Platform Disintermediation: Information Effects and Pricing Remedies

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Two-sided platforms typically generate revenue by matching prospective buyers and sellers and extracting commissions from completed transactions. Disintermediation, where sellers transact offline with buyers to bypass commission fees, can undermine the viability of these platforms. While transacting offline allows sellers to avoid commission fees, it also leaves them fully exposed to risky buyers given the absence of the platform's protections. In this paper, we examine how disintermediation and information quality – specifically, the accuracy of the signal sellers receive about a buyer's type – jointly impact a platform's revenue and optimal commission rate. In a setting where transactions occur online-only, an increase in information quality leads sellers to set more efficient prices and complete more transactions, which lifts platform revenue. However, if sellers can transact offline, this effect may be reversed – additional information about buyers can hurt platform revenue by reducing the risk sellers face offline, amplifying disintermediation. Further, while intuition suggests platforms should counter disintermediation by lowering commission rates, in a high-information environment a platform may be better off raising them. Lastly, while charging sellers platform-access fees can hedge against losses from disintermediation, it can fall short of the revenue attained under commission-based pricing when all transactions occur online. Overall, our findings provide insight into the mechanisms through which disintermediation disrupts platform operations and offers prescriptions to platforms seeking to counteract it.

1. Introduction

Online platforms that generate revenue through commission fees are vulnerable to *disintermediation*, where buyers and sellers matched by a platform transact off-platform to avoid paying the commission. Disintermediation can lead to significant revenue losses – the talent outsourcing platform ZBJ estimates that up to 90% of their service providers' transactions may occur off-platform (Zhu et al. 2018). In extreme cases, disintermediation can threaten the viability of the platform itself – for example, the demise of home-cleaning platform HomeJoy in 2015 has been partly attributed to disintermediation (Farr 2015). These risks are well-recognized by platforms: Airbnb explicitly warns hosts of buyers attempting to pay through alternative channels (Airbnb 2023b), and the freelance platform Upwork encourages users to report attempts at circumvention (UpWork 2023a). Although disintermediation is difficult to detect, there is growing empirical evidence of it occurring on multiple platforms (Lin et al. 2022, Karacaoglu et al. 2022, Gu 2022).

For sellers, the attractiveness of disintermediation depends on multiple features of the platform environment. Naturally, the commission rate the platform imposes on sellers may play a major role in their inclination to transact off-platform, as it can amount to a substantial share of sellers' earnings.[1] Although disintermediation allows sellers to bypass commission fees, it also entails giving up the benefits of transacting on the platform, such as insurance, receiving reviews, and the convenience of a digital transaction. Crucial among these services are the platform's policies that shield sellers from risky or even fraudulent behavior by buyers. Such protections are commonplace: Airbnb insures hosts against property damage by guests (Airbnb 2023a), Upwork holds payments in escrow to safeguard freelancers (UpWork 2023c), and eBay protects sellers against various forms of buyer fraud (eBay 2023). In deciding whether to disintermediate, sellers must therefore weigh the benefits of avoiding commission fees against full exposure to risky buyers.

Whether a seller decides to transact off-platform depends on their assessment of buyer riskiness. In most online marketplaces, the level of trust between a seller and buyer depends on the *quality* of information sellers obtain about buyers via the platform. Thus, to encourage on-platform transactions, many platforms include communication tools and reputation systems for both buyers and sellers. However, high information quality can also improve sellers' ability to screen risky buyers (Jin et al. 2018), diminishing the value of the platform's protections. Under the threat of disintermediation, the directional impact of information on platform revenue is therefore unclear – while to some extent necessary to facilitate on-platform transactions, high information quality may also increase the attractiveness of circumventing the platform entirely (Gu and Zhu 2021).

The quality of information that sellers obtain about buyers, whether through reputation systems or direct communication, varies by platform and context. Ratings may be unreliable or prone to inflation, reducing their usefulness in differentiating users (Nosko and Tadelis 2015), and users may also be imperfect in their ability to interpret ratings (Tadelis 2016). Additionally, communication-related policies differ across platforms: Airbnb algorithmically blocks email addresses and phone numbers in their on-platform chat until bookings are confirmed, while Upwork prohibits sharing contact information, but does not block it. As a consequence, vulnerability to disintermediation may vary across platforms depending on their choice of information architecture. Therefore, a key question is how platforms that are situated differently with respect to the level of information available to their sellers perform under the threat of disintermediation.

How should platforms respond to disintermediation? Reducing commission rates may encourage on-platform transactions, but may also needlessly sacrifice revenue if some degree of disintermediation is inevitable. Fundamentally, disintermediation poses a challenge to commission-based

[1] For example, Amazon charges sellers a 'referral fee' between 8% and 15% for most product categories, Airbnb hosts pay a commission of 14-16% under a host-only fee structure, and Upwork charges a commission rate between 5% and 20%, depending on transaction volume.

platforms due to a misalignment between the platform's value proposition (connecting sellers to buyers) and its pricing strategy (charging for completed transactions). Recognizing this gap, some platforms eschew commission fees and instead charge sellers for *access* to buyers – for example, the homeservices platform Thumbtack charges sellers for inquiries from potential buyers ("leads") (Thumbtack 2023), and caregiver platform Care.com charges service providers for the ability to exchange messages with prospective clients (Care.com 2023). Clearly, alternatives to commission fees such as charging sellers for access to buyers reduces the incentive to disintermediate, but their revenue implications are less clear as some sellers may be unwilling to pay upfront. More importantly, the efficacy of a given commission rate or pricing strategy may also depend on information quality, as a consequence of its effect on disintermediation.

Contributions. This paper examines how disintermediation and information quality jointly impact a platform's revenue, optimal commission rate, and choice of pricing strategy. In our model, heterogenous sellers set their prices in an *online* transaction channel prior to being matched to a buyer. The platform charges the seller a fixed fraction of the price if the transaction is completed in the online channel. Alternatively, a seller may attempt to bypass the commission by negotiating an off-platform price with the buyer and completing the transaction in an *offline* channel if doing so is mutually beneficial. Buyers' types are private information; in particular, "risky" buyers impose higher transaction costs on sellers in both channels, and additionally withhold payment in the offline channel. To capture information quality, we assume the platform has a technology that generates, with varying degrees of accuracy, a noisy signal of the buyer's type, which the seller observes after selecting their online price and prior to their choice of the transaction channel.

Our analysis focuses on the following three questions:

1. How does the threat of disintermediation (i.e., the presence of an offline transaction channel) alter the influence of information quality on platform revenue?
2. How does the threat of disintermediation impact the platform's optimal commission rate?
3. Under what conditions can platforms recover the revenue lost to disintermediation by switching from commission-based to access-based pricing?

This agenda complements an extensive body of empirical work on disintermediation that has emerged in recent years (Gu and Zhu 2021, Karacaoglu et al. 2022, Astashkina et al. 2022, He et al. 2020, Gu 2022). In particular, Gu and Zhu (2021) show using experiments that the presence of high quality information on a platform leads to more transactions, but simultaneously increases the likelihood of disintermediation. We further explore this trade-off by studying how information quality affects the platform's revenue and commission rate, and also analyze the efficacy of alternative pricing strategies proposed in the literature (Edelman and Hu 2016).

Our results are summarized in Table 1. First, in the absence of the offline channel, platform revenue weakly increases in information quality under any fixed commission rate. This occurs because an increase in information quality improves sellers' ability to screen buyers, which both increases transaction volume and leads sellers to set more efficient prices online. However, the threat of disintermediation can reverse this behavior: the presence of the offline channel can make platform revenue strictly *decrease* in information quality. The intuition is that when sellers can transact in an offline channel, an increase in information quality lowers the risk sellers face from disintermediating, leading to more offline transactions and an erosion of platform revenue.

Second, one might assume naively that the prospect of disintermediation would compel the platform to lower its commission rate. In some cases, however, it may be in the platform's interest to "double down" and *increase* its commission rate in response to users transacting offline. This result is a consequence of which transactions the platform chooses to capture value from. Specifically, in the presence of the offline channel, the platform must decide whether to prevent disintermediation entirely, which requires restricting its commission rate, or to permit disintermediation for some transactions and maximize revenue from those that remain online, which frees the platform to increase its commission rate. We provide a precise characterization of when the latter strategy is optimal.

Third, we examine the efficacy of *access-based pricing*, in which sellers are charged upfront to join the platform instead of paying commission fees. In contrast to commissions, access fees are robust to the potentially revenue-decreasing effect of information: platform revenue weakly increases in information quality, regardless of whether sellers can disintermediate. However, in a setting where disintermediation cannot occur, access fees can fall short of the revenue attainable under commissions, suggesting that upfront pricing alone cannot fully recoup revenue losses from disintermediation.

Our findings highlight how the threat of disintermediation can alter the role of information in platform operations. Conventional wisdom suggests that an open and information-rich environment can foster trust in the marketplace, and as a result, boost on-platform transactions (Resnick and Zeckhauser 2002, Tadelis 2016). However, when disintermediation is rife, additional information can backfire by encouraging buyers and sellers to transact off-platform, which has implications for a platform's revenue, choice of commission rate, and the pricing strategy itself. Unsurprisingly, our results suggest that platforms that do not account for disintermediation in their design choices can end up in outcomes that are highly sub-optimal.

1.1. Related Work

Disintermediation. In its most general sense, disintermediation refers to the circumvention of market intermediaries, and has been studied in a number of contexts, including supply

<table>
<tr><th></th><th>No Disintermediation</th><th>With Disintermediation</th></tr>
<tr><td>For fixed commission, platform revenue...</td><td>increases with α</td><td>decreases in α on $[\alpha_1, 1]$</td></tr>
<tr><td rowspan="2">Optimal commission...</td><td>increases in α</td><td>decreases in α on $[\frac{1}{2}, \alpha_2]$ if there are enough high-quality sellers</td></tr>
<tr><td colspan="2">is weakly higher under threat of disintermediation if $\alpha > \alpha_3$ and there are enough high-quality sellers</td></tr>
<tr><td>Access fees generate...</td><td>higher revenue than commissions if seller earnings are “balanced”</td><td>lower revenue than commissions</td></tr>
</table>

Table 1. Summary of results describing effect of information quality (α) on platform revenue and optimal commission rate with and without threat of disintermediation.

chains (Ritchie and Brindley 2000, Federgruen and Hu 2016). Our work bears some similarity to prior work on the cost-benefit trade-off of intermediation in supply chains and other operational settings (e.g., Agrawal and Seshadri (2000), Belavina and Girotra (2012)), although our focus is on platforms that mediate transactions between individual users, rather than firms.

Recently, there is a growing recognition of the threat posed by disintermediation (or *platform leakage*) to a variety of two-sided platforms[2]. For the most part, the extant literature on platform disintermediation is empirical, and uses novel identification approaches to quantify this phenomenon (He et al. 2020, Gu and Zhu 2021, Gu 2022, Astashkina et al. 2022, Karacaoglu et al. 2022, Lin et al. 2022). For example, Gu and Zhu (2021) use a randomized control trial to find evidence of disintermediation on a large outsourcing platform, Karacaoglu et al. (2022) use data from a home cleaning platform and estimate that the platform loses out on 24% of potential transactions due to disintermediation, and Lin et al. (2022) find the rate of disintermediation on Airbnb to be around 5.4% based on data from Austin, Texas. Our work is especially related to Gu and Zhu (2021), who find that providing more information about freelancer quality positively impacts the volume of transactions, but also increases the likelihood of disintermediation. Our model provides analytical support for the empirical results in Gu and Zhu (2021), and further sheds light on the impact of information quality on a platform's revenue and optimal commission rate.

On the modeling side, our paper builds on the framework developed in He et al. (2023) and complements their findings; we briefly outline the key differences. First, while He et al. (2023) study the causes of disintermediation, we are primarily interested in the role of information quality and how it influences platform operations when sellers have access to an offline transaction channel. Second, their paper investigates the impact of risky sellers underdelivering in two settings— when platforms provide perfect information or no information at all. In contrast, the risk in our model

[2] Edelman and Hu (2016) and Karacaoglu et al. (2022) outline the impact of disintermediation on a number of platforms, including Uber, Rover, Upwork, Handy, eHarmony, and Zeel.

stems from buyers reneging on payment, and we model information as a continuous parameter, which generates additional insights. Finally, He et al. (2023) propose a number of mechanisms that platforms can implement to avoid disintermediation (e.g., upskilling its sellers). We add to this discussion by studying pricing as an instrument to counteract disintermediation. Hagiu and Wright (2022) also present a model for platform disintermediation, although their setting differs in a few notable ways: there is no private information on either side of the market, buyers are homogenous and have zero bargaining power over the off-platform price, and sellers face no risks from transacting offline.

Information disclosure in platforms. Our paper is related to a growing literature on how information influences the decisions of platform users, which has consequences for social welfare or platform revenue. Papanastasiou et al. (2018) show that strategically withholding information from consumers can induce exploration of new or alternative products in a manner that ultimately improves consumer surplus; similarly, Gur et al. (2023) consider how information can be used as a lever to influence sellers' prices, also with the aim of improving consumer surplus. In a similar vein, Kanoria and Saban (2020) show that matching markets (e.g., dating platforms) can improve welfare by hiding the quality of users. With respect to platform revenue, Bimpikis et al. (2020) and Shi et al. (2022) describe mechanisms through which mislabelling high quality sellers can benefit the platform, and Jin et al. (2018) and Johari et al. (2019) identify conditions under which it is revenue-maximizing for platforms to filter out low quality users. More generally, there is a burgeoning literature on information design in a variety of operational contexts (Bimpikis and Papanastasiou 2019, Bimpikis et al. 2019, Lingenbrink and Iyer 2019, Candogan and Drakopoulos 2020, Drakopoulos et al. 2021, Liu et al. 2021, Ma et al. 2021, Anunrojwong et al. 2022, Bimpikis and Mantegazza 2022). Our paper contributes to this literature by considering a new mechanism through which information shapes platform revenue, namely, disintermediation.

Reputation systems. The success of many online platforms can be partially attributed to reputation (i.e., review) systems that build trust among users and allow for efficient matching (Resnick and Zeckhauser 2002, Cabral and Hortacsu 2010, Shi et al. 2022). At a high level, these systems help overcome the information asymmetry between buyers and sellers on the platform, which leads to more transactions and prices that accurately reflect quality (Moreno and Terwiesch 2014). However, an increase in trust on the platform can reduce the perceived importance of the platform's services, including policies that protect sellers from risky buyers or fraud (Edelman and Hu 2016, Gu and Zhu 2021). Further, reputation systems may be noisy due to bias or rating inflation (Garg and Johari 2021, Filippas et al. 2022). Our work aims to capture this interplay between information, risk, and disintermediation, and is also motivated by a recognition that buyer-side reputation systems are crucial for enabling sellers to operate efficiently at scale, including in online labor markets (Benson et al. 2020) and the sharing economy broadly (Fradkin et al. 2021, Jin et al. 2018).

Commissions vs. upfront pricing. Platform designers often have a wide range of pricing instruments at their disposal, and characterizing the trade-offs between the different mechanisms is an active area of study. For instance, there is a rich literature on advance selling that predates online marketplaces (Xie and Shugan 2001, Randhawa and Kumar 2008, Cachon and Feldman 2011), and it is now well known that a monopolist firm can often extract more revenue from subscriptions than per-use pricing. However, in the case of two-sided platforms with heterogeneous users, the effectiveness of upfront pricing may suffer by excluding users who are uncertainty averse (Edelman and Hu 2016) or derive low utility from the platform (Birge et al. 2021, Cui and Hamilton 2022). As a consequence, commissions remain the *de facto* pricing strategy in most modern marketplaces.

Naturally, a number of papers have looked at how platforms should set these commissions and whether they should coupled with other mechanisms, e.g., a fixed fee (Benjaafar et al. 2019, Hu and Zhou 2020, Feldman et al. 2022, Cachon et al. 2022). For instance, Cachon et al. (2022) consider how pricing control (i.e., whether the platform or sellers set prices) impacts the performance of commission and per-unit fees, and show that a two-part tariff that combines them performs well in both centralized and decentralized marketplaces. More generally, Birge et al. (2021) identify conditions under which it is optimal for platforms to use subscriptions or commissions and argue that platforms can lose out on revenue by not charging payments from both sides of the market. Our paper contributes to this literature by examining how the threat of disintermediation influences both the optimal commission rate and the efficacy of upfront pricing. Our focus on access fees is also motivated by recent interest in the interplay between pricing, information, and manipulation on platforms (Belavina et al. 2020, Mostagir and Siderius 2022, Papanastasiou et al. 2022).

2. Model

We consider a platform with a unit mass of sellers. Each seller is one of two types according to their quality. Let H and L denote the high- and low-quality seller types, respectively, and suppose their qualities are given by $q_H > q_L > 0$. Further, let $\mu \in [0,1]$ be the fraction of high-quality sellers.

Seller quality is public information and all sellers earn a reservation profit of 0 off the platform. As in many two-sided marketplaces (e.g., online labor or rental platforms), we assume each seller chooses their own price p for online transactions, which depends on their quality.

Each seller is randomly matched to a buyer. Buyers are heterogeneous in their quality sensitivity, θ, where a buyer with sensitivity θ has valuation $v = \theta q$ for a quality-q seller. We assume θ is distributed uniformly in $[0,1]$ and that the distribution is common knowledge; for convenience we denote the uniform cdf by $F(\theta)$. Buyers can be one of two types: we use $\{r, s\}$ (i.e., "risky" or "safe") to denote the buyer types, which is private information. The buyer type influences sellers' costs in two ways. First, a type-s buyer imposes lower transaction costs on sellers both online and offline, $c_s < c_r$. For simplicity, we assume $c_s = 0$ and $c_r = c > 0$. Second, a type-s buyer pays sellers in full in both transaction channels, whereas a type-r buyer withholds the entirety of the payment if the transaction occurs offline.[3] A buyer is type-s with probability λ, which is known to sellers and the platform and is independent of the buyer's quality sensitivity θ. We focus on a setting where a minority of buyers are risky by assuming $\lambda \in [\frac{1}{2}, 1]$.

