l)^2(b_v + \beta)]/[2b_l(b_l - g_l)(6b_l - g_l) + (2b_l - g_l)^2(3b_v + 2\beta)]$. The proof proceeds in four steps. First, we establish that: for $b_v > 0$, $\psi(b_v)$ strictly decreases in b_v and $\psi(b_v) \in (2\beta/3, \bar{\eta}\beta)$; and $(d/db_v)\Pi^I$ has the same sign as $\psi(b_v) - \gamma$. Second, we establish part (*i*); third, part (*iii*); and fourth, part (*ii*).

Step 1: Note $(\partial/\partial b_v)\psi(b_v) = -2(2b_l - g_l)^2\beta[4b_l(b_l - g_l)(2g_l + \beta) + g_l^2\beta)]/[(3b_v + 2\beta)(2b_l - g_l)^2 + 2b_l(6b_l^2 - 7b_l g_l + g_l^2)]^2 < 0$, $\lim_{b_v \to 0} \psi(b_v) = \bar{\eta}\beta$ and $\lim_{b_v \to \infty} \psi(b) = 2\beta/3$. Hence, for $b_v > 0$, $\psi(b_v)$ strictly decreases in b_v and $\psi(b_v) \in (2\beta/3, \bar{\eta}\beta)$. It can be shown algebraically that $(d/db_v)\Pi^I$ has the same sign as $\psi(b_v) - \gamma$; that is, $(d/db_v)\Pi^I < 0$ if and only if $\psi(b_v) < \gamma$, and $(d/db_v)\Pi^I > 0$ if and only if $\psi(b_v) > \gamma$.

Step 2: Suppose $\gamma/\beta \leq 2/3$. It follows from Step 1 that: $\gamma \leq 2\beta/3 < \psi(b_v)$ for $b_v > 0$; and $\psi(b_v) \geq \gamma$ implies $(d/db_v)\Pi^I \geq 0$ for $b_v > 0$. That is, part (i) holds.

Step 3: Suppose $\gamma/\beta \geq \bar{\eta}$. It follows by step 1 that: $\psi(b_v) < \bar{\eta}\beta \leq \gamma$ for $b_v > 0$; and $\psi(b_v) < \gamma$ implies $(d/db_v)\Pi^I < 0$ for $b_v > 0$. That is, part (iii) holds.

Step 4: Suppose $\gamma \in (2\beta/3, \bar{\eta}\beta)$. Because $\psi(b_v)$ strictly decreases in b_v on $b_v > 0$ and $\psi(b_v) \in (2\beta/3, \bar{\eta}\beta)$ (by step 1), there exists a unique solution on $b_v > 0$ to $\psi(b_v) = \gamma$, which we denote as $\underline{b}_v$; note $\underline{b}_v > 0$. It follows that $\psi(b_v) > \gamma$ if and only if $b_v \in (g, \underline{b}_v)$, and $\psi(b_v) < \gamma$ if and only if $b_v \in (\underline{b}_v, \infty)$. By step 1, this implies $(d/db_v)\Pi^I > 0$ if and only if $b_v \in (0, \underline{b}_v)$. Note $\underline{\Pi} = \lim_{b_v \to \infty} \Pi^I = \alpha^2\beta/(2\beta - \gamma)^2$ and $\Pi^0 = \alpha^2\beta b_l(b_l - g_l)^2(b_l + \beta)/[(2b_l - g_l)\beta(\beta - \gamma) + b_l(b_l - g_l)(2\beta - \gamma)]^2$. It is straightforward to show algebraically that $\underline{\Pi} > \Pi^0$ if and only if $\gamma/\beta < \underline{\eta}$. Next, suppose $\gamma/\beta \in (2/3, \underline{\eta}]$; we will show this implies $\Pi^I \geq \Pi^0$ for $b_v > 0$. For $b_v \in (0, \underline{b}_v)$, that Π^I strictly increases in b_v implies $\Pi^I > \Pi^0$. For $b_v \in (\underline{b}_v, \infty)$, $\Pi^I \geq \underline{\Pi} \geq \Pi^0$, where the first inequality follows because Π^I strictly decreases in b_v, and the second inequality follows because $\gamma/\beta \leq \underline{\eta}$. Finally, suppose $\gamma/\beta \in (\underline{\eta}, \bar{\eta})$. We will show that there exists $\bar{b}_v \in (\underline{b}_v, \infty)$ such that $\Pi^I < \Pi^0$ if and only if $b > \bar{b}_v$. For $b_v \in (0, \underline{b}_v)$, that Π^I strictly increases in b_v implies $\Pi^I > \Pi^0$. Because Π^I strictly decreases in b_v on $b_v \in (\underline{b}_v, \infty)$ and $\Pi^I|_{b_v = \underline{b}_v} > \Pi^0 > \underline{\Pi} = \lim_{b_v \to \infty} \Pi^I$, it follows that there exists $\bar{b}_v \in (\underline{b}_v, \infty)$ such that $\Pi^I < \Pi^0$ if and only if $b > \bar{b}_v$. □