The platform selects a commission rate of $\gamma \in [0, \gamma^{\max}]$; for simplicity we set $\gamma^{\max} = \frac{1}{2}$.[4] For a transaction completed online at price p, the seller and platform receive $(1-\gamma)p$ and γp, respectively. The platform has a technology that generates a noisy signal σ of the buyer's type, where $\sigma \in \{r, s\}$. To reflect variability in information quality, we assume the signal correctly reveals the buyer's type with probability $\alpha \in [\frac{1}{2}, 1]$.[5] We assume α is exogenous and known to all parties.

Instead of transacting online at the seller's posted price p, a buyer and seller can jointly decide to transact offline at an alternative price, b. The offline price b is given by the Nash bargaining solution to a cooperative game, discussed below in §2.1.

Timeline. The sequence of events is as follows:

1. The platform sets the commission rate γ.
2. Each seller chooses their online price p.
3. Each seller is randomly matched to a buyer. After observing the seller's price p, the buyer decides whether to contact the seller to initiate a transaction. (If the buyer does not make contact, no transaction occurs.)
4. Each contacted seller observes a noisy signal σ of the buyer's type, updates their belief of the buyer's type, and decides whether to accept or reject the buyer. (If the seller rejects, no transaction occurs.)
5. If a seller accepts a buyer, both parties attempt to negotiate an offline price, b. The transaction occurs offline at price b if the negotiation succeeds; otherwise, the transaction occurs online at price p.

In the third step, we assume buyers are individually rational in that they only contact sellers to initiate a transaction if the online price yields non-negative utility for the buyer (i.e., $\theta q \geq p$). This reflects the process commonly observed in service platforms (e.g., TaskRabbit or Airbnb) in which sellers post prices and are contacted by buyers only if their online price is acceptable.

[3] Our results extend to a setting where transacting offline also incurs other costs for the seller, including the inconvenience of transacting offline or the risk of being banned by the platform.

[4] Our results continue to hold using $\gamma^{\max} = 1 - \epsilon$ for any $\epsilon > 0$ if Assumption 1 is modified to include $q_H \geq 2c/\epsilon$.

[5] The assumption that $\alpha \geq \frac{1}{2}$ is without loss of generality, because a signal with accuracy $\alpha < \frac{1}{2}$ is equivalent to one with accuracy $1 - \alpha$ with the buyer type flipped.

2.1. Offline Price Bargaining

We now formalize a simple bargaining game between a buyer and seller that determines whether they disintermediate. Consider a seller with online price p who has accepted a buyer's request to transact. The buyer and seller then engage in cooperative bargaining to identify a mutually beneficial offline channel price b, if such a price exists. We assume the offline price b is given by the symmetric Nash bargaining solution (Nash 1953, Binmore et al. 1986), which is the price that maximizes the product of the buyer's and seller's surpluses from disintermediation. The outcome if price negotiation fails (i.e., the disagreement point) is to transact online at price p, which is set by the seller prior to accepting the the buyer and is thus fixed at the time of negotiation.

To derive the offline price, we first consider the buyer and seller's surplus from disintermediation under an online-offline price pair (p, b), starting with the seller. Note the seller's expected payment online is simply $(1-\gamma)p$. Because type-r buyers renege on payment in the offline channel, the seller's expected payment offline depends on their belief of the buyer's type – accordingly, let $\eta_{|\sigma} = \Pr(j = s|\sigma)$ be the seller's posterior belief the buyer is type-s after observing the signal σ. The seller's expected payment offline is then $\eta_{|\sigma} b$. Further, because the seller incurs cost c from type-r buyers only, their expected cost in both channels is $(1-\eta_{|\sigma})c$. It follows that the seller's surplus from disintermediation is $\eta_{|\sigma} b - (1-\gamma)p$.

On the buyer side, a type-s buyer's payoffs in the online and offline channels are $\theta q - p$ and $\theta q - b$, respectively, meaning their surplus from disintermediation is $p - b$. Further, although buyers' types are private information, type-r buyers have no incentive to signal their type during negotiation, because doing so would preclude an offline transaction entirely. Type-r buyers therefore mimic type-s buyers during bargaining.

The product of the buyer and seller's surpluses from disintermediation is then

$$(p-b)(\eta_{|\sigma} b - (1-\gamma)p).$$

Note that the function above is strictly concave and quadratic in b. Solving for the unique maximizer yields the offline price, which we denote by $b_\sigma(p)$ to highlight the dependence on the signal σ and the online price p:

$$b_\sigma(p) = \frac{p(1-\gamma+\eta_{|\sigma})}{2\eta_{|\sigma}}.$$

The expression for $b_\sigma(p)$ aligns with intuition: for a *fixed* price p, a higher commission rate strengthen the buyer's bargaining position and produce a lower offline price. Further, because $\eta_{|s} > \eta_{|r}$, the offline price is higher for $\sigma = r$ buyers than $\sigma = s$ buyers, reflecting the increased risk assumed by the seller.

2.2. Model Discussion

We now briefly discuss our key modeling choices and their limitations.

Single-period and matching dynamics. We use a single-period model to capture interactions on the platform, meaning buyers and sellers are assumed to have not previously interacted, and sellers rely exclusively on the platform's signal to assess buyer risk. This assumption allows us to isolate how the quality of information obtained *via the platform* influences disintermediation, which is the focus of our work. In practice, sellers may also obtain additional information based on previous interactions with a buyer. Further, while repeated interactions between the same buyer and seller are common in online platforms, empirical evidence suggests that a large portion of transactions are due to first-time interactions (Astashkina et al. 2022, Lin et al. 2022), and that the risk of disintermediation exists even if the buyer and seller have not previously met (Gu and Zhu 2021). Further, because disintermediation occurs after buyers and sellers have already been matched, we abstract away details of the platform's matching algorithm and related congestion effects. As a consequence, each seller in our model is matched to a buyer with probability 1. This assumption is not critical to our analysis, and we conjecture our main results would continue to hold under more general matching dynamics.

Model of buyer risk. We assume $\lambda \geq \frac{1}{2}$ to focus on a functioning marketplace that is not overwhelmed with risky buyers. Additionally, our model assumes that risky buyers pose two challenges to sellers: first, they are more costly for sellers to transact with online, and second, they withhold payment when the transaction occurs offline. In other words, we assume buyers who pose off-platform risks (e.g., delays in payment or fraud) are also more difficult for sellers to transact with on-platform (e.g., by posing ill-specified tasks or communicating poorly). We note that all of our results hold in a more abstracted setting where instead of withholding payment, risky buyers simply pose large, additional costs on sellers in the offline channel. Naturally, sellers cannot always identify such buyers because their type is private information and the platform's signal is imperfect.

Observable seller quality. We assume each seller's quality is public information. This assumption aligns with the notion that sellers typically engage more frequently with the platform than buyers, which gives the platform higher accuracy information about sellers (vs. buyers) that it can pass on to buyers (e.g., in the form of reviews). As a consequence of sellers' qualities being observable, buyers in our model always agree to disintermediate if the offline price is lower than the online price. In practice, it is conceivable that buyers also face risks when transacting offline – for example, a service provider may underdeliver on the agreed quality. That said, our model serves as a reasonable approximation for settings where payment occurs *after* service is provided, which creates disincentives for sellers to underdeliver, and more generally makes disintermediation riskier for sellers than buyers.

2.3. Preliminary Analysis

We conclude this section by characterizing the sellers' profit function, the platform's revenue, and the conditions under which disintermediation occurs.

Seller profit. Consider a seller with quality q. Because θ is uniformly distributed over $[0,1]$ and the buyer has a payoff of $\theta q - p$ for an online transaction, only buyers with $\theta \geq p/q$ contact a quality-q seller. A quality-q seller's expected profit from an online transaction conditioned on observing the signal σ is then

$$\pi_\sigma(p) = \left[\left((1-\gamma)p - (1-\eta_{|\sigma})c\right) \cdot \bar{F}\left(\frac{p}{q}\right)\right]^+. \tag{1}$$

In the above expression, $[x]^+ = \min(x, 0)$, which reflects the seller's ability to reject the buyer upon observing σ and collect their reservation profit of 0.

For an offline transaction, the seller's payment is random: conditional on observing σ, they receive the bargained price $b_\sigma(p)$ with probability $\Pr(s|\sigma) = \eta_{|\sigma}$ and nothing with probability $\Pr(r|\sigma) = 1 - \eta_{|\sigma}$. If the buyer accepts price $b_\sigma(p)$ offline, the seller's expected profit from an offline transaction as a function of p is

$$\tilde{\pi}_\sigma(p) = \left[\left(\eta_{|\sigma} b_\sigma(p) - (1-\eta_{|\sigma})c\right) \cdot \bar{F}\left(\frac{p}{q}\right)\right]^+. \tag{2}$$

It can be shown that $\tilde{\pi}_\sigma(p) \geq \pi_\sigma(p)$ if and only if $b_\sigma(p) \leq p$, meaning the bargaining process produces an offline price that is either favorable for the buyer and seller or unfavorable to both. Intuitively, if disintermediation generates surplus for either party, it can always be shared with the other to ensure an offline transaction occurs. As a consequence, the seller's expected profit under an online price of p and the signal σ can be written as

$$\Pi_\sigma(p) = \max\{\pi_\sigma(p), \tilde{\pi}_\sigma(p)\}.$$

Finally, because the seller sets their price before observing the signal σ, their expected profit is

$$\Pi(p) = \Pr(\sigma = r) \cdot \Pi_r(p) + \Pr(\sigma = s) \cdot \Pi_s(p),$$

and their optimal price is

$$p^*(\alpha, q, \gamma) = \underset{p \geq 0}{\operatorname{argmax}}\ \Pi(p).$$

Platform revenue. Let $\mathcal{S} \subseteq \{r, s\}$ be the set of signals that sellers transact with online, which depends on (α, q, γ). The platform's expected revenue from a unit mass of quality-q sellers is then

$$r(\alpha, q, \gamma) = \gamma \sum_{\sigma \in \mathcal{S}} \Pr(\sigma|\alpha) \cdot p^*(\alpha, q, \gamma) \cdot \bar{F}\left(\frac{p^*(\alpha, q, \gamma)}{q}\right). \tag{3}$$

Since μ denotes the fraction of sellers that are type-H, the platform's total expected commission revenue from all sellers is

$$R(\alpha) = \mu \cdot r(\alpha, q_H, \gamma) + (1-\mu) \cdot r(\alpha, q_L, \gamma).$$

Seller types. A seller with online price p accepts the request to transact from a buyer with signal σ if and only if doing so is profitable (i.e., $\Pi_\sigma(p) \geq 0$). In the remainder of the paper, we impose an assumption that ensures the two seller types are meaningfully different in terms of their behavior on the platform. In particular, we assume the seller types are well-separated with respect to quality:

ASSUMPTION 1 **(Separation)**. *The seller qualities q_L and q_H satisfy $q_L \leq c(1-\lambda)$ and $q_H \geq 4c$.*

The following lemma is an immediate consequence of Assumption 1:

LEMMA 1. *Under Assumption 1, for any value of $\gamma \in [0, \gamma^{\max}]$ and $\alpha \in [\frac{1}{2}, 1]$, the type-$H$ seller accepts $\sigma = r$ buyers and the type-L seller rejects $\sigma = r$ buyers.*

Assumption 1 restricts our attention to the interesting setting where transacting with a $\sigma = s$ buyer is profitable for both seller types but transacting with a $\sigma = r$ buyer is profitable only for one (i.e., the high-quality) seller type.

Disintermediation threshold. For a given online price p, the buyer and seller choose to disintermediate only if both prefer transacting offline at price $b_\sigma(p)$ to transacting online at price p. The following lemma characterizes when this occurs.

LEMMA 2. *For each $\sigma \in \{r, s\}$, both the buyer and seller prefer the offline channel at price $b_\sigma(p)$ over the online channel at price p if and only if $\gamma \geq \bar{\gamma}_\sigma$, where $\bar{\gamma}_\sigma = 1 - \eta_{|\sigma}$.*

Lemma 2 establishes that the offline channel is preferred by both the buyer and seller if and only if the commission rate is sufficiently high, where the threshold $\bar{\gamma}_\sigma$ is decreasing in the seller's posterior belief that the buyer is not risky (type-s).

It follows from the expression for $\bar{\gamma}_\sigma$ that $\bar{\gamma}_s < \bar{\gamma}_r$, which implies sellers disintermediate with either no buyers, only $\sigma = s$ buyers, or all buyers. Figure 1 illustrates how these two thresholds evolve with the signal accuracy α. In the extreme case where $\alpha = 1$, sellers are able to perfectly distinguish buyers, and thus always transact offline with type-s buyers (and always online with type-r buyers). Since the platform trivially generates zero revenue in the case where sellers disintermediate with all buyers, we assume in the remainder of the paper that $\gamma \leq \bar{\gamma}_r$.

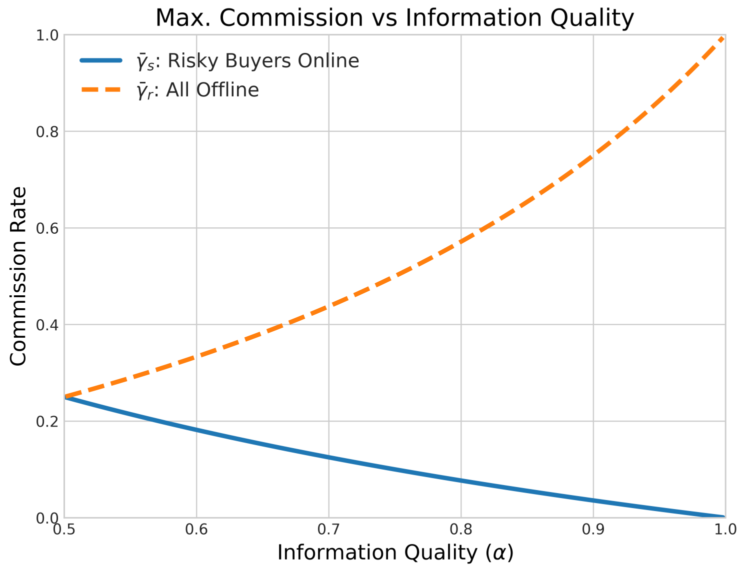

FIGURE 1. The largest commission rate (γ) at which transactions remain online as a function of information quality (α) – i.e., the probability the signal σ matches the buyer's true type – when 25% of buyers are risky ($\lambda = 0.75$). Sellers transact offline with low-risk ($\sigma = s$) buyers for $\gamma > \bar{\gamma}_s$ and high-risk ($\sigma = r$) buyers for $\gamma > \bar{\gamma}_r$, where $\bar{\gamma}_s < \bar{\gamma}_r$. As information quality α increases, sellers can more accurately distinguish between low- and high-risk buyers, causing $\bar{\gamma}_s$ to decrease (due to sellers' increased confidence that a $\sigma = s$ buyer will pay) and causing $\bar{\gamma}_r$ to increase (due to sellers' increased confidence that a $\sigma = r$ buyer will withhold payment).

3. Information and Platform Revenue Under Fixed Commission

We first consider how disintermediation and information quality jointly shape platform revenue. To establish a baseline, §3.1 addresses a setting where transactions occur online-only. In §3.2, we introduce the threat of disintermediation by assuming sellers can also transact in an offline channel. To isolate the role of information quality, we assume in this section that the platform's commission rate γ is fixed. Accordingly, we suppress dependence on γ in the notation.

3.1. Online-Only

Our first result establishes that in the total absence of disintermediation, the platform benefits from sellers having more accurate information about buyer types.

LEMMA 3. *Suppose transactions occur online-only. For any $\gamma \in [0, \gamma^{\max}]$, platform revenue $R(\alpha)$ weakly increases on $\alpha \in [\frac{1}{2}, 1]$.*

The reliability of the buyer signal σ increases in information quality α, which generates two effects on platform revenue. First, as α increases, a greater share of type-s buyers are correctly identified as such to the sellers, which we call the *trust effect*. Second, as α increases, sellers set more efficient prices, which we call the *price effect*. Next, we describe how these two information-related effects combine to impact platform revenue.

When only the online channel exists, the trust effect *lifts* revenue by increasing the volume of on-platform transactions. To see why, note that the high-quality seller transacts with both buyer types (Lemma 1), so their behavior is invariant to α. However, because the low-quality seller transacts with type-s buyers only, the trust effect increases the likelihood a low-quality seller completes a transaction by increasing the share of type-s buyers correctly labeled with $\sigma = s$. As a result, total transaction volume from all sellers increases with information quality α.

Next, to understand the price effect, it is helpful to examine the optimal price of type-L sellers (see Lemma 9 in the Appendix):

$$p^* = \frac{1}{2}\left(q_L + \underbrace{\frac{\mathbb{E}[c|\sigma = s]}{1-\gamma}}_{\psi}\right).$$

The first component in the optimal price, $\frac{1}{2}q_L$, reflects that buyers' valuations increase in a seller's quality. The second term, $\frac{1}{2}\psi$, is a "premium" charged by sellers due to the risk of transacting with a type-r buyer mislabeled with the signal $\sigma = s$. The premium $\frac{1}{2}\psi$ is a pricing inefficiency that stems from the information asymmetry faced by the seller, which hurts platform revenue.[6] As information quality α increases, the signal σ becomes more informative to sellers, compelling them to reduce the premium to the benefit of the platform.

In the baseline setting where transactions occur online-only, both the trust and price effects boost platform revenue, resulting in Lemma 3. Next, we examine how the threat of disintermediation – represented by the existence of a second, offline transaction channel – alters each of these information-related effects.

3.2. Online and Offline Channels

Our main result in this section is that the availability of the offline channel can reverse Lemma 3: for a fixed commission rate γ, an increase in information quality α can *hurt* platform revenue.

PROPOSITION 1. *Suppose transactions can occur online or offline. Then for any $\gamma \in [0, \gamma^{\max}]$, there exists $\bar{\alpha} \in (\frac{1}{2}, 1)$, and $\bar{\lambda} \in (\frac{1}{2}, 1)$ such that if $\lambda \geq \bar{\lambda}$, the platform's revenue $R(\alpha)$ strictly decreases on $(\bar{\alpha}, 1)$.*

Proposition 1 states that the platform's revenue decreases with information quality in high information environments ($\alpha \geq \bar{\alpha}$) without many risky buyers ($\lambda \geq \bar{\lambda}$). In the following corollary, we more precisely characterize these two thresholds. Surprisingly, Proposition 1 holds for almost all values of $\lambda \in [\frac{1}{2}, 1]$ – that is, even when there are many risky buyers.

[6] To see why the premium hurts platform revenue, note the platform's expected revenue from the type-L sellers is $(1-\mu)\gamma p(1 - \frac{p}{q_L})$. The price that maximizes platform revenue is thus $\frac{q_L}{2}$, which is the first component in the seller's chosen price p^*, and thus strictly lower than p^*.

Corollary 1. *The thresholds* $\bar{\lambda}$ *and* $\bar{\alpha}$ *in Proposition 1 satisfy* $\bar{\lambda} < 0.54$, $\bar{\alpha} = \frac{(1-\gamma)(1-\lambda)}{(1-\lambda)(1-\gamma)+\lambda\gamma}$ *for* $\gamma \leq 1-\lambda$ *and* $\bar{\alpha} = \frac{\lambda\gamma}{(1-\gamma)(1-\lambda)+\lambda\gamma}$ *for* $\gamma > 1-\lambda$.

Proposition 1 can similarly be explained by the trust and price effects described in §3.1. In the presence of the offline channel, the direction of the price effect is unchanged: An increase in information quality pushes sellers toward more efficient (i.e., lower) prices, increasing platform revenue. In contrast, when information quality is sufficiently high ($\alpha \geq \bar{\alpha}$), the direction of the trust effect is *flipped*, which pushes down on platform revenue.

To understand how the presence of the offline channel changes the trust effect, first note that when α is low, all sellers transact online due to the unreliability of the buyer signal. In other words, when there is sufficient risk associated with each buyer, sellers place a high value on the protections (e.g., insurance or payment guarantees) afforded by the platform. In this environment, the behavior in Lemma 3 is preserved, as shown in Figure 2.[7] However, when information quality is high ($\alpha \geq \bar{\alpha}$), sellers transact offline with $\sigma = s$ due to increased confidence they will not renege on payment, at which point further increases in α only pull a greater share of transactions off the platform. As a consequence, when the offline channel exists *and* information quality is high, the trust effect hurts platform revenue by amplifying disintermediation, putting it in tension with the price effect. Proposition 1 indicates that in the high information setting where sellers disintermediate with $\sigma = s$ buyers, the (revenue-decreasing) trust effect dominates the (revenue-increasing) price effect, provided the share of non-risky buyers is not excessively low ($\lambda \geq \bar{\lambda}$).

Combining Lemma 3 and Proposition 1 allows us to compare how disintermediation impacts platforms with different information environments. In particular, platforms that are associated with a more informative reputation system (larger α) may suffer significant revenue losses due to disintermediation, and may consider lowering their commission rate in response (e.g., see Figure 2 for $\alpha \in [0.63, 0.8]$); we explore this further in §4. Individual platforms may also become more informative over time—this can happen organically as more data on buyers are collected or if the platform's underlying reputation technology improves (e.g., see Fradkin et al. (2021)). Our results suggest that disintermediation, and the ensuing revenue losses, may be a by-product of this process. Finally, we remark that under low information quality, even intermediate commission rates can lead to market failure as all transactions will occur offline (e.g., see Figure 1); this may provide some insight into the collapse of platforms such as Homejoy.

[7] See Lemma 13 in Appendix D for a complete characterization of how platform revenue depends on information quality, including the case where $\alpha \leq \bar{\alpha}$.

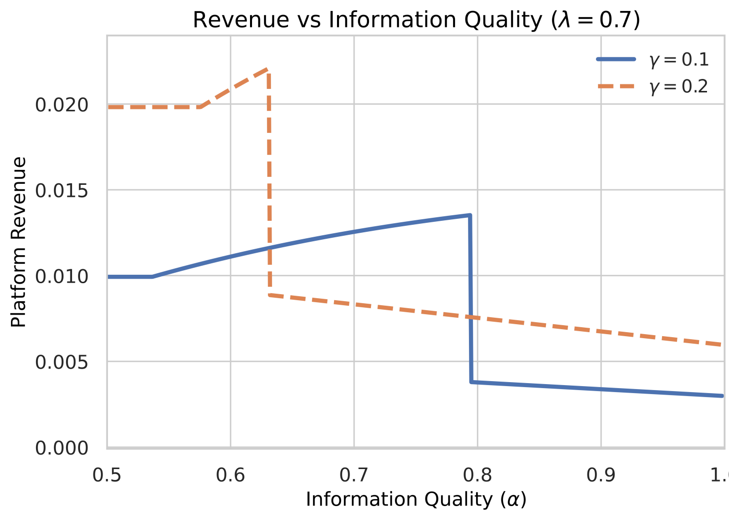

FIGURE 2. The platform's revenue as a function of information quality (α) for two values of the commission rate (γ), under parameters $\lambda = 0.7$, $c = 1$, $q_H = 4c$, $q_L = (1-\lambda)c$, and $\mu = 0.1$. At $\lambda = 0.7$, 30% of buyers on the platform are risky (type-r), and so an increase in information quality initially leads to an improvement in the platform's revenue as sellers are able to more accurately identify low-risk buyers to transact with. However, once $\alpha \geq \bar{\alpha}$ (where $\bar{\alpha} \approx 0.8$ for $\gamma = 0.1$ and $\bar{\alpha} \approx 0.63$ for $\gamma = 0.2$), sellers disintermediate with $\sigma = s$ buyers, which causes revenue to drop sharply and continue to decline as α increases further. Note for intermediate values of α, the smaller commission rate yields higher revenue by pushing the disintermediation threshold $\bar{\alpha}$ out further.

4. Disintermediation and Optimal Commission

In §3, we isolated the effect of information quality α on platform revenue by assuming the commission rate γ was fixed. Given that a platform's commission rate is one of its primary levers in shaping revenue, one might naturally ask how a platform should adjust commissions in response to disintermediation. This section focuses precisely on that question, with a spotlight on the role of information quality.

Our main result, presented in §4.1, shows that the optimal commission rate when sellers can transact offline can be *higher* than if disintermediation posed no threat at all. In §4.2, we examine how disintermediation alters the role that information quality plays in shaping the platform's optimal commission rate and optimal revenue, analogous to §3.

4.1. Optimal Commission Under Disintermediation

By Lemma 2, sellers disintermediate if and only if the commission rate is sufficiently high. Intuition would then suggest that the prospect of disintermediation should compel the platform to choose a strictly lower commission rate (compared to the online-only setting) to encourage sellers to remain on-platform. Our next result reveals that this prescription does not hold universally:

Proposition 2. *Let* γ^* *and* γ_0 *denote the platform's optimal commission rate with and without the offline channel, respectively. There exists* $\underline{\alpha} \in (\frac{1}{2}, 1)$ *and* $\mu_{\min} \in (0,1)$ *such that if* $\alpha \geq \underline{\alpha}$ *and* $\mu \geq \mu_{\min}$*, the optimal commission rate is weakly higher under the threat of disintermediation:*

$$\gamma_0 \leq \gamma^*.$$

In the presence of the offline channel, the platform can respond to disintermediation using one of two strategies, which determines the optimal commission rate γ^*:

1. A *back down* strategy, in which the platform lowers the commission rate to prevent platform participants from transacting offline.
2. A *double down* strategy, in which the platform raises the commission rate to maximize revenue from participants that remain online (even if some decide to transact offline).

Proposition 2 states that the platform should adopt the high-commission, double down strategy when information quality α and the share of high-quality sellers μ are both sufficiently high.

For the intuition behind Proposition 2, first suppose information quality is high ($\alpha \geq \underline{\alpha}$). In this setting, the signal σ is highly reliable, and thus sellers face minimal risk in transacting offline with $\sigma = s$ buyers. As a result, the back down strategy can lead to significant revenue losses since, in a high information environment, the maximum commission rate under which no disintermediation occurs is small (see Figure 1). In contrast, under the double down strategy, the platform forfeits revenue from $\sigma = s$ buyers and relies exclusively on $\sigma = r$ buyers transacting online (who both seller types are unwilling to transact with offline), and is thus free to choose a higher rate. Further, under the double down strategy, the platform's revenue is generated exclusively by high-quality sellers transacting online with $\sigma = r$ buyers. Therefore, for the double down strategy to collect enough revenue, the fraction of high quality sellers cannot be too small, which produces the additional condition $\mu \geq \mu_{\min}$.

In summary, if information quality is high and there are enough high-quality sellers, the platform is better off setting a commission rate at least as high as in the online-only setting, extracting the maximal revenue from the transactions that occur online, and absorbing the losses from disintermediation. Next, we show that Proposition 2 can be sharpened to a strict inequality under stronger conditions.

Corollary 2. *There exists* $\bar{\lambda} \in (\frac{1}{2}, 1)$ *such that if* $\lambda < \bar{\lambda}$ *and* $q_L \leq (1-\lambda)c/2$*, then* $\gamma_0 < \gamma^*$ *over some intervals* $\alpha \in (\underline{\alpha}, \bar{\alpha})$ *and* $\mu \in (\mu_{\min}, \mu_{\max})$*.*

The intuition behind Corollary 2 is subtle, but the main thrust is that it describes an environment where the platform sets a low commission rate γ_0 in the online-only setting to ensure the profitability of low-quality sellers. This is not a concern in the presence of disintermediation because low-quality sellers transact fully offline when information quality is sufficiently high.

What do our results thus far imply for platforms facing disintermediation? First, operating with a large degree of trust (large α) and featuring many high quality sellers (large μ) are often considered as markers of successful online platforms. Our analysis indicates that it is precisely these platforms that are most vulnerable to disintermediation (Proposition 1) and, accordingly, more likely to *increase* their commission rate in response (Proposition 2 and Corollary 2). Second, many platforms have been criticized for their high commission rates (e.g., see Gurley (2013)), and some have argued that high commission rates may cause disintermediation (Edelman and Hu 2016). Our results lend support to the alternative view by showing that a high commission rate may in fact be the optimal response to disintermediation.

4.2. Effect of Information Quality on Optimal Commission and Revenue

Analogous to the analysis in §3, we now examine how the threat of disintermediation alters the role of information in shaping platform's optimal commission rate and *optimal* revenue. We again begin by considering a benchmark setting in which disintermediation cannot occur.

Lemma 4. *Suppose transactions occur online-only. Then there exist thresholds* $\alpha_1 \in (\frac{1}{2}, 1)$ *and* $\alpha_2 \in [\alpha_1, 1)$ *such that (i)* $\gamma_0 = \gamma^{\max}$ *for* $\alpha \leq \alpha_1$ *and* $\alpha \geq \alpha_2$ *and (ii)* $\gamma_0 < \gamma^{\max}$ *and is strictly increasing for* $\alpha \in (\alpha_1, \alpha_2)$*. Further, the platform's optimal revenue* $R^*(\alpha)$ *weakly increases in* $\alpha \in [\frac{1}{2}, 1]$*.*

In the online-only setting, the platform's optimal commission rate weakly increases in α almost everywhere.[8] This result is intuitive – as information quality increases, sellers set more efficient prices and complete more transactions, due to the price and trust effects (described in §3), respectively. These two effects lift platform revenue and allow the platform to set a higher commission rate. Similarly, the platform's optimal revenue $R^*(\alpha)$ weakly increases in α, analogous to Lemma 3. Our next result, Proposition 3, shows that Lemma 4 is potentially reversed by the threat of disintermediation:

Proposition 3. *Suppose transactions can occur online or offline. There exists* $\bar{\mu}$ *and* $\bar{\alpha}$ *such that if* $\mu \geq \bar{\mu}$*, then* $\gamma^*(\alpha)$ *and* $R^*(\alpha)$ *strictly decrease in* α *on* $\alpha \in [\frac{1}{2}, \bar{\alpha}]$*.*

In a low-information setting ($\alpha \leq \bar{\alpha}$), the low-quality seller is unprofitable and does not participate, so the platform targets the high-quality seller only. Because the high-quality seller generates ample demand, the optimal strategy for the platform is to set the commission rate as high as possible while keeping the high-quality seller online, which is precisely the threshold $\gamma^* = \bar{\gamma}_s$ given in Lemma 2. Next, as information quality α increases, so does the attractiveness of transacting offline with

[8] The exception is $\alpha = \alpha_1$, which is the point at which low-quality sellers becomes profitable. At this point, the optimal commission rate γ_0 drops sharply to capitalize on transactions from low-quality sellers.

$\sigma = s$ buyers, due to the higher reliability of the buyer signal. As a consequence, the threshold for disintermediation $\bar{\gamma}_s$ – and therefore the optimal commission γ^* – decreases with α. This decrease in the commission rate to prevent disintermediation also drives down optimal revenue $R^*(\alpha)$.

5. Platform Pricing: Commissions vs. Access Fees

Our results thus far suggest that under commission-based pricing, the threat of disintermediation can make sellers' access to buyer information costly for the platform, contrary to a setting where transactions can only occur online. This invites the question of whether there exists an alternative pricing strategy that is resistant to disintermediation and the revenue-decreasing effects of information. In this section, we examine a natural candidate: charging sellers an upfront *access fee* to join the platform, instead of extracting commissions from on-platform transactions.[9]

Intuitively, access fees can mitigate revenue losses from disintermediation by collecting payments upfront and reducing sellers' incentives to transact offline. What is less clear is their efficiency: can access fees *fully* recoup the revenue that would otherwise be lost to disintermediation under commission-based pricing? Our main result in this section answers in the negative – while access fees dominate commissions in the setting where sellers can disintermediate, they can fall short of the maximum possible commission revenue when transactions can only occur online. Below, we provide a precise characterization of when access fees fail in this regard.

Under access-based pricing, each seller must pay the platform a fixed fee of $\phi > 0$ to join the platform. The use of a common fee ϕ for all sellers aligns with practice, and also follows naturally from an assumption that sellers' qualities are private information prior to joining the platform, which precludes price discrimination by the platform. Sellers set their prices after joining, and the remainder of the game proceeds as described in §2 (with $\gamma = 0$).

Let Π_0^i be the profit of a type-i seller on the platform in the absence of any commission or access fees; for convenience, we call Π_0^i a type-i seller's *maximum potential earnings*. Because sellers' outside options are normalized to 0, a type-i seller pays the access fee ϕ if and only if $\Pi_0^i \geq \phi$. Further, $\Pi_0^H > \Pi_0^L$ because $q_H > q_L$. Therefore, for any $\phi \leq \Pi_0^H$, either both seller types join the platform, or only the type-H sellers joins. The platform's revenue under an access fee of ϕ is then

$$R_\phi(\alpha, \phi) = \begin{cases} \phi, & \text{if } \phi \in [0, \Pi_0^L], \\ \mu\phi, & \text{if } \phi \in (\Pi_0^L, \Pi_0^H], \\ 0, & \text{if } \phi > \Pi_0^H. \end{cases}$$

[9] This pricing mechanism approximates the use of similar fees in practice, which aim to monetize matches between sellers and buyers instead of transactions. For example, Thumbtack charges service providers whenever they are contacted by prospective clients, and Care.com charges caregivers for the right to send messages to potential employers. For consistency with the commission-based model in §2, we assume that the access fee allows the seller to complete at most one transaction on the platform.

It follows that the platform's optimal access fee satisfies

$$\phi^* \in \{\Pi_0^L, \Pi_0^H\}, \tag{4}$$

and the platform's optimal revenue is

$$R_\phi^* = \max\{\Pi_0^L, \mu\Pi_0^H\}.$$

Let R_γ^* be the platform's optimal revenue under commission-based pricing. Our first result establishes that access fees generate higher revenue than commission fees when sellers can disintermediate.

LEMMA 5. *Suppose transactions can occur online or offline. Then access fees always generate higher revenue than commission fees:*

$$R_\phi^* > R_\gamma^*.$$

Further, for any $\phi > 0$, the platform's revenue $R_\phi(\alpha, \phi)$ weakly increases on $\alpha \in [\frac{1}{2}, 1]$.

Whether commissions or access fees generate higher revenue depends on which mechanism extracts more of the sellers' total potential earnings on the platform. For the intuition behind Lemma 5, note that under the optimal commission rate γ^*, either both seller types transact online, or only high-quality sellers do (which occurs when the low-quality seller either disintermediates or is unprofitable). If only high-quality sellers transact online under γ^*, the result is straightforward – the platform can extract the entirety of high-quality sellers' potential earnings by simply setting ϕ to that amount, which commission fees cannot match.

The result is less immediate when both seller types transact online. At a high level, if both seller types transact online under γ^*, then the commission rate γ^* must be small enough such that the low-quality seller is profitable and neither seller type disintermediates. This yields an upper bound on the optimal commission revenue, which is shown to be exceeded by the revenue attained under the optimal access fee. Lemma 5 also confirms that platform revenue never decreases in information quality under access fees, in sharp contrast to commission fees as shown in Proposition 1.

Having confirmed that access fees outperform commissions when the offline channel exists, we now address whether they can fully recover the revenue losses due to disintermediation. Our analysis relies on the following characterization of the platform's sellers:

DEFINITION 1 (RELATIVE EARNINGS). Let Π_0^i be the maximum potential earnings for a type-i seller on the platform in the absence of any commission or access fees. The *relative earnings* on the seller-side of the platform are then given by:

$$\beta = \frac{(1-\mu)\Pi_0^L}{\mu\Pi_0^H}.$$

When the ratio β is close to 1, we say the sellers' earnings are *balanced*. We now present the main result of this section: in the setting where buyers and sellers cannot disintermediate, access fees generate *lower* revenue than commission fees when relative earnings are moderately balanced and the information quality α is not too low.

Proposition 4. *Suppose transactions can occur online-only. There exist thresholds $\underline{\alpha} \in (\frac{1}{2}, 1)$, β_1, and β_2 such that for any $\alpha \geq \underline{\alpha}$ and $\beta \in (\beta_1, \beta_2)$, the platform generates strictly higher revenue through commissions than access fees:*

$$R^*_\gamma > R^*_\phi.$$

Further, $\beta_1 \geq 1/4$ and $\beta_2 \leq 5$.