Proof of Proposition 4: If $\gamma/\beta < \underline{\eta}$, then $\Pi^I \geq \Pi^0$ (by Lemma 1(i)). If $\gamma/\beta \geq \bar{\eta}$, then $\Pi^I < \Pi^0$ (by Lemma 1(ii)-(iii)). If $\gamma/\beta \in (\underline{\eta}, \bar{\eta})$, then there exists $\bar{b}_v \in (\underline{b}_v, \infty)$, where $\underline{b}_v > 0$, such that $\Pi^I < \Pi^0$ if and only if $b_v > \bar{b}_v$ (by Lemma 1(ii). We can state the preceding results as follows: There exists $\bar{b}_v \geq 0$ such that access to individually-owned AVs decreases equilibrium profit $\Pi^I < \Pi^0$ if and only if $b_v > \bar{b}_v$; further, if $\gamma/\beta \leq \underline{\eta}$, then $\bar{b}_v = \infty$; if $\gamma/\beta \in (\underline{\eta}, \bar{\eta})$, then $\underline{b}_v < \bar{b}_v < \infty$; and if $\gamma/\beta \geq \bar{\eta}$, then $\bar{b}_v = 0$. Let $\bar{\phi} = 1/\bar{b}_v$, and recall $\phi = 1/b_v$. The result follows. □

Lemma 2 *Suppose there is no competition in the labor market or the AV market $g_l = g_v = 0$ and platform i's labor sourcing cost is $c_l(L_i)$ and AV sourcing cost is $\phi c_v(V_i)$, where $c_l(\cdot)$ and $c_v(\cdot)$ are strictly convex with $c_l(0) = c_v(0) = 0$. There exist $\breve{\phi} > 0$ and $\breve{\eta} < 1$ such that if the relative price sensitivity of demand is high $\gamma/\beta > \breve{\eta}$, platform profit under access to individually owned AVs Π^I increases in the AV cost ϕ on $\phi \in (0, \breve{\phi})$.*

Proof of Lemma 2: Platform i's profit under prices $\mathbf{p}$, AV supply $V_i \in [0, D_i(\mathbf{p})]$, labor sourcing cost $c_l(L_i)$ and AV sourcing cost $\phi c_v(V_i)$ can be written as $\Pi_i(\mathbf{p}, V_i) = p_i D_i(\mathbf{p}) - c_l(D_i(\mathbf{p}) - V_i) - \phi c_v(V_i)$. With some abuse of notation, let $V_i(\mathbf{p})$ denote platform i's profit-maximizing AV supply under prices $\mathbf{p}$. We use $c'_v(V_i)$ to denote $(d/dV_i)c_v(V_i)$ and $c''_v(V_i)$ to denote $(d^2/d^2V_i)c_v(V_i)$; $c'_l(\cdot)$ is defined similarly. Because platform i's profit maximizing price p_i and AV supply V_i satisfy the first order conditions $(\partial/\partial p_i)\Pi_i(\mathbf{p}, V_i) = D_i(\mathbf{p}) - \beta[p_i - c'_l(D_i(\mathbf{p}) - V_i)] = 0$ and $(\partial/\partial V_i)\Pi_i(\mathbf{p}, V_i) = c'_l(D_i(\mathbf{p}) - V_i) - \phi c'_v(V_i) = 0$, the equilibrium prices $\mathbf{p}^*$ satisfy $D_i(\mathbf{p}^*) - \beta[p_i^* - \phi c'_v(V_i(\mathbf{p}^*))] = 0$. Because the equilibrium prices are symmetric $p_1^* = p_2^* = p^*$, as are the equilibrium AV supplies $V_1^* = V_2^* = V^* = V_i(\mathbf{p}^*)$, the equilibrium price and AV supply (p^*, V^*) satisfy $D_i(p^*, p^*) - \beta[p^* - \phi c'_v(V^*)] = 0$. Applying the implicit function theorem yields $\partial p^*/\partial\phi = \beta c'_v(V^*)/[2\beta - \gamma - \beta\phi c''_v(V^*)(\partial V^*/\partial p^*)]$. Further, $(d/d\phi)\Pi_i(\mathbf{p}^*, V_i(\mathbf{p}^*)) = (\partial p_j^*/\partial\phi)(\gamma/\beta)D_i(\mathbf{p}^*) - c_v(V_i(\mathbf{p}^*))$. This, together with the previous expression for $\partial p^*/\partial\phi$ and the observations that $\partial p_j^*/\partial\phi = \partial p^*/\partial\phi$ and $\Pi^I = \Pi_i(\mathbf{p}^*, V_i(\mathbf{p}^*))$, implies $\lim_{\phi\to 0}(d/d\phi)\Pi^I = \alpha\beta\gamma c'_v(\alpha\beta/(2\beta-\gamma))/(2\beta-\gamma)^2 - c_v(\alpha\beta/(2\beta-\gamma))$. Further, $\lim_{\gamma\to\beta}\lim_{\phi\to 0}(d/d\phi)\Pi^I = \alpha c'_v(\alpha) - c_v(\alpha)$. Because $c_v(\cdot)$ is strictly convex, $[c_v(\alpha) - c_v(0)]/\alpha$ strictly increases in α for $\alpha > 0$; therefore, $c_v(0) = 0$ implies $c_v(\alpha)/\alpha$ strictly increases in α, which implies $\alpha c'_v(\alpha) - c_v(\alpha) > 0$. Because $(d/d\phi)\Pi^I$ is continuous in γ and ϕ, $\lim_{\gamma\to\beta}\lim_{\phi\to 0}(d/d\phi)\Pi^I > 0$ implies that there exist $\breve{\gamma} < \beta$ and $\breve{\phi} > 0$ such that $(d/d\phi)\Pi^I > 0$ for $\phi < \breve{\phi}$ and $\gamma > \breve{\gamma}$. It follows that the result holds with $\breve{\eta} = \breve{\gamma}/\beta$.□