The effectiveness of any pricing strategy depends on how much of the sellers' potential earnings it can extract. Therefore, to understand Proposition 4, it is helpful to consider the platform's "uncaptured" revenue, i.e., the portion of the sellers' potential earnings that is not extracted by the access fee.

As shown in (4), the platform's optimal access fee is either Π_0^L or Π_0^H. Under a fee of $\phi = \Pi_0^L$, the platform fully extracts the earnings of low-quality sellers, but not high-quality sellers, who collectively enjoy an uncaptured surplus of $\mu(\Pi_0^H - \phi)$. Conversely, if $\phi = \Pi_0^H$, then the platform fully extracts high-quality sellers' earnings, but low-quality sellers do not join the platform – in this case, the platform misses out on the potential earnings of low-quality sellers, $(1-\mu)\Pi_0^L$. When β is moderate, the uncaptured revenue is large regardless of how the platform chooses ϕ, which makes access fees less revenue-efficient than commission fees, resulting in Proposition 4. Lastly, to see the role of the condition $\alpha \geq \underline{\alpha}$, note that when α is low, low-quality sellers are unprofitable on the platform and thus do not join. In this case, the platform can fully extract the earnings of all participating (high-quality) sellers by setting $\phi = \Pi_0^H$.

In summary, combining Lemma 5 and Proposition 4 yields the following conclusion: charging sellers access fees instead of commissions can help the platform recover some – but not all – of the revenue that would otherwise be lost to disintermediation under commission-based pricing. Notably, the platform may benefit from excluding low-quality sellers by setting a high access fee, which is at odds with the notion that platforms should accommodate a diversity of sellers at varying quality levels and price points.

Upfront pricing mechanisms such as subscriptions and access fees have drawn significant attention in recent years, and pose a variety of advantages over commissions (Hu and Zhou 2020, Feldman et al. 2022, Cachon et al. 2022, Cui and Hamilton 2022). Our study complements this line of work by highlighting how access fees can correct for misalignment between a platform's pricing strategy and its value proposition. Our results also shed light on the choice of different pricing strategies by platforms in practice. Platforms that charge sellers for access to buyers (e.g., Thumbtack and Care.com) may perceive disintermediation to be highly likely, whereas platforms that rely on commission (e.g., Etsy) may believe the revenue losses from disintermediation to be minimal. Finally, our work helps explain why the same platform may adopt different pricing strategies in different markets – for example, Uber charges drivers commission fees in North America, but in 2022 unveiled access-based pricing with 0% commission for drivers in South Asia (Uber 2023). This may be due to variability in the threat of disintermediation and the level of social trust across markets (Astashkina et al. 2022) – for example, platforms may witness higher levels of disintermediation in developing countries (Rampal 2023).

Although our focus has been on platform revenue, the platform's choice of pricing strategy also has implications for the welfare of platform participants. For labor platforms like Thumbtack that charge for access to buyers, one might naturally assume it is in their best interest to have workers of different quality levels join the platform in order to cater to a variety of jobs. However, Thumbtack's pricing strategy of charging sellers for "leads" (i.e., messages from prospective buyers) also results in sellers paying fees that are not backed by a guaranteed transaction, which may generate discontent from sellers who are not sufficiently profitable (Better Business Bureau 2023). Access fees may also be a natural choice in dating (e.g., eHarmony) and caregiving (e.g., Care.com) platforms, especially if the platform can benefit by keeping out low quality users (Johari et al. 2019).

Our findings are obtained under a model that abstracts away from additional factors that, in practice, may influence a seller's willingness to pay upfront for platform access. In particular, we assume sellers engage in a single transaction at most, know their own quality, and face no uncertainty about the demand state on the platform. We conjecture that our main insight – that access fees are only a partial solution to disintermediation – is likely to persist when accounting for additional features of a platform environment.

6. Discussion

In this paper, we considered how information quality and disintermediation jointly impact the operations of a two-sided platform. We showed how an informative environment can be costly for platforms – in particular, when risky buyers are scarce, having informed sellers can harm revenue by promoting disintermediation. More generally, our results suggest that whether platforms should strive for a high- or low-information environment depends on the ease with which platform participants can disintermediate and the perceived risks of doing so.

In responding to disintermediation, commission-based platforms might consider adjusting rates or adopting an alternative pricing strategy entirely. Despite the obvious costs of disintermediation, lowering commission rates in an attempt to thwart off-platform transactions may be ill-advised – if disintermediation is inevitable, platforms may be better off setting high commission rates to capitalize on the transactions that remain online. Alternatively, platforms may benefit by charging sellers for access to buyers instead of levying commission fees; however, this can generate *less* revenue than commission-based pricing if disintermediation poses little risk. Further, our finding that neither commission fees nor access fees dominate the other may help explain why both arise in practice, and also suggests that a platform's choice of pricing strategy depends partly on the extent to which disintermediation threatens its bottom line.

Our work modeled information quality (α) as exogenous to understand how disintermediation impacts different platforms, given that some platforms have more reliable information signals than others (Donaker et al. 2019). In practice, marketplace designers may have some limited ability to influence the information available to sellers (Fradkin et al. 2021), for example, by enhancing the platform's reputation system or moderating sellers' ability to communicate with buyers. Our results suggest that in the face of disintermediation, platforms have an incentive to withhold information – for instance, by making reputation systems uninformative so to discourage sellers from disintermediating. Similarly, our results suggest platforms have a strong incentive to warn sellers about the possibility of risky buyers (see, e.g., UpWork (2023b)), or even deliberately exaggerate the risk of buyer fraud to deter sellers from disintermediating. Finally, our findings contribute to the emerging literature on the use of information as an operational lever (Bimpikis et al. 2020, Drakopoulos et al. 2021, Shi et al. 2022) by identifying a new rationale for controlling information, namely, to reduce the attractiveness of disintermediation.

Our work abstracted away from several features of the platform environment that are likely to produce additional insights if addressed formally. First, we simplified the mechanism through which buyers search for sellers and assumed buyers and sellers are matched randomly. Although disintermediation occurs after matching, it is conceivable that the matching mechanism impacts a seller's propensity for disintermediation (e.g., if high-quality sellers receive more requests from buyers). While incorporating more sophisticated matching dynamics may alter the precise conditions for disintermediation, we conjecture our qualitative insights would remain unchanged. Second, a seller's quality may be initially unknown but learned by a buyer following a transaction, and buyers may be more likely to disintermediate with sellers they have previously transacted with. Finally, our model was independent of context; in practice, the likelihood of disintermediation may also depend on the service or product in question. More broadly, as intermediary platforms become increasingly prevalent, new research questions regarding the role of information in platform operations will inevitably emerge, and our work can serve as a basis for their investigation.

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Appendix

A. Exhibit: Disintermediation in the Wild

As discussed in §1, there has been significant interest in the topic of platform disintermediation in recent years. Prior work has focused on quantifying the extent of platform disintermediation in a variety of contexts, including online labor (Gu and Zhu 2021), the sharing economy (Lin et al. 2022), e-commerce (He et al. 2020), cargo delivery (Xie and Zhu 2023), home cleaning (Karacaoglu et al. 2022), and pet sitting (Farronato et al. 2022).

In this section, we complement the empirical evidence in the work cited above by providing anecdotal evidence of disintermediation on specific platforms. The purpose of this section is to convey the pervasiveness of disintermediation and shed light on the manner in which buyers and sellers disintermediate from various platforms.

- **Airbnb**: Airbnb is an online marketplace for short-term rentals. Broadly speaking, the platform offers two alternative fee structures: (*i*) *Split Fee*: Hosts pay 3% of the total booking value and guests pay 14% of the same; (*ii*) *Host-only Fee*: Hosts pay 14-16% of the booking value. Although Airbnb uses algorithmic filters to block users from sharing contact details prior to booking the property, hosts and guests often find creative ways to circumvent the algorithm, as illustrated in Figure 3a.

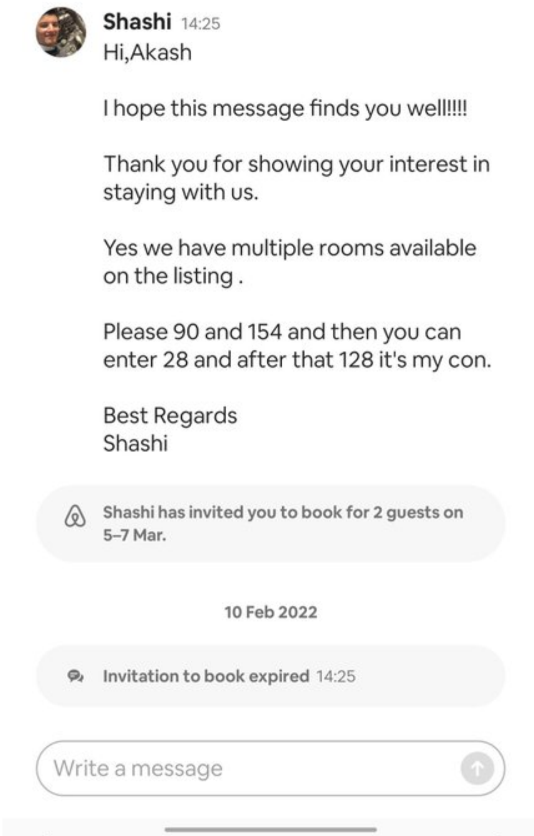
- **Upwork**: Formerly known as Elance-oDesk, Upwork is one of the world's largest freelancing platforms. As of March 2023, Upwork charges at flat commission of 10% to all freelancers. The negotiations between freelancers and employers provide ample opportunity for taking transactions offline, although, as Figure 3b indicates, the request to disintermediate can be rather explicit in some cases.

- **Uber/Lyft**: Ridesharing platforms such as Uber and Lyft can take up to 25% of the rider's payment as commission. When the rider's cancellation fee is smaller than the commission[11], both parties can benefit by disintermediating. As illustrated in Ghose (2022), transacting off-platform can be particularly rewarding on longer trips.

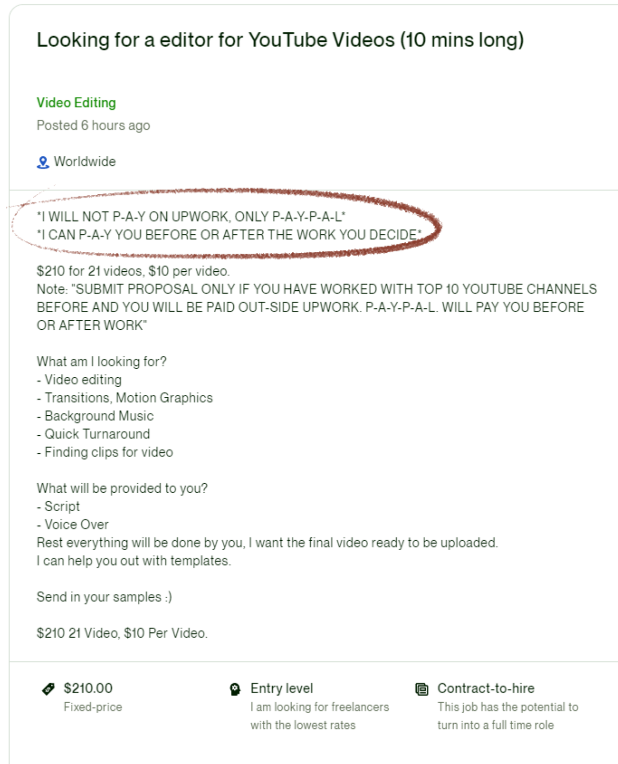
- **GoFundMe**: GoFundMe is a fundraising platform that is typically used for personal or charitable causes. The platform extracts a 2.9% commission from each donation and along with a flat fee of $0.30. Given the relatively low fee, there is limited incentive for disintermediation. However, in important causes or

[10] https://twitter.com/Akashsakatn/status/1505179413658607622.

[11] Unlike riders who pay a flat cancellation fee, drivers do not incur penalties for cancelling. However, if drivers exceed a maximum cancellation rate, their account may get suspended.



(a) **Airbnb**: A host messages a guest and shares his mobile number. The host circumvents the platform filters by separating the digits with some additional text (e.g., "90 and 154"). Source: Twitter[10].



(b) **Upwork**: A job posting on Upwork where the employer explicitly states that the payment will be made outside the platform. Multiple attempts are made (e.g., "P-A-Y-P-A-L") to bypass Upwork's filters.

Figure 3. Disintermediation attempts on Airbnb and UpWork.

when funding is tight, fundraisers may urge donors to donate outside the platform to maximize their funds, as seen in Figure 4.

- **Rover**: Rover is a pet-sitting platform that charges a commission rate of 20%. Rover's high fees have been the source of significant discontent among its sitters[12]. Naturally, disintermediation appears to be commonplace on the platform, sometimes even for the first transaction[13]. The following quote from a 2018 article on Rover sums up sitters' concerns: "They take a huge cut from us but then don't really have our interests at heart . . . so I don't mind taking their customers" (Lieber 2018).
- **Openbay**: Openbay is a two-sided marketplace that connects vehicle owners with mechanics and charges a commission rate of 13%. The following comment by the Openbay CEO Rob Infantino in a 2017 summit at the Harvard Business School hints at the extent of disintermediation on the platform (Karacaoglu et al. 2022):

[12] https://www.rover.com/community/question/48718/why-do-you-guys-take-such-high-fees/.

[13] https://www.reddit.com/r/RoverPetSitting/comments/10qd89i/clients_dont_want_to_pay_me_on_rover/, https://www.reddit.com/r/RoverPetSitting/comments/qpm7zv/first_sitter_wants_payment_outside_of_rover/, https://twitter.com/vc/status/1409270449884651521.

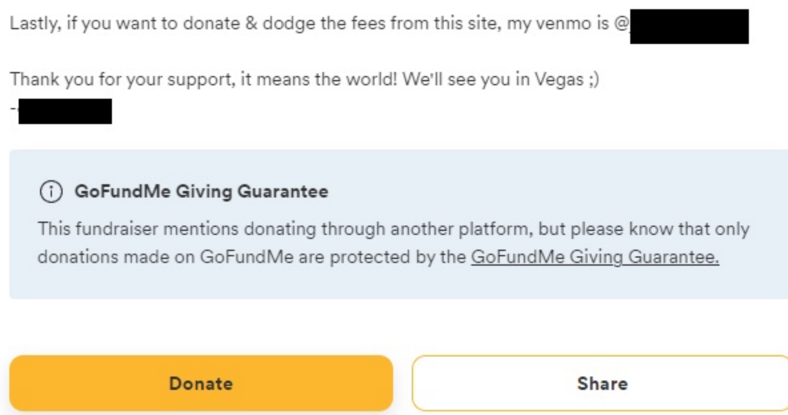


FIGURE 4. **GoFundMe**: A fundraiser shares an alternative channel (Venmo) for commission-free donations. GoFundMe algorithmically detects the request to disintermediate and appends a reminder of the platform's protections (i.e., a money-back guarantee). Identifying details have been redacted to protect privacy.

> "We worry about this problem every single, day. [...] How do you prevent that from happening? You build value on both sides of the marketplace. [...] But the point is, disintermediation is big; it's a big problem. And we are constantly fighting it. We are working on it [...] but that is a big, big problem for us."

- **Reverb**: Reverb is an online marketplace for buying and selling musical equipment that charges sellers a 5% commission fee. Anecdotal evidence[14] suggests that disintermediation is common on the platform particularly when a) the instrument is expensive; b) the transaction is local, i.e., in-person; c) sellers put the listing on multiple channels and buyer simply use Reverb for discovering items.

B. Supporting Lemmas

Here we present several auxiliary lemmas useful for the proofs of the results in §2-§5.

LEMMA 6. *For $\alpha \in [\frac{1}{2}, 1]$, define $\eta_\sigma = Pr(\sigma)$ and $\eta_{|\sigma} = Pr(j = s|\sigma)$. Then $\eta_r = (1-\alpha)\lambda + \alpha(1-\lambda)$, $\eta_s = \alpha\lambda + (1-\alpha)(1-\lambda)$, $\eta_{|r} = (1-\alpha)\lambda/\eta_r$, and $\eta_{|s} = \alpha\lambda/\eta_s$. Further, $\frac{d}{d\alpha}\eta_r < 0$, $\frac{d}{d\alpha}\eta_s > 0$, $\frac{d}{d\alpha}\eta_{|r} < 0$, and $\frac{d}{d\alpha}\eta_{|s} > 0$.*

Proof. Note $\lambda = \Pr(j = s)$ and $\alpha = \Pr(j = s|\sigma = s) = \Pr(j = r|\sigma = r)$ by definition. The expressions for η_σ and $\eta_{|\sigma}$ follow from the total probability rule and Bayes' rule, respectively:

$$\eta_{|s} = \Pr(j = s|\sigma = s) = \frac{\alpha\lambda}{\alpha\lambda + (1-\alpha)(1-\lambda)},$$
$$\eta_{|r} = \Pr(j = s|\sigma = r) = \frac{(1-\alpha)\lambda}{(1-\alpha)\lambda + \alpha(1-\lambda)}.$$

[14] https://www.acousticguitarforum.com/forums/showthread.php?t=594676,
https://www.thegearpage.net/board/index.php?threads/is-reverb-cracking-down-on-offsite-selling.2384673/.

Next, with some effort it can be shown that

$$\frac{\partial \eta_r}{d\alpha} = 1 - 2\lambda, \qquad \frac{\partial \eta_s}{d\alpha} = 2\lambda - 1,$$

and

$$\frac{d\eta_{|r}}{d\alpha} = -\frac{(1-\lambda)\lambda}{(\alpha(1-\lambda)+(1-\alpha)\lambda)^2}, \qquad \frac{d\eta_{|s}}{d\alpha} = \frac{(1-\lambda)\lambda}{((1-\alpha)(1-\lambda)+\alpha\lambda)^2}.$$

The result follows because $\lambda \in [\frac{1}{2}, 1]$. $\square$

Lemma 7. *(i) For any $\lambda \in (0,1]$, $0 < \bar{\gamma}_s < \bar{\gamma}_r < 1$ for $\alpha \in (\frac{1}{2}, 1]$ and $\bar{\gamma}_r = \bar{\gamma}_s$ for $\alpha = \frac{1}{2}$. (ii) For any $\gamma \in (0,1]$, there exists $\bar{\alpha}_r \in (0,1]$ and $\bar{\alpha}_s \in (0,1]$ such that $\gamma \geq \bar{\gamma}_r$ if and only if $\alpha \leq \bar{\alpha}_r$ and $\gamma \geq \bar{\gamma}_s$ if and only if $\alpha \geq \bar{\alpha}_s$. (iii) If $\gamma < 1-\lambda$, then $\bar{\alpha}_r < \frac{1}{2} < \bar{\alpha}_s$, if $\gamma > 1-\lambda$, then $\bar{\alpha}_s < \frac{1}{2} < \bar{\alpha}_r$, and if $\gamma = 1-\lambda$, then $\bar{\alpha}_s = \bar{\alpha}_r = \frac{1}{2}$.*

Proof. (i). Note $\bar{\gamma}_\sigma \in (0,1]$ for each $\sigma \in \{r,s\}$ because $\alpha \in [\frac{1}{2}, 1]$ and $\lambda \in (0,1]$. It remains to show $\bar{\gamma}_r > \bar{\gamma}_s$. Note

$$\bar{\gamma}_r - \bar{\gamma}_s = \eta_{|s} - \eta_{|r} = \lambda \left(\frac{\alpha}{\eta_s} - \frac{(1-\alpha)}{\eta_r} \right). \tag{5}$$

It remains to show $\alpha\eta_r - (1-\alpha)\eta_s > 0$. Using the definitions of η_σ (Lemma 6), we have

$$\begin{aligned} \alpha\eta_r - (1-\alpha)\eta_s &= \alpha\left((1-\alpha)\lambda + \alpha(1-\lambda)\right) - (1-\alpha)\left(\alpha\lambda - (1-\alpha)(1-\lambda)\right) \\ &= (1-\lambda)(\alpha^2 - (1-\alpha)^2) \\ &> 0, \end{aligned}$$

where the inequality follows because $\alpha \in [\frac{1}{2}, 1]$. It is straightforward to verify that $\alpha = \frac{1}{2}$ implies $\eta_r = \eta_s$, which combined with (5) implies $\bar{\gamma}_r = \bar{\gamma}_s$. (ii). Solving $\bar{\gamma}_\sigma = \gamma$ for each $\sigma \in \{r,s\}$ yields

$$\bar{\alpha}_r = \frac{\lambda\gamma}{(1-\lambda)(1-\gamma)+\lambda\gamma}, \qquad \bar{\alpha}_s = \frac{(1-\lambda)(1-\gamma)}{(1-\lambda)(1-\gamma)+\lambda\gamma}. \tag{7}$$

It is straightforward to verify that $\bar{\alpha}_\sigma \in (0,1]$ for all $\gamma \in (0,1]$ for each $\sigma \in \{r,s\}$. Next, note

$$\frac{\partial \bar{\gamma}_r}{\partial \alpha} = \frac{(1-\lambda)\lambda}{\eta_r^2} > 0, \quad \frac{\partial \bar{\gamma}_s}{\partial \alpha} = -\frac{(1-\lambda)\lambda}{\eta_s^2} < 0,$$

which implies $\bar{\gamma}_r$ is strictly increasing in α on $(0,1]$ and $\bar{\gamma}_s$ is strictly decreasing in α on $(0,1]$. Further, because $\bar{\alpha}_\sigma \in (0,1]$ for $\sigma \in \{r,s\}$, it follows that $\bar{\gamma}_\sigma$ that for any $\gamma \in (0,1]$, $\gamma \geq \bar{\gamma}_r$ if and only if $\alpha \leq \bar{\alpha}_r$ and $\gamma \geq \bar{\gamma}_s$ if and only if $\alpha \geq \bar{\alpha}_s$. (iii) The result can be shown algebraically using the expressions for $\bar{\alpha}_r$ and $\bar{\alpha}_s$ given in (7). $\square$

For use in Lemma 8 below, define

$$v_\sigma(p) = ((1-\gamma)p - (1-\eta_{|\sigma})c)\left(1 - \frac{p}{q}\right), \quad \tilde{v}_\sigma(p) = (\eta_{|\sigma} b_\sigma(p) - (1-\eta_{|\sigma})c)\left(1 - \frac{p}{q}\right), \tag{8}$$

and

$$p_\sigma^0 = \frac{c(1-\eta_{|\sigma})}{1-\gamma}, \quad \tilde{p}_\sigma^0 = \frac{2c(1-\eta_{|\sigma})}{1-\gamma+\eta_{|\sigma}}. \tag{9}$$

LEMMA 8. *(i) The seller's expected profit from trading online at price p with a type-σ buyer is*

$$\pi_\sigma(p) = \begin{cases} 0, & \text{if } p \le p^0_\sigma, \\ v_\sigma(p), & \text{if } p > p^0_\sigma, \end{cases}$$

where $p^0_s < p^0_r$ for all $\alpha \in [\frac{1}{2}, 1]$, $\gamma \in (0,1]$ and $\lambda \in (0,1]$. Further, for each $\sigma \in \{r,s\}$, $\pi_\sigma(p) > 0$ for $p \ge p^0_\sigma$.
(ii). The seller's expected profit from trading offline at price $b_\sigma(p)$ with a type-σ buyer is

$$\tilde{\pi}_\sigma(p) = \begin{cases} 0, & \text{if } p \le \tilde{p}^0_\sigma, \\ v_\sigma(p), & \text{if } p > \tilde{p}^0_\sigma, \end{cases}$$

where $\tilde{p}^0_s < \tilde{p}^0_r$ holds for all $\alpha \in [\frac{1}{2}, 1]$, $\gamma \in (0,1]$ and $\lambda \in (0,1]$. Further, for each $\sigma \in \{r,s\}$, $\tilde{\pi}_\sigma(p) > 0$ for $p \ge \tilde{p}^0_\sigma$.

Proof. We prove the statements in order. For (i), note (1) can be written as

$$\pi_\sigma(p) = \left[((1-\gamma)p - (1-\eta_{|\sigma})c) \left(1 - \frac{p}{q}\right) \right]^+ = [v_\sigma(p)]^+.$$

It is straightforward to verify that $v_\sigma(p)$ is strictly concave and quadratic in p and that $p = q$ is the larger solution to $v_\sigma(p) = 0$. It can be shown that p^0_σ is the smaller solution to $v_\sigma(p) = 0$ by plugging the expression for $\eta_{|\sigma}$ given in Lemma 6 into $v_\sigma(p)$ and verifying algebraically that $v_\sigma(p^0_\sigma) = 0$ for each $\sigma \in \{r,s\}$. It follows that $\pi_\sigma(p) = 0$ on $p \in [0, p^0_\sigma]$ and $\pi_\sigma(p) = v_\sigma(p) > 0$ on $p \in (p^0_\sigma, 1]$, as desired. Next, to show $p^0_s < p^0_r$ holds for all $\alpha \in [\frac{1}{2}, 1]$, $\gamma \in (0,1]$ and $\lambda \in (0,1]$, note

$$p^0_r - p^0_s = \frac{c(1-\eta_{|r})}{1-\gamma} - \frac{c(1-\eta_{|s})}{1-\gamma} = \frac{c}{1-\gamma}\left(\bar{\gamma}_r - \bar{\gamma}_s\right) > 0,$$

where the strict inequality follows because $\bar{\gamma}_r > \bar{\gamma}_s$ by Lemma 7. We prove (ii) next. Note (2) can be written equivalently as

$$\tilde{\pi}_\sigma(p) = \left[(\eta_{|\sigma} b_\sigma(p) - (1-\eta_{|\sigma})c) \left(1 - \frac{p}{q}\right) \right]^+ = [\tilde{v}_\sigma(p)]^+.$$

It is straightforward to verify that $v_\sigma(p)$ is strictly quadratic and concave in p, and that $p = q$ is the larger solution to $v_\sigma(p) = 0$. It can be shown that $\tilde{p}^0_\sigma$ is the smaller solution to $v_\sigma(p) = 0$ by plugging the expression for $\eta_{|\sigma}$ given in Lemma 6 into $v_\sigma(p)$ and verifying algebraically that $v_\sigma(p^0_\sigma) = 0$ for each $\sigma \in \{r,s\}$. It follows that $\tilde{\pi}_\sigma(p) = 0$ on $p \in [0, \tilde{p}^0_\sigma]$ and $\tilde{\pi}_\sigma(p) = \tilde{v}_\sigma(p) > 0$ on $p \in (\tilde{p}^0_\sigma, 1]$, as desired. Next, to show $\tilde{p}^0_s < \tilde{p}^0_r$ holds for all $\alpha[\frac{1}{2}, 1]$, $\gamma \in (0,1]$ and $\lambda \in (0,1]$, note

$$\tilde{p}^0_r - \tilde{p}^0_s = \frac{2c(1-\eta_{|r})}{1-\gamma+\eta_{|r}} - \frac{2c(1-\eta_{|s})}{1-\gamma+\eta_{|s}}.$$

It suffices to show that $(1-\eta_{|r})(1-\gamma+\eta_{|s}) > (1-\eta_{|s})(1-\gamma+\eta_{|r})$. Note

$$(1-\eta_{|r})(1-\gamma+\eta_{|s}) - (1-\eta_{|s})(1-\gamma+\eta_{|r}) = (2-\gamma)(\eta_{|s} - \eta_{|r}) > 0,$$

where the strict inequality follows because $\eta_{|\sigma} = 1 - \bar{\gamma}_\sigma$ for $\sigma \in \{r,s\}$ by definition of $\bar{\gamma}_\sigma$ and $\bar{\gamma}_r > \bar{\gamma}_s$ by Lemma 7. $\square$

LEMMA 9. *Consider a unit mass of sellers with common quality $q > 0$. Under the platform's commission rate of γ, the sellers' profit function $\Pi(p)$, optimal price p^*, optimal profit Π^*, and contribution to platform revenue $r(\alpha, q, \gamma)$ under p^*, are characterized as follows.*

(a) If the seller accepts both $\sigma = r$ and $\sigma = s$ online, then

$$\Pi(p) = \pi^a(p) = ((1-\gamma)p - (1-\lambda)c)\left(1 - \frac{p}{q}\right),$$
$$p^* = p^a = \frac{1}{2}\left(q + \frac{(1-\lambda)c}{1-\gamma}\right),$$
$$\Pi^* = \pi^a(p^a) = (1-\gamma)q\left(\frac{1}{2} - \frac{(1-\lambda)c}{2q(1-\gamma)}\right)^2,$$
$$r(\alpha, q, \gamma) = r^a(\alpha, q, \gamma) = \gamma p^a \bar{F}\left(\frac{p^a}{q}\right).$$

(b) If the seller rejects $\sigma = r$ and accepts $\sigma = s$ online, then

$$\Pi(p) = \pi^b(p) = \eta_s\left((1-\gamma)p - (1-\eta_{|s})c\right)\left(1 - \frac{p}{q}\right),$$
$$p^* = p^b = \frac{1}{2}\left(q + \frac{(1-\eta_{|s})c}{1-\gamma}\right),$$
$$\Pi^* = \pi^b(p^b) = \eta_s(1-\gamma)q\left(\frac{1}{2} - \frac{(1-\eta_{|s})c}{2q(1-\gamma)}\right)^2,$$
$$r(\alpha, q, \gamma) = r^b(\alpha, q, \gamma) = \gamma\eta_s p^b \bar{F}\left(\frac{p^b}{q}\right).$$

(c) If the seller accepts $\sigma = r$ online and accepts $\sigma = s$ offline, then

$$\Pi(p) = \pi^c(p) = \left(\zeta p - (1-\lambda)c\right)\left(1 - \frac{p}{q}\right),$$
$$p^* = p^c = \frac{1}{2}\left(q + \frac{(1-\lambda)c}{\zeta}\right),$$
$$\Pi^* = \pi^c(p^c) = \zeta q\left(\frac{1}{2} - \frac{(1-\lambda)c}{2q\zeta}\right)^2,$$
$$r(\alpha, q, \gamma) = r^c(\alpha, q, \gamma) = \gamma\eta_r p^c \bar{F}\left(\frac{p^c}{q}\right),$$

where $\zeta = \eta_r(1-\gamma) + \frac{1}{2}\eta_s(1-\gamma+\eta_{|s})$.

(d) If the seller rejects $\sigma = r$ and accepts $\sigma = s$ offline, then

$$\Pi(p) = \pi^d(p) = \eta_s\left(1 - \frac{p}{q}\right)\left(\frac{p(1-\gamma+\eta_{|s})}{2} - (1-\eta_{|s})c\right),$$
$$p^* = p^d = \frac{q}{2} + \frac{(1-\eta_{|s})c}{1-\gamma+\eta_{|s}},$$
$$\Pi^* = \pi^d(p^d) = \eta_s\frac{(1-\gamma+\eta_{|s})}{2}q\left(\frac{1}{2} - \frac{(1-\eta_{|s})c}{q(1-\gamma+\eta_{|s})}\right)^2,$$
$$r(\alpha, q, \gamma) = r^d(\alpha, q, \gamma) = 0.$$

Proof. We prove statements (a) through (d) in order. The revenue expressions $r(\alpha, q, \gamma)$ follow immediately from combining (3) with the expressions for the optimal price p^* in the lemma statement. We therefore focus on characterizing $\Pi(p)$, p^*, and Π^*. We will initially assume the first-order condition $\frac{\partial}{\partial p}\Pi(p) = 0$ holds at p^* in all cases and confirm this to be true at the end of the proof.

(a). Because the seller accepts the buyer regardless of the signal, their expected cost is simply $(1-\lambda)c$. Because the transaction is online, the seller's profit is then $\Pi(p) = \pi^a(p)$, where

$$\pi^a(p) = ((1-\gamma)p - (1-\lambda)c)\left(1 - \frac{p}{q}\right).$$

Because the profit maximizing price p^* satisfies $\frac{\partial}{\partial p}\pi^a(p)|_{p=p^*} = 0$, we have

$$\frac{\partial}{\partial p}\pi^a(p)\Big|_{p=p^*} = (1-\gamma)\left(1 - \frac{p^*}{q}\right) - \frac{1}{q}((1-\gamma)p^* - (1-\lambda)c) = 0,$$

which implies $p^* = p^a$, where

$$p^a = \frac{1}{2}\left(q + c\frac{1-\lambda}{1-\gamma}\right).$$

It follows that the optimal profit is

$$\begin{aligned}
\Pi^* = \pi^a(p^a) &= \left(1 - \frac{p^a}{q}\right)((1-\gamma)p^a - (1-\lambda)c), \\
&= \left(\frac{1}{2} - \frac{(1-\lambda)c}{2q(1-\gamma)}\right)\left((1-\gamma)\frac{q}{2} + \frac{(1-\lambda)c}{2} - (1-\lambda)c\right), \\
&= (1-\gamma)q\left(\frac{1}{2} - \frac{(1-\lambda)c}{2q(1-\gamma)}\right)^2.
\end{aligned}$$

(b). In this scenario, the seller simply rejects the buyer if $\sigma = r$. The seller's profit in this case is then $\Pi(p) = \pi^b(p)$, where

$$\pi^b(p) = \eta_s((1-\gamma)p - (1-\eta_{|s})c)\left(1 - \frac{p}{q}\right).$$

For the profit maximizing price, we have

$$\frac{\partial}{\partial p}\pi^b(p)\Big|_{p=p^*} = \eta_s\left((1-\gamma)\left(1 - \frac{p^*}{q}\right) - \frac{1}{q}((1-\gamma)p^* - (1-\eta_{|s})c)\right) = 0$$

which implies $p^* = p^b$, where

$$p^b = \frac{1}{2}\left(q + c\frac{1-\eta_{|s}}{1-\gamma}\right).$$

The optimal profit is then given by

$$\begin{aligned}
\Pi^* = \pi^b(p^b) &= \left(1 - \frac{p^b}{q}\right)((1-\gamma)p^b - (1-\eta_{|s})c), \\
&= \left(\frac{1}{2} - \frac{(1-\eta_{|s})c}{2q(1-\gamma)}\right)\left((1-\gamma)\frac{q}{2} + \frac{(1-\eta_{|s})c}{2} - (1-\eta_{|s})c\right), \\
&= (1-\gamma)q\left(\frac{1}{2} - \frac{(1-\eta_{|s})c}{2q(1-\gamma)}\right)^2.
\end{aligned}$$

(c). In this setting, the seller's profit is given by $\Pi(p) = \pi^c(p)$, where

$$\pi^c(p) = \left(1 - \frac{p}{q}\right)\left(\eta_r((1-\gamma)p - (1-\eta_{|r})c) + \eta_s\left(\frac{p}{2}(1-\gamma+\eta_{|s}) - (1-\eta_{|s})c\right)\right) = \left(1 - \frac{p}{q}\right)\Big(\zeta p - (1-\lambda)c\Big),$$

where the second equality follows by setting $\zeta = \eta_r(1-\gamma) + \eta_s(1-\gamma+\eta_{|s})/2$ and noting

$$\sum_{\sigma\in\{r,s\}} \eta_\sigma(1-\eta_{|\sigma})c = \eta_r(1-\eta_{|r})c + \eta_s(1-\eta_{|s})c = (1-\lambda)c,$$

with the final equation following by definition of η_σ and $\eta_{|\sigma}$ (Lemma 6). Note that because the seller accepts $\sigma = s$ offline and $\sigma = r$ online, we have $\gamma \in (\bar{\gamma}_s, \bar{\gamma}_r)$ by Lemma 7. Because $\bar{\gamma}_\sigma = 1 - \eta_{|\sigma}$ by definition, this implies $\eta_{|s} \geq 1-\gamma$ and thus $\zeta \geq 1-\gamma$. Next, for the optimal price we have

$$\frac{\partial}{\partial p}\pi^c(p)\Big|_{p=p^*} = \zeta\left(1 - \frac{p^*}{q}\right) - \frac{1}{q}(\zeta p^* - (1-\lambda)c) = 0,$$

which yields an optimal price of $p^* = p^c$, where

$$p^c = \frac{1}{2}\left(q + c\frac{1-\lambda}{\zeta}\right).$$

The optimal profit is then

$$\begin{aligned}
\Pi^* = \pi^c(p^c) &= \left(1 - \frac{p^c}{q}\right)(\zeta p^c - (1-\lambda)c), \\
&= \left(\frac{1}{2} - \frac{(1-\lambda)c}{2q\zeta}\right)\left(\zeta\frac{q}{2} + \frac{(1-\lambda)c}{2} - (1-\lambda)c\right), \\
&= \zeta q \left(\frac{1}{2} - \frac{(1-\lambda)c}{2q\zeta}\right)^2.
\end{aligned}$$

(d). In this setting, because the seller rejects $\sigma = r$ and accepts $\sigma = s$ offline, the seller's profit is $\Pi(p) = \pi^d(p)$, where

$$\pi^d(p) = \eta_s\left(1 - \frac{p}{q}\right)\left(\frac{p(1-\gamma+\eta_{|s})}{2} - (1-\eta_{|s})c\right).$$

For the optimal price, we have

$$\frac{\partial}{\partial p}\pi^d(p)\Big|_{p=p^*} = \eta_s\left(\frac{(1-\gamma+\eta_{|s})}{2}\left(1 - \frac{p^*}{q}\right) - \frac{1}{q}\left(\frac{1-\gamma+\eta_{|s}}{2}p^* - (1-\eta_{|s})c\right)\right) = 0$$

which implies $p^* = p^d$, where

$$p^d = \frac{q}{2} + c\frac{1-\eta_{|s}}{1-\gamma+\eta_{|s}}.$$

The optimal profit is then

$$\begin{aligned}
\Pi^* = \pi^d(p^d) &= \left(1 - \frac{p^d}{q}\right)\left(p^d\frac{1-\gamma+\eta_{|s}}{2} - (1-\eta_{|s})c\right), \\
&= \left(\frac{1}{2} - \frac{(1-\eta_{|s})c}{q(1-\gamma+\eta_{|s})}\right)\left((1-\gamma+\eta_{|s})\frac{q}{4} + \frac{(1-\eta_{|s})c}{2} - (1-\eta_{|s})c\right), \\
&= \frac{(1-\gamma+\eta_{|s})}{2}q\left(\frac{1}{2} - \frac{(1-\eta_{|s})c}{q(1-\gamma+\eta_{|s})}\right)^2.
\end{aligned}$$

Finally, because no transactions occur online, the platform's revenue is $r^d(p^d) = 0$.