Let AW^I denote equilibrium agent welfare under own-wage sensitivity of AV supply $b_v > 0$; that is, AW^I denotes equilibrium agent welfare under access to individually-owned AVs. Although Proposition 5 addresses the case where $g_v = 0$, the proof establishes the result for the more general case where $g_v \in [0, b_v)$. Let AW^0 denote equilibrium platform profit under $b_v = g_v = 0$; that is, AW^0 denotes equilibrium agent welfare under no access to AVs.

Proof of Proposition 5: At the outset, note that it is straightforward to verify that $\lim_{b_v\to g_v} AW^I = AW^0$ for any $g_v \geq 0$, and define $\tilde{\phi} = 1/\tilde{b}_v$. It suffices to show: if $\gamma/\beta \geq g_l/b_l$, then $AW^I \geq \lim_{b_v\to g_v} AW^I$ for any $g_v \geq 0$ and $b_v \in (g_v, \infty)$; and if $\gamma/\beta < g_l/b_l$, then there exists $\tilde{b}_v > g_v$ such that $AW^I < \lim_{b_v\to g_v} AW^I$ if and only if $b_v \in (g_v, \tilde{b}_v)$. Note $(\partial/\partial b_v)AW^I = 2\alpha^2\beta^2(2b_l - g_l)[(b_v - g_v)^2 + b_v^2]\xi(b_v, g_v)/\zeta(b_v, g_v)^3$ for some functions ξ and ζ, where $\xi(b_v, g_v) = \beta b_v(2b_l - g_l)^2(b_v - g_v) + b_l(b_l - g_l)(2b_v - g_v)(\gamma b_l - \beta g_l)$. It is straightforward to verify algebraically that $\zeta(b_v, g_v) > 0$ for any $b_v \in (g_v, \infty)$; thus $(\partial/\partial b_v)AW^I < 0$ if and only if $\xi(b_v, g_v) < 0$. We consider two cases: $\gamma/\beta \geq g_l/b_l$ and $\gamma/\beta < g_l/b_l$. Case 1: $\gamma/\beta \geq g_l/b_l$. It is straightforward to verify that $\gamma/\beta \geq g_l/b_l$ implies $\xi(b_v, g_v) > 0$ for any $g_v \geq 0$ and $b_v \in (g_v, \infty)$. Hence $\gamma/\beta \geq g_l/b_l$ implies $(\partial/\partial b_v)AW^I > 0$ for any $g_v \geq 0$ and $b_v \in (g_v, \infty)$. It follows that $\lim_{b_v\to g_v} AW^I < AW^I$ for any $g_v \geq 0$ and $b_v \in (g_v, \infty)$. Case 2: $\gamma/\beta < g_l/b_l$. Note $\lim_{b_v\to g_v}\xi(b_v, g_v) = b_l g_v(b_l - g_l)(\gamma b_l - \beta g_l) < 0$, where the inequality holds because $\gamma/\beta < g_l/b_l$. Further, $\lim_{b_v\to\infty}\xi(b_v, g_v) = \infty > 0$ and $(d^2/db_v^2)\xi(b_v, g_v) = 2(2b_l - g_l)^2\beta > 0$. It follows that there exists $\hat{b}_v > g_v$ such that $\xi(b_v, g_v) < 0$ if and only if $b_v \in (g_v, \hat{b}_v)$, and thus $(d/db_v)AW^I < 0$ if and only if $b_v \in (g_v, \hat{b}_v)$. It follows that there exists $\tilde{b}_v > g_v$ such that $AW^I < \lim_{b_v\to g_v} AW^I = AW^0$ if and only if $b_v \in (g_v, \tilde{b}_v)$.□

Let SW^I denote equilibrium social welfare under $b_v > 0$; that is, SW^I denotes equilibrium social welfare under access to individually-owned AVs. Let SW^0 denote equilibrium platform profit under $b_v = g_v = 0$; that is, SW^0 denotes equilibrium social welfare under no access to AVs.

Proof of Proposition 6: Note that $(d/db_v)SW = 2\alpha^2\beta^2(2b_l - g_l)\vartheta/\{2(2b_l-g_l)\beta(\beta-\gamma)+[2b_l(b_l - g_l)+b_v(2b_l-g_l)](2\beta-\gamma)\}^3$, where $\vartheta = 2\beta[4b_l^2(b_l-g_l)+(2b_l-g_l)^2(2b_v+\beta)]-\gamma[(2b_l-g_l)^2(3b_v+2\beta)+2b_l(b_l-g_l)(4b_l-g_l)]$. Note ϑ is strictly decreasing in γ and $\lim_{\gamma\to\beta}\vartheta = \beta[b_v g_l^2+2b_l(b_l-g_l)(2b_v+g_l)] > 0$. The result follows. $\square$

Appendix D: Proofs for Extensions: Non-Linear Platform AV Cost, Platform- and Individually-Owned AVs, and Competition over Individually-Owned AV

Proof of Lemma 3: (i) Because the AV cost function $c_k(K)$ is weakly convex and strictly increasing, uniqueness of the symmetric equilibrium follows by Lemma 9, and $(d/d\theta)K^* < 0$ for $K^* > 0$ follows by Lemma 10.

(ii). Similar to Lemma 9, we refer to $K_i \in [0, \alpha/(2-\gamma/\beta)]$ for $i \in \{1,2\}$ as the *truncated strategy space* and $K_i \in [0,\infty)$ for $i \in \{1,2\}$ as the *full strategy space.* The proof proceeds in three steps. First, we first show that for any $K_j \geq 0$ and $\theta \in (0,\theta_m)$, the profit function $\Pi_i(\mathbf{K})$ is strictly quasi-concave in K_i on $K_i \in [0,\infty)$, for $i \neq j$. Second, we show that there exists a symmetric equilibrium on the truncated strategy space. Third, we show that this is the only symmetric equilibrium on the full strategy space.

Step 1: Fix $K_j \geq 0$ and $\theta \in (0,\theta_m)$. We consider two cases: $K_i \in [0,\bar{\bar{K}}_i)$ and $K_i \in [\bar{\bar{K}}_i,\infty)$. Suppose $K_i \in [0,\bar{\bar{K}}_i)$. Then

$$\begin{aligned}(\partial^2/\partial K_i^2)\Pi_i(\mathbf{K}) &= (\partial^2/\partial K_i^2)r_i(\mathbf{K}) - \theta(\partial^2/\partial K_i^2)c_k(K_i)\\ &< (\partial^2/\partial K_i^2)r_i(\mathbf{K}) - \theta_m(\partial^2/\partial K_i^2)c_k(K_i)\\ &= (\partial^2/\partial K_i^2)r_i(\mathbf{K}) - [(\partial/\partial K_i)r_i(\mathbf{K})/(\partial/\partial K_i)c_k(K_i)]|_{K_i=K_j=0}(\partial^2/\partial K_i^2)c_k(K_i)\\ &< (\partial^2/\partial K_i\partial K_j)r_i(\mathbf{K})\\ &\leq 0.\end{aligned}$$