We now verify that $\frac{\partial}{\partial p}\Pi(p) = 0$ must hold at p^* in cases (a)-(d). Note p^x solves $\frac{\partial}{\partial p}\Pi(p) = 0$ for some $x \in \{a, b, c, d\}$, as established above. It suffices to show that if $\Pi(p^*) > 0$, then $p^* = p^x$ for some $x \in \{a, b, c, d\}$. We consider two cases: $\gamma < \bar{\gamma}_s$, and $\gamma \in [\bar{\gamma}_s, \bar{\gamma}_r)$. If $\gamma < \bar{\gamma}_s$, then by Lemmas 2 and 8 the seller's profit function can be written as

$$\Pi(p) = \begin{cases} 0, & \text{if } p < p_s^0, \\ \pi^b(p), & \text{if } p_s^0 \leq p < p_r^0, \\ \pi^a(p), & \text{if } p_r^0 \leq p, \end{cases}$$

where p^0_σ is defined in (9). Further, using the expressions for $v_\sigma(p)$ in (8), we have $\pi^a(p) = \eta_s v_s(p) + \eta_r v_r(p)$ and $\pi^b(p) = \eta_s v_s(p)$. It is straightforward to verify that $v_\sigma(p)$ is strictly concave in p for $\sigma \in \{r, s\}$, and thus so are $\pi^a(p)$ and $\pi^b(p)$. It follows that there are four candidate prices for p^*: p^a, p^b, p^0_s, or p^0_r. By Lemma 8, $\pi^a(p^0_s) = \pi^b(p^0_s) = 0$, which eliminates p^0_s. Further, for $p^* = p^0_r$ to hold, p^0_r must be a local maximizer of $\Pi(p)$. Because $\pi^a(p^0_r) = \pi^b(p^0_r)$, $p^* = p^0_r$ implies

$$\frac{\partial}{\partial p}\pi^b(p)\big|_{p=p^0_r} > 0 > \frac{\partial}{\partial p}\pi^a(p)\big|_{p=p^0_r}.$$

By concavity of π^a and π^b, the inequalities above imply $p^a < p^b$. However, because $\lambda \le \eta_{|s}$, it follows from the expressions for p^a and p^b given in the lemma statement that $p^a \ge p^b$. It follows that $p^* = p^0_r$ cannot hold. Therefore, we must have $p^* = p^a$ or $p^* = p^b$ if $\gamma < \bar{\gamma}_s$. Next, for the case where $\gamma \in [\bar{\gamma}_s, \bar{\gamma}_r)$, it follows from Lemmas 2 and 8 that the seller's profit function is

$$\Pi(p) = \begin{cases} 0, & \text{if } p < \tilde{p}^0_s, \\ \pi^d(p), & \text{if } \tilde{p}^0_s \le p < p^0_r, \\ \pi^c(p), & \text{if } p^0_r \le p, \end{cases} \tag{17}$$

where $\pi^d(p) = \eta_s \tilde{v}_s(p)$ and $\pi^c(p) = \eta_s \tilde{v}_s(p) + \eta_r v_r(p)$, and $\tilde{p}_\sigma$ and $\tilde{v}_\sigma$ are defined in (8) and (9), respectively. Because $v_s(p)$ and $\tilde{v}_s(p)$ are concave, so are $\pi^c(p)$ and $\pi^d(p)$, which implies p^* must be one of p^c, p^d, $\tilde{p}^0_s$, or p^0_r. By definition of π^c and π^d it can be shown that $\Pi(p)$ is continuous and that $\pi^c(\tilde{p}^0_s) = \pi^d(\tilde{p}^0_s) = 0$, which implies $p^* = \tilde{p}^0_s$ cannot hold. It remains to show $p^* = p^0_r$ cannot hold. By parallel argument to the $\gamma < \bar{\gamma}_s$ case, $p^* = p^0_r$ implies

$$\frac{\partial}{\partial p}\pi^d(p)\big|_{p=p^0_r} \ge 0 \ge \frac{\partial}{\partial p}\pi^c(p)\big|_{p=p^0_r} \tag{18}$$

and

$$\pi^c(p^0_r) = \pi^d(p^0_r) > 0. \tag{19}$$

We show that (18) and (19) cannot hold simultaneously. Note $\pi^d(p) - \pi^c(p) = -\eta_r v_r(p)$. Then

$$\frac{\partial}{\partial p}\left(\pi^d(p) - \pi^c(p)\right)\big|_{p=p^0_r} = -\frac{\partial}{\partial p}\eta_r v_r(p)\big|_{p=p^0_r} = -\eta_r\left(1 - \gamma - \frac{c(1-\eta_{|r})}{q}\right).$$

It follows that (18) implies

$$q < \frac{c(1-\eta_{|r})}{1-\gamma} = p^0_r,$$

where the equality follows by definition of p^0_r. It remains to show that if $q < p^0_r$, then $\pi^d(p^0_r) < 0$. The result follows algebraically by plugging $p^0_r = \frac{c(1-\eta_{|r})}{1-\gamma}$ into the expression for $\pi^d(p)$ given in the lemma statement and using $q < \frac{c(1-\eta_{|r})}{1-\gamma}$. Therefore, if $\gamma \in [\bar{\gamma}_s, \bar{\gamma}_r)$, then either $p^* = p^c$ or $p^* = p^d$. $\square$

Lemma 10. *Consider a seller with quality $q > 0$. Define the thresholds*

$$\underline{q} = \frac{c(1-\eta_{|s})}{1-\gamma}, \qquad \bar{q} = \frac{c(1-\lambda-k_1(1-\eta_{|s}))}{(1-\gamma)(1-k_1)},$$

$$q' = \frac{2c(1-\eta_{|s})}{1+\eta_{|s}-\gamma}, \qquad q'' = \frac{c}{1-k_2}\left(\frac{1-\lambda}{\zeta} - \frac{2k_2(1-\eta_{|s})}{1-\gamma+\eta_{|s}}\right),$$

where $k_1 = \sqrt{\eta_s}$, $k_2 = \sqrt{\eta_s(1-\gamma+\eta_{|s})/2\zeta}$, and ζ is as defined in Lemma 9. Suppose transactions can occur only in the online channel for both $\sigma = r$ and $\sigma = s$ buyers. Then the seller *(i)* *rejects* $\sigma \in \{r, s\}$ *if and only if* $q \le \underline{q}$,

(ii) *rejects* $\sigma = r$ *and accepts* $\sigma = s$ *if and only if* $\underline{q} < q \le \bar{q}$, *and*

(iii) *accepts both* $\sigma \in \{r, s\}$ *if and only if* $\bar{q} < q$.

Suppose transactions with the $\sigma = r$ *and* $\sigma = s$ *buyer occur online and offline, respectively. Then the seller*

(iv) *rejects both* $\sigma \in \{r, s\}$ *if and only if* $q \le q'$,

(v) *rejects* $\sigma = r$ *and accepts* $\sigma = s$ *if and only if* $q' < q \le q''$, *and*

(vi) *accepts both* $\sigma \in \{r, s\}$ *if and only if* $q > q''$.

Proof. We first show statements $(i) - (iii)$. For statement (i), because p^a and p^b are each maximizers of strictly concave functions, it follows that $p^b \in [p_s^0, p_r^0)$ and $\pi^b(p^b) > 0$ are necessary conditions for $p^* = p^b$ and $p^a \ge p_r^0$ and $\pi^a(p^a) > 0$ are necessary conditions for $p^* = p^a$. Next, solving $p^a = p_r^0$, $p^b = p_r^0$ and $p^b = p_s^0$ in q yields q^a, q^b, and $\underline{q}$, respectively, where

$$q^a = \frac{c(1 - 2\eta_{|r} + \lambda)}{1 - \gamma}, \qquad q^b = \frac{c(1 - 2\eta_{|r} + \eta_{|s})}{1 - \gamma}.$$

Further, using the fact that $\lambda \in [\frac{1}{2}, 1]$ and $\alpha \in [\frac{1}{2}, 1]$, it can be shown that $0 < \underline{q} < q^a < q^b$. Next, because p^a and p^b are both increasing in q, it follows that neither $p^* = p^a$ nor $p^* = p^b$ can hold if and only if $q \le \underline{q}$, which implies the seller rejects both buyer types. Statement (i) follows. Next, for (ii) and (iii), it suffices to show there exists $\bar{q} > 0$ such that $p^* = p^a$ if $q \ge \bar{q}$ and $p^* = p^b$ if $q \in (\underline{q}, \bar{q})$. Note that for $q \ge \underline{q}$, $p^* = p^a$ if and only if $\pi^a(p^a) \ge \pi^b(p^b)$ and $p^* = p^b$ if and only if $\pi^a(p^a) < \pi^b(p^b)$. Using the profit expressions in Lemma 9, $\pi^a(p^a) \ge \pi^b(p^b)$ is equivalent to

$$(1 - \gamma)q \left(\frac{1}{2} - \frac{(1 - \lambda)c}{2q(1 - \gamma)} \right)^2 \ge \eta_s (1 - \gamma) q \left(\frac{1}{2} - \frac{(1 - \eta_{|s})c}{2q(1 - \gamma)} \right)^2.$$

Simplifying the inequality and setting $k_1 = \sqrt{\eta_s}$ yields

$$\left(\frac{1}{2} - \frac{(1 - \lambda)c}{2q(1 - \gamma)} \right) \ge k_1 \left(\frac{1}{2} - \frac{(1 - \eta_{|s})c}{2q(1 - \gamma)} \right).$$

Finally, rearranging for q gives us

$$q \ge \frac{c(1 - \lambda - k_1(1 - \eta_{|s}))}{(1 - \gamma)(1 - k_1)} = \bar{q}.$$

Statements (ii) and (iii) follow. Finally, we show $\bar{q} > \underline{q}$. Because $\underline{q} < q^a$ as established above, it suffices to show $q^a \le \bar{q}$. By a similar argument to part (i) we have $p^* = p^b$ if $q \in (\underline{q}, q^a)$ and $p^* = p^a$ if $q \ge q^b$. It follows that $q > q^b$ implies $\pi^a(p^a) > \pi^b(p^b)$ and $q < q^a$ implies $\pi^a(p^a) < \pi^b(p^b)$. Because $\pi^a(p^a) = \pi^b(p^b)$ at $\bar{q}$, we conclude $q^a \le \bar{q}$.

The proof for $(iv) - (vi)$ follows by parallel argument to $(i) - (iii)$. Solving $p^c = p_r^0$ and $p^d = \tilde{p}_s^0$ in q yields q^c and q'', respectively, where

$$q^c = \frac{2c(1 - \eta_{|r})}{1 - \gamma} - \frac{c(1 - \lambda)}{\zeta}.$$

Using the expression for ζ in Lemma 9, it is straightforward to verify that $q' < q^c$. Next, by parallel argument to Lemma 10, it can be shown that $\Pi(p) \le 0$ for all $p \ge 0$ if and only if $q < q'$. Statement (iv) follows.

For statements (v) and (vi), we wish to show q'' such that $\pi^c(p^c) \geq \pi^d(p^d)$ if and only if $q \geq q''$. Using the expressions from Lemma 9, $\pi^c(p^c) \geq \pi^d(p^d)$ is equivalent to

$$\zeta q \left(\frac{1}{2} - \frac{(1-\lambda)c}{2q\zeta}\right)^2 \geq \eta_s \frac{(1-\gamma+\eta_{|s})}{2} q \left(\frac{1}{2} - \frac{(1-\eta_{|s})c}{q(1-\gamma+\eta_{|s})}\right)^2.$$

Letting $k_2 = \sqrt{\eta_s(1-\gamma+\eta_{|s})/2\zeta}$ and simplifying yields

$$\left(\frac{1}{2} - \frac{(1-\lambda)c}{2q\zeta}\right) \geq k_2 \left(\frac{1}{2} - \frac{(1-\eta_{|s})c}{q(1-\gamma+\eta_{|s})}\right).$$

Re-arranging for q gives us

$$q \geq \frac{c}{1-k_2}\left(\frac{1-\lambda}{\zeta} - \frac{2k_2(1-\eta_{|s})}{1-\gamma+\eta_{|s}}\right) = q''.$$

Statements (v) and (vi) follow. Finally, we show $q'' \geq q'$. Because $q' < q^c$, it suffices to show $q^c \leq q''$. Note that p^c is increasing in q (Lemma 9) and $p^c(q^c) = p_r^0$ by definition of q^c. It follows that if $q < q^c$, then the profit function $\Pi(p)$ is strictly decreasing on $p \geq p_r^0$, and thus we cannot have $p^* = p^c$ by (17). Therefore, $q > q^c$ is necessary for $p^* = p^c$, or equivalently, $\pi^c(p^c) > \pi^d(p^d)$. Because $\pi^c(p^c) > \pi^d(p^d)$ if and only if $q > q''$, it follows that $q^c \leq q''$. □

For the following lemma, the functions $r^a(\alpha,q,\gamma)$, $r^b(\alpha,q,\gamma)$, and $r^c(\alpha,q,\gamma)$ are as defined in the statement of Lemma 9.