The first line follows by definition of $\Pi_i(\mathbf{K})$. The second line follows because $\theta < \theta_m$ and $(\partial^2/\partial K_i^2)c_k(K_i) < 0$. The third line follows by definition of θ_m. The fourth line follows from inequality (10), and the fifth line follows from Lemma 6. Therefore, $(\partial^2/\partial K_i^2)\Pi_i(\mathbf{K}) < 0$ for all $K_i \in [0,\bar{\bar{K}}_i)$. Next, suppose $K_i \in [\bar{\bar{K}}_i,\infty)$. Because $(\partial^2/\partial K_i^2)r_i(\mathbf{K}) = 0$ (by Lemma 6), $\theta > 0$, and $c_k(K_i)$ is strictly increasing, $\Pi_i(\mathbf{K})$ is strictly decreasing in K_i on $K_i \in [\bar{\bar{K}}_i,\infty)$. Because $\Pi_i(\mathbf{K})$ is strictly concave on $K_i \in [0,\bar{\bar{K}}_i)$ and strictly decreasing on $K_i \in [\bar{\bar{K}}_i,\infty)$, it follows that $\Pi_i(\mathbf{K})$ is quasi-concave in K_i on $K_i \in [0,\infty)$.

Step 2: Because $\Pi_i(\mathbf{K})$, $i \in \{1,2\}$ are quasi-concave, the profit functions are symmetric, and the truncated strategy space is compact and convex, there exists at least one symmetric equilibrium on the truncated strategy space (Cachon and Netessine 2004).

Step 3: To show that there is at most one symmetric equilibrium on the truncated strategy space, it suffices to show that the magnitude of the slopes of the best response functions $\tilde{K}_i(K_j)$, $i = \{1,2\}$ are strictly less than one everywhere on the truncated strategy space (Cachon and Netessine 2004). By the implicit function theorem, the slope of $\tilde{K}_i(K_j)$ is given by $|(d/dK_j)\tilde{K}_i(K_j)| = |[(\partial^2/\partial K_i\partial K_j)r_i(\mathbf{K})]/[(\partial^2/\partial K_i^2)r_i(\mathbf{K}) - \theta(\partial^2/\partial K_i^2)c_k(K_i)]_{K_i=\tilde{K}_i(K_j)}|$. Because $\tilde{K}_i(K_j) < \bar{\bar{K}}_i$ and $\theta < \theta_m$, it can be verified that inequality (10) implies $|[(\partial^2/\partial K_i\partial K_j)r_i(\mathbf{K})]/[(\partial^2/\partial K_i^2)r_i(\mathbf{K}) -$

$\theta(\partial^2/\partial K_i^2)c_k(K_i)]_{K_i=\tilde{K}_i(K_j)}| < 1$. Therefore, there exists exactly one symmetric equilibrium on the truncated strategy space, $K_i \in [0, \alpha/(2-\gamma/\beta)]$. It can be shown using a parallel argument to the proof of Lemma 9 that there cannot exist a symmetric equilibrium K^* where $K^* > \alpha/(2-\gamma/\beta)$. We conclude that there exists exactly one symmetric equilibrium on the full strategy space $K_i \in [0, \infty)$. Lastly, to see that $(d/d\theta)K^* < 0$, note that the proof of Lemma 10, which establishes this inequality, continues to hold when $c_k(K)$ is not weakly convex, provided that $(\partial^2/\partial K_j^2)\Pi_j + (\partial^2/\partial K_i \partial K_j)\Pi_i < 0$ holds at $K_i = K_j = K^*$. It remains to show the preceding inequality holds at $K_i = K_j = K^*$ under inequality (10). First, note that by Lemma 8, $D_j(\mathbf{p}^*(\mathbf{K})) > K_j$ at $K_i = K_j = K^*$, which implies $K^* < \bar{\bar{K}}_j$ by definition of $\bar{\bar{K}}_j$. Because $K^* < \bar{\bar{K}}_j$, and $(\partial^2/\partial K_j^2)\Pi_j < 0$ for $K_j < \bar{\bar{K}}_j$ by Step 1 of this proof, $(\partial^2/\partial K_j^2)\Pi_j < 0$ at $K_i = K_j = K^*$. Next, note $(\partial^2/\partial K_i \partial K_j)\Pi_i = (\partial^2/\partial K_i \partial K_j)r_i \leq 0$ at $K_i = K_j = K^*$, where the equality follows from the definition of Π_i, and the inequality follows from Lemma 6.□