Lemma 11. *Let Assumption 1 hold. Then $r^a(\alpha,q_H,\gamma)$ and $r^c(\alpha,q_H,\gamma)$ both strictly increase in $\gamma \in [0,\gamma^{\max}]$ for all $\alpha \in [\frac{1}{2},1]$. Further, for any $q>0$, $\frac{d}{d\gamma}r^b(\alpha,q,\gamma)$ strictly decreases in $\gamma \in [0,\gamma^{\max}]$ and strictly increases in α on $\alpha \in [\frac{1}{2},1]$.*

Proof. We show $\frac{d}{d\gamma}r^a > 0$ first. Using the expression for $r^a(\alpha,q,\gamma)$ from Lemma 9 and differentiating in γ, we have

$$\frac{dr^a}{d\gamma} = \frac{\partial r^a}{\partial p}\frac{dp^a}{d\gamma} + \frac{\partial r^a}{\partial \gamma} = \gamma\left(1 - \frac{2p^a}{q_H}\right)\frac{c(1-\lambda)}{2(1-\gamma)^2} + p^a\left(1 - \frac{p^a}{q_H}\right).$$

Next, plugging in the expression for p^a from Lemma 9 and simplifying yields

$$\frac{dr^a}{d\gamma} = \frac{1}{4q_H}\left(q_H^2 - \frac{c^2(1-\lambda)^2(1+\gamma)}{(1-\gamma)^3}\right).$$

It follows that $\frac{d}{d\gamma}r^a > 0$ if and only if

$$\sqrt{\frac{c^2(1-\lambda)(1+\gamma)}{(1-\gamma)^3}} < q_H.$$

Using the fact that $\gamma^{\max} = \frac{1}{2}$ and $q_H \geq 4c$ by Assumption 1, we have

$$\sqrt{\frac{c^2(1-\lambda)(1+\gamma)}{(1-\gamma)^3}} \leq \sqrt{\frac{c^2(1-\lambda)(1+\gamma^{\max})}{(1-\gamma^{\max})^3}} = c\sqrt{12(1-\lambda)} < 4c \leq q_H,$$

as desired. Next, we show $\frac{d}{d\gamma}r^c > 0$. Consider the expression for $r^c(\alpha,q_H,\gamma)$ from Lemma 9:

$$r^c(\alpha,q_H,\gamma) = \frac{\gamma\eta_r}{4}q_H\left(1 - \frac{(1-\lambda)^2c^2}{q_H^2\zeta^2}\right),$$

where $\zeta = \eta_r(1-\gamma) + \frac{1}{2}\eta_s(1-\gamma+\eta_{|s})$. Differentiating in γ, we have

$$\frac{dr^c}{d\gamma} = \frac{\eta_r}{4}q_H\left(\left(1-\frac{(1-\lambda)^2c^2}{q_H^2\zeta^2}\right) - 2\gamma\frac{(1-\lambda)^2c^2}{q_H^2\zeta^3}\left(1-\frac{\eta_s}{2}\right)\right) = \frac{\eta_r}{4}q_H\left(1-\frac{(1-\lambda)^2c^2}{q_H^2\zeta^3}\left(\zeta+2\gamma\left(1-\frac{\eta_s}{2}\right)\right)\right).$$

In the above derivative, we used the fact that $\frac{d}{d\gamma}\zeta = -\eta_r - \eta_s/2 = -1+\eta_s/2$. Further, we know that $\gamma \le \gamma^{\max} = \frac{1}{2}$. Since $\eta_{|s} \ge \frac{1}{2}$ as well, this implies

$$\zeta = \eta_r(1-\gamma) + \frac{1}{2}\eta_s(1-\gamma+\eta_{|s}) \ge \frac{1}{2}.$$

Further, we know $\zeta \le 1$ and $\eta_s \ge \lambda \ge \frac{1}{2}$. Plugging these bounds into our expression for $\frac{d}{d\gamma}r^c$ and using the fact that $q_H \ge 4c$ yields

$$\frac{dr^c}{d\gamma} \ge \frac{\eta_r}{4}q_H\left(1-\frac{(1-\lambda)^2c^2}{16c^2\frac{1}{2^3}}\left(\zeta+2\gamma\left(1-\frac{\eta_s}{2}\right)\right)\right) \ge \frac{\eta_r}{4}q_H\left(1-\frac{(1-\lambda)^2}{2}\left(1+\frac{3}{4}\right)\right) > 0.$$

Lastly, we address $\frac{d}{d\gamma}r^b$. By Lemma 9,

$$r^b(\alpha,q,\gamma) = q\frac{\gamma\eta_s}{4}\left(1-\left(\frac{c^2(1-\eta_{|s})^2}{q^2(1-\gamma)^2}\right)\right).$$

Differentiating with respect to γ gives us

$$\frac{dr^b}{d\gamma} = \frac{q\eta_s}{4}\left(1-\left(\frac{(1-\eta_{|s})^2c^2}{q(1-\gamma)^2}\right)\right) - \frac{q^2\eta_s}{4}2\gamma\frac{(1-\eta_{|s})^2c^2}{q^2(1-\gamma)^3} = \frac{q\eta_s}{4}\left(1-(1+\gamma)\frac{(1-\eta_{|s})^2c^2}{q^2(1-\gamma)^3}.\right)$$

Using the final expression on the right hand side above, it is straightforward to verify that $\frac{d}{d\gamma}r^b$ strictly decreases in γ. Similarly, $(1-\eta_{|s})$ strictly decreases in α (Lemma 6), which implies $\frac{d}{d\gamma}r^b$ strictly increases in α. $\square$

LEMMA 12. *Let Assumption 1 hold and suppose only the online channel exists. For all $\gamma \in [0,\gamma^{\max}]$, the type-H seller accepts both buyer types $\sigma \in \{r,s\}$ and the type-L seller rejects the $\sigma = s$ buyer.*

Proof. We address the type-H seller first. By Lemma 10, the type-H seller accepts both buyer types if and only if

$$q_H \ge \frac{c(1-\lambda-k_1(1-\eta_{|s}))}{(1-\gamma)(1-k_1)},$$

where $k_1 = \sqrt{\eta_s}$. We show that for any $q_H \ge 4c$ (i.e., satisfying Assumption 1), the above inequality holds and that the seller's profit is strictly positive for all $\gamma \in [0,\gamma^{\max}]$. Recall $\gamma^{\max} = \frac{1}{2}$. Beginning with the term in the right hand side, we have

$$\frac{c(1-\lambda-k_1(1-\eta_{|s}))}{(1-\gamma)(1-k_1)} \le \frac{c(1-\lambda)}{(1-\gamma^{\max})(1-k_1)} \le 2c\frac{(1-\lambda)}{1-\sqrt{\lambda}} = 2c(1+\sqrt{\lambda}) < 4c \le q_H,$$

as desired. In the above expressions, we used the fact that $k_1 = \sqrt{\eta_s}$ and $\eta_s \le \lambda$ for any $\alpha \in [\frac{1}{2},1]$. It remains to show the type-H seller's profit is positive over $\gamma \in [0,\gamma^{\max}]$; that is, $\pi^a(p^a) > 0$ when $q_H \ge 4c$ for all $\gamma \in [0,\gamma^{\max}]$. Using the expression for $\pi^a(p^a)$ in Lemma 9, we have

$$\pi^a(p^a) = (1-\gamma)q_H\left(\frac{1}{2}-\frac{(1-\lambda)c}{2q_H(1-\gamma)}\right)^2 \ge (1-\gamma)q_H\left(\frac{1}{2}-\frac{(1-\lambda)c}{8c(1-\gamma^{\max})}\right)^2 \ge (1-\gamma)q_H\left(\frac{1}{2}-\frac{(1-\lambda)}{4}\right)^2 > 0.$$

In the above expressions, we used the facts that $\lambda \geq \frac{1}{2}$ and $\gamma^{\max} = \frac{1}{2}$. Next, we show the type-L seller always rejects the $\sigma = s$ buyer. By Lemma 10, it suffices to show that if $q_L \leq c(1-\lambda)$ (i.e., Assumption 1 holds), then

$$q_L \leq \frac{c(1-\lambda-k_1(1-\eta_{|s}))}{(1-\gamma)(1-k_1)}$$

for all $\gamma \in [0, \gamma^{\max}]$. Using the expression for $\eta_{|s}$ in Lemma 6, it can be shown that $\eta_{|s} \geq \lambda$. It follows that

$$\frac{c(1-\lambda-k_1(1-\eta_{|s}))}{(1-\gamma)(1-k_1)} \geq \frac{c(1-\lambda-k_1(1-\lambda))}{(1-k_1)} = \frac{c(1-\lambda)(1-k_1)}{(1-k_1)} = c(1-\lambda) \geq q_L,$$

as desired. □

C. Proofs for Section 2

For convenience, we first prove Lemma 2, and then use the result to prove Lemma 1.

Proof of Lemma 2. We show that trading offline at price $b_\sigma(p)$ yields a higher payoff for both buyer and seller than trading online at price p if and only if $\gamma \geq \bar{\gamma}_\sigma$, where $\bar{\gamma}_\sigma = 1 - \eta_{|\sigma}$ by definition. Pick any $\alpha \in [\frac{1}{2}, 1]$ and $p \geq 0$. Next, using the expression for $b_\sigma(p)$, the seller's surplus from disintermediation is

$$\eta_{|\sigma}\left(\frac{p(1-\gamma+\eta_{|\sigma})}{2\eta_{|\sigma}}\right) - (1-\gamma)p = \frac{p(\gamma+\eta_{|\sigma}-1)}{2}.$$

It follows that the seller's surplus is positive if and only if $\gamma \geq 1-\eta_{|\sigma}$. Next, for the buyer's surplus, we have

$$p - b_\sigma(p) = p - \frac{p(1-\gamma+\eta_{|\sigma})}{2\eta_{|\sigma}} = p\left(\frac{\eta_{|\sigma}-1+\gamma}{2\eta_{|\sigma}}\right).$$

Because $p \geq 0$ and $\eta_{|\sigma} \geq 0$, it follows that $p - b_\sigma(p) \geq 0$ also holds if and only if $\gamma \geq 1-\eta_{|\sigma}$. The result follows. □

Proof of Lemma 1. We consider two cases: $\gamma \leq \bar{\gamma}_s$ and $\gamma \in (\bar{\gamma}_s, \bar{\gamma}_r]$. If $\gamma \leq \bar{\gamma}_s$, then by Lemma 2 all transactions occur in the online channel. The proof then follows by an identical argument to the proof of Lemma 12. It remains to address the case where $\gamma \in (\bar{\gamma}_s, \bar{\gamma}_r]$. By Lemma 2, in this setting all transactions with the $\sigma = s$ buyer occur offline. We start with the type-L seller first. By Lemma 10, we know the type-L seller rejects the $\sigma = r$ buyer if

$$q_L \leq \frac{c}{1-k_2}\left(\frac{1-\lambda}{\zeta} - \frac{2k_2(1-\eta_{|s})}{1-\gamma+\eta_{|s}}\right).$$

Manipulating the left hand side yields

$$\frac{c}{1-k_2}\left(\frac{1-\lambda}{\zeta} - \frac{2k_2(1-\eta_{|s})}{1-\gamma+\eta_{|s}}\right) \geq \frac{c}{1-k_2}\left(\frac{1-\lambda}{\zeta} - k_2\frac{1-\lambda}{\zeta}\right) = c\frac{(1-\lambda)}{\zeta} \geq c(1-\lambda) \geq q_L,$$

where above we have used $\eta_{|s} \geq \lambda$ and $\zeta \leq (1-\gamma+\eta_{|s})/2$ as per Lemma 9. We follow a similar approach for the type-H seller. By Lemma 10, the type-H seller accepts the $\sigma = r$ buyer if and only if

$$q_H \geq \frac{c}{1-k_2}\left(\frac{1-\lambda}{\zeta} - \frac{2k_2(1-\eta_{|s})}{1-\gamma+\eta_{|s}}\right),$$

where $k_2 = \sqrt{\eta_s(1-\gamma+\eta_{|s})/2\zeta}$. Because $q_H \geq 4c$ by Assumption 1, it suffices to show the right hand side of the inequality above does not exceed $4c$. First, following the definitions given in Lemma 6, we have $\eta_s \geq \eta_r$ and $\eta_{|s} \geq 1-\gamma$ for $\gamma \geq \bar{\gamma}_s$. Using these inequalities, we can write

$$k_2 = \sqrt{\frac{\eta_s(1-\gamma+\eta_{|s})}{2\zeta}} = \sqrt{\frac{\eta_s(1-\gamma+\eta_{|s})}{2\eta_r(1-\gamma)+\eta_s(1-\gamma+\eta_{|s})}} \geq \sqrt{\frac{1}{2}}. \tag{20}$$

Next, because $\gamma \leq \gamma^{\max} = \frac{1}{2}$ and $\eta_{|s} \geq \frac{1}{2}$, we have $\zeta \geq \frac{1}{2}$. Combining with (20), it follows that

$$\frac{c}{1-k_2}\left(\frac{1-\lambda}{\zeta} - \frac{2k_2(1-\eta_{|s})}{1-\gamma+\eta_{|s}}\right) \leq c\frac{(1-\lambda)}{\zeta(1-k_2)} \leq \frac{c(1-\lambda)}{\frac{1}{2}(1-k_2)} \leq 2c < q_H.$$

We complete the proof by showing that the type-H seller's profit in this setting is strictly positive. By Lemma 9, we have

$$\pi^c(p^c) = \zeta q_H \left(\frac{1}{2} - \frac{(1-\lambda)c}{2q_H\zeta}\right)^2 > 0$$

where the strictly inequality follows because $\zeta \geq \frac{1}{2}$ and $q_H \geq 4c$. □

D. Proofs for Section 3

For simplicity, throughout the remainder of the Appendix we use $\pi^x(\alpha, q)$ to denote $\pi^x(p^x)$ under the parameters (α, q) for each $x \in \{a, b, c, d\}$, where $\pi^x(p^x)$ is defined in Lemma 9. Additionally, because γ is held fixed in Lemma 3 and Proposition 1, we suppress dependence on it in this section. Since the commission rate γ is fixed in Section 3, we abuse notation and use $r^a(\alpha, q), r^b(\alpha, q), r^c(\alpha, q), r^d(\alpha, q)$ to describe the platform's revenue in the four cases listed in Lemma 9.

Proof of Lemma 3. The proof proceeds in two steps. First, we show that in the absence of the offline channel, the platform's revenue for any fixed γ is given by

$$R(\alpha) = \begin{cases} \mu r^a(\alpha, q_H), & \text{if } \alpha \in [\frac{1}{2}, \alpha_L), \\ \mu r^a(\alpha, q_H) + (1-\mu) r^b(\alpha, q_L), & \text{if } \alpha \in [\alpha_L, 1], \end{cases}$$

for some $\alpha_L \in [\frac{1}{2}, 1)$. Second, we show $r^a(\alpha, q_H)$ is invariant to α and therefore non-decreasing in α, and that $r^b(\alpha, q_L)$ is strictly increasing in α, which proves the lemma statement when combined with the result from Step 1.

Step 1. By Lemma 12, type-H sellers transact with both buyer types. Because all transactions occur online, it follows from Lemma 9 that the platform's revenue from the type-H seller is $r^a(\alpha, q_H)$ for all $\alpha \in [\frac{1}{2}, 1]$. It remains to show there exists $\alpha_L \in [\frac{1}{2}, 1)$ such that the type-L seller's contribution to platform revenue is $r^b(\alpha, q_L)$ if $\alpha \geq \alpha_L$ and 0 otherwise. By Lemma 12 type-L sellers always reject the $\sigma = r$ buyer. The seller's expected profit is then

$$\pi^b(\alpha, q_L) = \eta_s(1-\gamma)q_L\left(\frac{1}{2} - \frac{(1-\eta_{|s})c}{2q_L(1-\gamma)}\right)^2 = \eta_s(1-\gamma)q_L \underbrace{\left(\frac{1}{2} - \frac{(1-\alpha)(1-\lambda)c}{2\alpha\lambda q_L(1-\gamma)}\right)^2}_{\omega}.$$

Using the expression for η_s given in Lemma 6, it can be shown that η_s strictly increases in α. Further, because the expression ω also strictly increases in α, it follows there exists some threshold α_u such that

$\pi^b(\alpha, q_L) \geq 0$ if and only if $\alpha \geq \alpha_u$. Further, because $\pi^b(\alpha, q_L)|_{\alpha=1} > 0$, we have $\alpha_u < 1$. It follows that type-L seller's contribution to platform revenue is $r^b(\alpha, q_L)$ if $\alpha \geq \alpha_L$ and 0 otherwise. This completes the first step.

Step 2. By Lemma 9, the revenue $r^a(\alpha, q_H)$ can be written as

$$r^a(\alpha, q_H) = \gamma p^a \bar{F}\left(\frac{p^a}{q_H}\right) = \gamma \frac{1}{2}\left(q_H + \frac{(1-\lambda)c}{1-\gamma}\right)\frac{1}{2}\left(1 - \frac{(1-\lambda)c}{q_H(1-\gamma)}\right).$$

Clearly, $\frac{d}{d\alpha} r^a(\alpha, q_H) = 0$. Next, using the expression for revenue from Lemma 9 we have

$$\frac{dr^b(\alpha, q)}{d\alpha} = \gamma\left(\frac{\partial \eta_s}{\partial \alpha} p^b\left(1 - \frac{p^b}{q}\right) + \eta_s \frac{\partial}{\partial \eta_{|s}}\left(p^b\left(1 - \frac{p^b}{q}\right)\right)\frac{\partial \eta_{|s}}{\partial \alpha}\right).$$

Using the expression for p^b from Lemma 9 it can be shown that

$$\frac{\partial}{\partial \eta_{|s}}\left(p^b\left(1 - \frac{p^b}{q}\right)\right) = \frac{c^2(1-\eta_{|s})}{q(1-\gamma)^2} > 0. \tag{21}$$

Combining (21) with the fact that $\frac{\partial}{\partial \alpha}\eta_{|s} > 0$ and $\frac{d}{d\alpha}\eta_s > 0$ implies $\frac{d}{d\alpha} r^b(\alpha, q) > 0$, as desired. □

The following critical lemma provides a complete characterization of how platform revenue $R(\alpha)$ changes in α for any fixed γ, from which Proposition 1 immediately follows.

Lemma 13. *For each $\gamma \in (0, \gamma^{\max}]$, there exists $\bar{\alpha} \in [\frac{1}{2}, 1]$, $\underline{\lambda} \in [\frac{1}{2}, 1]$ and $\bar{\lambda} \in (\underline{\lambda}, 1]$ such that the following statements hold.*

(i) If $\gamma < 1 - \lambda$, the platform's revenue $R(\alpha)$ weakly increases on $[\frac{1}{2}, \bar{\alpha})$, decreases sharply at $\alpha = \bar{\alpha}$, and strictly increases (strictly decreases) on $(\bar{\alpha}, 1]$ if $\lambda \leq \underline{\lambda}$ ($\lambda \geq \bar{\lambda}$).

(ii) If $\gamma \geq 1 - \lambda$, the platform's revenue $R(\alpha)$ is zero on $\alpha \in [\frac{1}{2}, \bar{\alpha})$, increases sharply at $\alpha = \bar{\alpha}$ and strictly increases (strictly decreases) on $\alpha \in (\bar{\alpha}, 1]$ if $\lambda \leq \underline{\lambda}$ ($\lambda \geq \bar{\lambda}$).

Proof. We focus on proving statement (i). The proof for (ii) follows by parallel argument and is briefly discussed at the end. In what follows, let $r^a(\alpha, q)$, $r^b(\alpha, q)$ and $r^c(\alpha, q)$ be as defined in Lemma 9, where dependence on γ is suppressed for clarity. The proof for (i) proceeds in three steps: First, we establish that $r^a(\alpha, q_H)$ and $r^b(\alpha, q_L)$ both weakly increase on $\alpha \in [\frac{1}{2}, \bar{\alpha}_s)$, where $\bar{\alpha}_s$ is defined in Lemma 7. Second, we show that for q_H satisfying Assumption 1, there exists $\underline{\lambda} \in [\frac{1}{2}, 1]$ and $\bar{\lambda} \in (\underline{\lambda}, 1]$ such that $r^c(\alpha, q_H)$ strictly increases on $(\bar{\alpha}_s, 1]$ if $\lambda \leq \underline{\lambda}$ and decreases on $(\bar{\alpha}_s, 1]$ if $\lambda \geq \underline{\lambda}$. Third, we use the results of the first two steps to prove statement (i).