Proof of Proposition 7: We prove the statements in order. (i). Define $\tilde{\pi}(\theta) = \pi(\theta) + c_k(0)$. Because $(d/d\theta)K^*(\theta) < 0$ for $K^* > 0$ by Assumption 1, it follows by parallel argument to Step 1 of the proof of Proposition 1 that $\lim_{\theta \uparrow \theta_m}(d/d\theta)\tilde{\pi}(\theta) > 0$ if and only if $\gamma/\beta > g/b$. Suppose $\gamma/\beta > g/b$. It follows from Step 2 of the proof of Proposition 1 that there exists $\bar{\theta} < \theta_m$ such that $\tilde{\pi}(\theta) < \Pi^0$ for all $\theta \in (\bar{\theta}, \theta_m)$. Because $c_k(0) \geq 0$, $\pi(\theta) \leq \tilde{\pi}(\theta)$. The result follows.

(ii). The proof of Proposition 3 continues to hold when the assumption that $c_k(K) = K$ is replaced by Assumption 1, which ensures that $(d/d\theta)K^*(\theta) < 0$.

(iii). Define $\tilde{sw}(\theta) = sw(\theta) + 2c_k(0)$. Because $(d/d\theta)K^*(\theta) < 0$ for $K^* > 0$ by Assumption 1, it follows by parallel argument to Step 1 of the proof of Proposition 3 that $\lim_{\theta \uparrow \theta_m}(d/d\theta)\tilde{sw}(\theta) < 0$ if and only if $\gamma/\beta > g/b$. It follows from Step 2 of the proof of Proposition 3 that if $\gamma/\beta > g/b$, then there exists $\hat{\theta} < \theta_m$ such that $\tilde{sw}(\theta) < SW^0$ for all $\theta \in (\hat{\theta}, \theta_m)$. Because $c_k(0) \geq 0$, $sw(\theta) \leq \tilde{sw}(\theta)$. The result follows. □

Proof of Proposition 8: We prove the statements in order. (i) Let $\pi(\theta, b_v)$ denote equilibrium platform profit under (θ, b_v), where $b_v > 0$. This implies: $\Pi^I = \pi(\theta_m(1/b_v), b_v)$; and $\Pi^M = \pi(\theta, b_v)$ for $\theta < \theta_m(1/b_v)$. Suppose $\gamma/\beta > \underline{\eta}$. Then, because $\phi = 1/b_v$, by Proposition 4 there exists $\bar{b}_v > 0$ such that $\pi(\theta_m(1/b_v), b_v) < \Pi^0$ for all $b_v > \bar{b}_v$. Fix $b_v > \bar{b}_v$. By parallel argument to the proof of Lemma 11, it can be shown that $\pi(\theta, b_v)$ is strictly convex in θ for all $b_v \geq 0$. Because $\pi(\theta_m(1/b_v), b_v) < \Pi^0$, $\pi(\theta, b_v)$ is strictly convex in θ, and Π^0 is invariant to θ, there exists $\bar{\theta} < \theta_m(1/b_v)$ such that $\pi(\theta, b_v) < \Pi^0$ if and only if $\theta \in (\underline{\theta}, \theta_m(1/b_v))$. The result follows by setting $\bar{\phi} = 1/\bar{b}_v$ and noting $\theta_m(1/b_v) = \theta_m(\phi)$.

(ii). Let $AW(K, K, b_v)$ denote equilibrium agent welfare under (K, K) and b_v, where $b_v > 0$. This implies: $AW^I = AW(0, 0, b_v)$; and $AW^M = AW(K^*(\theta), K^*(\theta), b_v)$ for $\theta < \theta_m(1/b_v)$. Suppose $\gamma/\beta < g_l/b_l$. Then by Proposition 5, there exists $\tilde{b}_v > 0$ such that $AW(0, 0, b_v) < AW^0$ for all $b_v < \tilde{b}_v$. Next, it can be verified algebraically that $AW(K, K, b_v)$ is strictly convex in K for any $b_v \geq 0$. Fix $b_v < \tilde{b}_v$. Because $AW(0, 0, b_v) < AW^0$, $AW(K, K, b_v)$ is strictly convex in K, and AW^0 is invariant to K, there exists $\bar{K} > 0$ such that $AW(K, K, b_v) < AW^0$ if and only if $K \in (0, \bar{K})$. Next, it can be shown by parallel argument to Lemma 10 that $K^*(\theta)$ is strictly decreasing in θ on $\theta \in (0, \theta_m(1/b_v))$. It follows that there exists $\bar{\theta} < \theta_m(1/b_v)$ such that $K^*(\theta) \in (0, \bar{K})$ if and only if $\theta \in (\bar{\theta}, \theta_m(b_v))$. Therefore, $AW(K^*(\theta), K^*(\theta), b_v) < AW^0$ if and only if $\theta \in (\bar{\theta}, \theta_m(b_v))$. The result follows by letting $\tilde{\phi} = 1/\tilde{b}_v$ and noting $\theta_m(1/b_v) = \theta_m(\phi)$.

(iii). Let $sw(\theta, b_v)$ denote equilibrium social welfare under (θ, b_v). Define $\theta'_m = \lim_{b_v \to 0} \theta_m(1/b_v)$ and $\theta^-(b_v) = \arg\inf_\theta \, sw(\theta, b_v)$. Note $SW^P = sw(\theta, 0)$ for $\theta < \theta'_m$, $SW^I = sw(\theta_m(1/b_v), b_v)$ for $b_v > 0$, and $SW^M = sw(\theta, b_v)$ for $b_v > 0$ and $\theta < \theta_m(1/b_v)$. Suppose $\gamma/\beta > g_l/b_l$ and $b_v = 0$. Then by Proposition 3, there exists $\hat{\theta} < \theta'_m$ such that $sw(\theta, 0) < SW^0$ for all $\theta \in (\hat{\theta}, \theta'_m)$. It follows that $sw(\theta^-(0), 0) < SW^0$. By parallel argument to the proof of Lemma 11, it can be shown that $sw(\theta, b_v)$ is strictly convex in θ. Because $sw(\theta^-(0), 0) < SW^0$, $sw(\theta, b_v)$ is strictly convex in θ, and $sw(\theta'_m, 0) = SW^0$, (by definition of θ'_m), we have $\theta^-(0) < \theta'_m$. Because $\theta^-(0) < \theta'_m$, by continuity of $\theta^-(b_v)$ and $\theta_m(b_v)$ in b_v there exists $\hat{b}_v > 0$ such that $sw(\theta^-(b_v), b_v) < SW^0$ and $\theta^-(b_v) < \theta_m(1/b_v)$ for all $b_v < \hat{b}_v$. Select any $b_v < \hat{b}_v$. Because $sw(\theta^-(b_v), b_v) < SW^0$, $sw(\theta, b_v)$ is strictly convex in θ, and SW^0 is invariant to θ, there exists $\underline{\theta} < \theta^-(b_v)$ and $\underline{\theta} > \theta^-(b_v)$ such that $sw(\theta, b_v) < SW^0$ if and only if $\theta \in (\underline{\theta}, \bar{\theta})$. Note that $\underline{\theta} < \theta_m(1/b_v)$ because $\underline{\theta} < \theta^-(b_v)$ and $\theta^-(b_v) < \theta_m(1/b_v)$. Further, because $sw(\theta_m(1/b_v), b_v) > SW^0$ by Proposition 6, it must be that $\bar{\theta} < \theta_m(1/b_v)$. The result follows by letting $\hat{\phi} = 1/\hat{b}_v$ and noting $\theta_m(1/b_v) = \theta_m(\phi)$. □

Proof of Proposition 9: We prove the statements in order. (i) Let Π^I denote equilibrium platform profit under $b_v > 0$ and $g_v \in [0, b_v)$; that is, Π^I represents equilibrium profit under access to AVs. Let Π^0 denote equilibrium platform profit under $b_v = g_v = 0$; that is, Π^0 represents equilibrium profit under no access to AVs. We show that if $\gamma/\beta > g_l/b_l$, then there exists $\bar{b}_v > g_v$ such that $\Pi^I < \Pi^0$ for all $b_v < \bar{b}_v$, where we define $\bar{\phi} = 1/\bar{b}_v$. Suppose $\gamma/\beta > g_l/b_l$. Note $\lim_{b_v \to g_v}(\partial/\partial b_v)\Pi^I = (\gamma b_l - \beta g_l) \cdot \xi(\beta, \gamma, g_l, b_l)$, where $\xi(\beta, \gamma, g_l, b_l) = -2\alpha^2\beta^2 b_l(b_l - g_l)(2b_l - g_l)/\left[\beta(\beta - \gamma)(2b_l - g_l) + (2\beta - \gamma)b_l(b_l - g_l)\right]^3$. Because $\xi(\beta, \gamma, g_l, b_l) < 0$, $\gamma/\beta > g_l/b_l$ implies that $\lim_{b_v \to g_v}(\partial/\partial b_v)\Pi^I < 0$. By continuity of $(\partial/\partial b_v)\Pi^I$ in b_v, for any $g_v \geq 0$ there exists $\bar{b}_v > g_v$ such that $(\partial/\partial b_v)\Pi^I < 0$ if $b_v \in (g_v, \bar{b}_v)$. Further, it is straightforward to verify that $\lim_{b_v \to g_v} \Pi^I = \Pi^0$. It follows that $\Pi^I < \Pi^0$ for all $b_v \in (g_v, \bar{b}_v)$. Because $\bar{\phi} = 1/\bar{b}_v$, $\phi_m = 1/g_v$ and $\bar{b}_v > g_v$, it follows that $\bar{\phi} < \phi_m$.

(ii). The proof is identical to that of Proposition 5, which considers the case where $g_v \in [0, b_v)$.

(iii). It can be verified algebraically that $\lim_{b_v \to g_v}(\partial/\partial b_v)SW^I = \lim_{b_v \to g_v}(\partial/\partial b_v)\Pi^I$ and $\lim_{b_v \to g_v} SW^I = SW^0$. The remainder of the proof follows by parallel argument to the proof of part (i), with SW^I in place of Π^I and SW^0 in place of Π^0. □

Appendix E: Derivation of Consumer Surplus and Agent Welfare

Consumer surplus. The representative consumer chooses (D_1, D_2) to maximize her utility $\tau D_1 + \tau D_2 - (\chi D_1^2 + 2\mu D_1 D_2 + \chi D_2^2)/2 - (p_1 D_1 + p_2 D_2)$. Using the first order conditions of the consumer utility function, platform i's inverse demand function can be written as $p_i = \tau - \chi D_i - \mu D_j$ for $i \in \{1, 2\}$ and $j \neq i$. Let $\alpha = \tau(\chi - \mu)/(\chi^2 - \mu^2) = \tau/(\chi + \mu)$, $\beta = \chi/(\chi^2 - \mu^2)$ and $\gamma = \mu/(\chi^2 - \mu^2)$. Rearranging yields platform i's demand (1) for $i \in \{1, 2\}$ and $j \neq i$. Under the symmetric equilibrium prices $p_1^* = p_2^* = p^*$, platform i's demand is $D_i(\mathbf{p}^*)$ for $i \in \{1, 2\}$ and the consumer's utility (equivalently, consumer surplus) is $CS = 2(\tau - p^*)D_i(\mathbf{p}^*) - (\chi + \mu)D_i(\mathbf{p}^*)^2$. Further, note $a = \alpha/(\beta - \gamma)$, $\chi = \beta/(\beta^2 - \gamma^2)$ and $\mu = \gamma/(\beta^2 - \gamma^2)$. Therefore, $CS = 2[\alpha/(\beta - \gamma) - p^*]D_i(\mathbf{p}^*) - D_i(\mathbf{p}^*)^2/(\beta - \gamma)$. Noting that $D_i(\mathbf{p}^*) = \alpha - (\beta - \gamma)p^*$ and simplifying further yields $CS = D_i(\mathbf{p}^*)^2/(\beta - \gamma)$.

Labor welfare. The representative worker chooses (L_1, L_2) to maximize her utility $w_{l,1}L_1 + w_{l,2}L_2 - (xL_1^2 + 2mL_1L_2 + xL_2^2)/2$. Using the first order conditions of the worker utility function, platform i's inverse supply function can be written as $w_{l,i} = xL_i + mL_j$ for $i \in \{1,2\}$ and $j \neq i$. Let $b_l = x/(x^2 - m^2)$ and $g_l = m/(x^2 - m^2)$. Rearranging yields platform i's labor supply (2) for $i \in \{1,2\}$ and $j \neq i$. Under the symmetric equilibrium wages $w_{l,1}^* = w_{l,2}^* = w_l^*$, platform i's labor supply is $L_i(\mathbf{w}_l^*)$ for $i \in \{1,2\}$ and the worker's utility (equivalently, labor welfare) is $LW = 2w_l^* L_i(\mathbf{w}_l^*) - (x+m)L_i(\mathbf{w}_l^*)^2$. Further, note $x = b_l/(b_l^2 - g_l^2)$ and $m = g_l/(b_l^2 - g_l^2)$. Therefore $LW = 2w_l^* L_i(\mathbf{w}_l^*) - L_i(\mathbf{w}_l^*)^2/(b_l - g_l)$. Noting that $L_i(\mathbf{w}_l^*) = (b_l - g_l)w_l^*$ and simplifying further yields $LW = L_i(\mathbf{w}_l^*)^2/(b_l - g_l)$.