Step 1. Following the proof of Proposition 3, we have $\frac{d}{d\alpha} r^a(\alpha, q) = 0$ and $\frac{d}{d\alpha} r^b(\alpha, q) > 0$ for all $\alpha \in [\frac{1}{2}, 1]$. It follows that $\frac{d}{d\alpha} r^a(\alpha, q) \geq 0$ and $\frac{d}{d\alpha} r^b(\alpha, q) \geq 0$ on $\alpha \in [\frac{1}{2}, \bar{\alpha}_s)$.

Step 2. Differentiating $r^c(\alpha, q)$ in α yields

$$\begin{aligned}
\frac{dr^c(\alpha, q)}{d\alpha} &= \gamma\left(\frac{\partial \eta_r}{\partial \alpha} p^c\left(1 - \frac{p^c}{q}\right) + \eta_r \frac{\partial}{\partial \zeta}\left(p^c\left(1 - \frac{p^c}{q}\right)\right)\frac{\partial \zeta}{\partial \alpha}\right), \\
&= \gamma\left((1-2\lambda)\frac{(q\zeta)^2 - (c(1-\lambda))^2}{4q\zeta^2} + \eta_r \frac{c^2(1-\lambda)^2}{2q\zeta^3}\frac{\partial \zeta}{\partial \alpha}\right), \\
&= \gamma\left((1-2\lambda)\frac{q}{4} - \frac{(c(1-\lambda))^2}{4q\zeta^2} + \eta_r \frac{c^2(1-\lambda)^2}{2q\zeta^3}\frac{\partial \zeta}{\partial \alpha}\right).
\end{aligned} \tag{22}$$

By the definition of ζ in Lemma 9, we have

$$\zeta = \eta_r(1-\gamma) + \frac{1}{2}\eta_s(1-\gamma+\eta_{|s}(\alpha)) = \frac{1}{2}\left((1+\eta_r)(1-\gamma)+\alpha\lambda\right),$$

and therefore

$$\frac{\partial \zeta}{\partial \alpha} = \frac{1}{2}\left((1-\gamma)\frac{\partial \eta_r}{\partial \alpha}+\lambda\right) = \frac{1}{2}\left((1-\gamma)(1-2\lambda)+\lambda\right).$$

Next, we determine the sign of $(d/d\alpha)r^c(\alpha,q)$. We show that there exists $\underline{\lambda} \in (\frac{1}{2},1)$ and $\bar{\lambda} \in (\underline{\lambda},1)$ such that $\frac{d}{d\alpha}r^c(\alpha,q) > 0$ for all $\alpha \in [\frac{1}{2},1]$ if $\lambda \le \underline{\lambda}$ and $\frac{d}{d\alpha}r^c(\alpha,q) < 0$ for all $\alpha \in [\frac{1}{2},1]$ if $\lambda \ge \bar{\lambda}$. Using the expressions for ζ and $\frac{\partial \zeta}{\partial \alpha}$ above, we have

$$\begin{aligned}
&\lim_{\lambda\to\frac{1}{2}} \zeta = \frac{1}{4}(\alpha+3(1-\gamma)),\\
&\lim_{\lambda\to\frac{1}{2}} \frac{\partial \zeta}{\partial \alpha} = \frac{1}{4},\\
&\lim_{\lambda\to 1} \zeta = 1-\frac{1}{2}(2-\alpha)\gamma\\
&\lim_{\lambda\to 1} \frac{\partial \zeta}{\partial \alpha} = \frac{\gamma}{2}.
\end{aligned}$$

Combining the limits above with (22) then yields

$$\lim_{\lambda\to 1/2}\frac{dr^c(\alpha,q)}{d\alpha} = \frac{c^2\gamma}{q(\alpha+3(1-\gamma))^3} > 0 \quad \text{and} \quad \lim_{\lambda\to 1}\frac{dr^c(\alpha,q)}{d\alpha} = -\frac{q\gamma}{4} < 0$$

for all $\alpha \in [\frac{1}{2},1]$. The existence of $\underline{\lambda}$ and $\bar{\lambda}$ follow by continuity of $\frac{d}{d\alpha}r^c(\alpha,q)$ in λ.

Step 3. We now prove statement (i). By Lemma 1, the type-H seller accepts $\sigma = r$ for all $\alpha \in [\frac{1}{2},1]$, which implies they accept $\sigma = s$ as well. Next, the proof of Lemma 10 establishes that there exists a threshold $\bar{q}_s$ such that the type-L seller accepts $\sigma = s$ if and only if $q_L \ge \bar{q}_s$, where $\bar{q}_s$ strictly decreases in α. It follows that there exists $\alpha_\ell \in [\frac{1}{2},1]$ such that $q_L \ge \bar{q}_s$ if and only if $\alpha \ge \alpha_\ell$. Further, because $\gamma < 1-\lambda$, by Lemma 7 we have $\bar{\alpha}_r < \frac{1}{2} < \bar{\alpha}_s < 1$. Thus, there are two cases to consider: $\alpha_\ell < \bar{\alpha}_s$ and $\alpha_\ell \ge \bar{\alpha}_s$. We prove statement (i) for both of these cases. In the first case, by Assumption 1 and Lemma 7, the platform's revenue can be written as

$$R(\alpha) = \begin{cases} \mu r^a(\alpha,q_H), & \text{if } \alpha \in [\frac{1}{2},\alpha_\ell), \\ \mu r^a(\alpha,q_H)+(1-\mu)r^b(\alpha,q_L), & \text{if } \alpha \in [\alpha_\ell,\bar{\alpha}_s), \\ \mu r^c(\alpha,q_H)+(1-\mu)r^d(\alpha,q_L), & \text{if } \alpha \in [\bar{\alpha}_s,1]. \end{cases} \tag{23}$$

Using the expressions in Lemma 9, it can be verified that $r^a(\alpha,q_H) > 0$ and $r^c(\alpha,q_H) > 0$ for all $\alpha \in [\frac{1}{2},1]$. Next, because $r^a(\alpha,q)$ is continuous in α and $r^b(\alpha,q) \ge 0$ for $\alpha \ge \alpha_\ell$, we have

$$\lim_{\alpha\to\alpha_\ell^-} R(\alpha) = \mu r^a(\alpha_\ell,q_H) \le \mu r^a(\alpha_\ell,q_H) + (1-\mu)r^b(\alpha_\ell,q_L) = \lim_{\alpha\to\alpha_\ell^+} R(\alpha).$$

Further, as established in Step 1, $\frac{d}{d\alpha}r^b(\alpha,q_L) > 0$ and $\frac{d}{d\alpha}r^a(\alpha,q_H) = 0$ on $\alpha \in [\frac{1}{2},\bar{\alpha}_s)$. It follows that $R(\alpha)$ weakly increases on $\alpha \in [\frac{1}{2},\bar{\alpha}_s)$. We now wish to show that $R(\alpha)$ jumps down at $\bar{\alpha}$, that is, $\lim_{\alpha\to\bar{\alpha}_s^-} R(\alpha) > \lim_{\alpha\to\bar{\alpha}_s^+} R(\alpha)$. Because $r^d(\alpha,q_L) = 0$ (Lemma 9), it suffices to show $\lim_{\alpha\to\bar{\alpha}_s}(r^a(\alpha,q_H) - r^c(\alpha,q_H)) > 0$. First, it is straightforward to verify using the expressions for $\bar{\alpha}_s$ from (7) and ζ from Lemma 9 that $\lim_{\alpha\to\bar{\alpha}_s}\zeta = 1-\gamma$. Therefore, by Lemma 9,

$$\lim_{\alpha\to\bar{\alpha}_s} p^c = \lim_{\alpha\to\bar{\alpha}_s}\frac{1}{2}\left(q+\frac{c(1-\lambda)}{\zeta}\right) = \frac{1}{2}\left(q+\frac{c(1-\lambda)}{1-\gamma}\right) = p^a.$$

Because $\lim_{\alpha\to\bar{\alpha}_s} p^c = p^a$, plugging p^a into the expressions for $r^a(\alpha,q)$ and $r^c(\alpha,q)$ from Lemma 9 yields

$$\begin{aligned}\lim_{\alpha\to\bar{\alpha}_s}(r^a(\alpha,q)-r^c(\alpha,q)) &= \gamma\left(p^a\left(1-\frac{p^a}{q}\right)-\eta_r p^a\left(1-\frac{p^a}{q}\right)\right)\\ &= \gamma p^a\left(1-\frac{p^a}{q}\right)(1-\eta_r)\\ &> 0,\end{aligned}$$

where the final inequality follows because $\eta_r < 1$ for $\lambda \in [\frac{1}{2},1]$. Therefore, $\lim_{\alpha\to\bar{\alpha}_s^-} R(\alpha) > \lim_{\alpha\to\bar{\alpha}_s^+} R(\alpha)$, as desired. The final component of statement (i) follows immediately by applying Step 2 to the expression for $R(\alpha)$ given in (23). This completes the proof of statement (i) for the case where $\alpha_\ell < \bar{\alpha}_s$. For the case where $\alpha_\ell \geq \bar{\alpha}_s$, we can write the platform's revenue as

$$R(\alpha)=\begin{cases}\mu r^a(\alpha,q_H), & \text{if } \alpha\in[\frac{1}{2},\bar{\alpha}_s),\\ \mu r^c(\alpha,q_H)+(1-\mu)r^d(\alpha,q_L), & \text{if } \alpha\in[\bar{\alpha}_s,1],\end{cases}$$

and the result follows by parallel argument to the $\alpha_\ell < \bar{\alpha}_s$ case.

We now prove statement (ii). By Lemma 7, $\bar{\alpha}_s < \frac{1}{2} < \bar{\alpha}_r < 1$. It follows from Lemmas 1 and 7 that the type-H seller accepts $\sigma = r$ offline for $\alpha \in [\frac{1}{2},\bar{\alpha}_r)$, the type-$H$ seller accepts $\sigma = r$ online for $\alpha \in [\bar{\alpha}_r,1]$, the type-$H$ seller accepts $\sigma = s$ offline for all $\alpha \in [\frac{1}{2},1]$, the type-$L$ seller either accepts offline or rejects $\sigma = s$ for all $\alpha \in [\frac{1}{2},1]$, and the type-$L$ seller rejects $\sigma = r$ for all $\alpha \in [\frac{1}{2},1]$. Combining these statements allows us to write the platform's revenue as

$$R(\alpha)=\begin{cases}0, & \text{if } \alpha\in[\frac{1}{2},\bar{\alpha}_r),\\ \mu r^c(\alpha,q_H)+(1-\mu)r^d(\alpha,q_L), & \text{if } \alpha\in[\bar{\alpha}_r,1].\end{cases}$$

The result follows by an identical argument to the proof of Step 2, where $\bar{\alpha}_s$ is substituted with $\bar{\alpha}_r$. □

Proof of Proposition 1. The proof follows from combining statements (i) and (ii) in Lemma 13. □

Proof of Corollary 1. The expression for $\bar{\alpha}$ follows immediately from Lemma 13. Therefore, we focus on proving the bound $\bar{\lambda} < 0.54$. It suffices for us to show that $\frac{dr^c(\alpha,q)}{d\alpha} < 0$ for all α and γ if $\lambda \geq 0.54$. Consider the term $\frac{dr^c(\alpha,q)}{d\alpha} < 0$ from equation (22). We have

$$\frac{dr^c(\alpha)}{d\alpha} = \gamma\left((1-2\lambda)\frac{q_H}{4}+\frac{(c(1-\lambda))^2}{4q_H\zeta^2}+\eta_r\frac{c^2(1-\lambda)^2}{2q_H\zeta^3}\frac{d\zeta}{d\alpha}\right),$$

where $\frac{\partial\zeta}{\partial\alpha} = \frac{1}{2}\left((1-\gamma)(1-2\lambda)+\lambda\right)$. Clearly, the above expression is maximized when q_H takes on its smallest value and γ takes on its largest value (and hence ζ is minimized). Substituting $q_H = 4c$ and $\gamma = \gamma^{\max}$ yields

$$\begin{aligned}\frac{dr^c(\alpha)}{d\alpha} &\leq \gamma^{\max}\left((1-2\lambda)c+\frac{(c(1-\lambda))^2}{16c\zeta^2}+\eta_r\frac{c^2(1-\lambda)^2}{8c\zeta^3}\cdot\frac{1}{4}\right)\\ &= c\gamma^{\max}\left((1-2\lambda)+\frac{(1-\lambda)^2}{16\zeta^2}+\eta_r\frac{(1-\lambda)^2}{32\zeta^3}\right).\end{aligned} \tag{25}$$

Recall that $\zeta = \eta_r(1-\gamma)+\frac{\eta_s}{2}(1-\gamma+\eta_{|s})$, where $\eta_{|s} = \frac{\alpha\lambda}{\alpha\lambda+(1-\lambda)(1-\alpha)}$. For a fixed λ, $\eta_{|s}$ is increasing in α and attains its minimum value at $\alpha = 0.5$. Using this, we get

$$\zeta \geq \eta_r(1-\gamma^{\max})+\frac{\eta_s}{2}(1-\gamma^{\max}+\lambda) \geq \frac{3}{8}+\frac{\lambda}{4}.$$

Combining this inequality with equation (25) and using the bound that $\eta_r \leq \frac{1}{2}$ yields

$$\frac{dr^c(\alpha)}{d\alpha} \leq c\gamma^{\max}\left((1-2\lambda) + \frac{(1-\lambda)^2}{16\left(\frac{3}{8}+\frac{\lambda}{4}\right)^2} + \frac{(1-\lambda)^2}{64\left(\frac{3}{8}+\frac{\lambda}{4}\right)^3}\right).$$

Finally, substituting $\lambda = 0.54$ above, we get $\frac{dr^c(\alpha)}{d\alpha} < 0$, which completes the proof. □

E. Proofs for Section 4

Proof of Proposition 2. We prove the result by constructing a subset of the parameter space (λ, μ, α) where $\gamma^* = \gamma^{\max}$ and $\gamma_0 = \gamma^{\max}$, which implies $\gamma_0 \leq \gamma^*$. The proof proceeds in the following four steps. First, we choose the thresholds $\bar{\lambda}$ and $\underline{\alpha}$ and show $2r^a(\underline{\alpha}, q_H, \bar{\gamma}_s) \leq r^c(\underline{\alpha}, q_H, \gamma^{\max})$, where $r^a(\alpha, q, \gamma)$ and $r^c(\alpha, q, \gamma)$ denote the platform's revenue from sellers who transact fully online and sellers who only transact online with risky buyers, respectively, as defined in Lemma 9. Second, we compare the platform's revenues at two candidate solutions, $\gamma = \bar{\gamma}_s$ and $\gamma = \gamma^{\max}$, and we choose $\mu_{\min}$ so that $R(\underline{\alpha}, \bar{\gamma}_s) \leq R(\underline{\alpha}, \gamma^{\max})$ at $\mu = \mu_{\min}$. Third, we prove that $\gamma^* = \gamma^{\max}$ at $\alpha = \underline{\alpha}$ and $\mu = \mu_{\min}$, which implies $\gamma_0 \leq \gamma^*$. Fourth, we show $\gamma_0 \leq \gamma^*$ continues to hold for all $\alpha \geq \underline{\alpha}$ and all $\mu \geq \mu_{\min}$.

Step 1. For any fixed λ, set $\underline{\alpha}$ such that $\bar{\gamma}_s = \min\left(\frac{1-\lambda}{5}, \frac{q_L}{2c}\right)$. Note $\bar{\gamma}_s$ is decreasing in α (Lemma 6). Our choice of $\bar{\gamma}_s$ implies that

$$\underline{\alpha} = \max\left(\frac{4+\lambda}{4+2\lambda}, \frac{(1-\lambda)(2c-q_L)}{q_L\lambda + (2c-q_L)(1-\lambda)}\right).$$

Next, it is straightforward to verify using the definition of $\bar{\gamma}_r$ in Lemma 6 that $\bar{\gamma}_r$ decreases in λ. Pick $\bar{\lambda}$ to be the solution to $\bar{\gamma}_r = \gamma^{\max} = \frac{1}{2}$. It follows that $\gamma^{\max} \leq \bar{\gamma}_r$ for $\lambda \leq \bar{\lambda}$. In the remainder of the proof, we assume $\lambda \leq \bar{\lambda}$ is fixed and that $\underline{\alpha}$ is chosen so that $\bar{\gamma}_s \leq \min\left(\frac{1-\lambda}{5}, \frac{q_L}{2c}\right)$.

Next, we show $2r^a(\underline{\alpha}, q_H, \bar{\gamma}_s) \leq r^c(\underline{\alpha}, q_H, \gamma^{\max})$. As an intermediate step, we show for $\gamma > 0$ and $\alpha \in [\frac{1}{2}, 1]$ that

$$1 - \left(\frac{(1-\lambda)c}{q_H\zeta}\right)^2 \geq \frac{4}{5}\left(1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma)}\right)^2\right). \tag{26}$$

where $\zeta = \eta_r(1-\gamma^{\max}) + \frac{\eta_s}{2}(1-\gamma^{\max}+\eta_{|s})$, as defined in Lemma 9. Note $\zeta \geq 1-\gamma^{\max} = \frac{1}{2}$. We then have

$$\frac{1 - \left(\frac{(1-\lambda)c}{q_H\zeta}\right)^2}{1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma)}\right)^2} \geq \frac{1 - 4\left(\frac{(1-\lambda)c}{q_H}\right)^2}{1 - \left(\frac{(1-\lambda)c}{q_H}\right)^2} = \frac{1-4z}{1-z}, \tag{27}$$

where $z = \left(\frac{(1-\lambda)c}{q_H}\right)^2$. It is straightforward to verify that the expression on the right hand side of (27) decreases in z on $z \in [0, \frac{1}{4}]$. Further, because $\lambda \in [\frac{1}{2}, 1]$ and $q_H \geq 4c$. we have

$$z = \left(\frac{(1-\lambda)c}{q_H}\right)^2 \leq \left(\frac{(\frac{1}{2})c}{4c}\right)^2 = \frac{1}{64}.$$

Plugging this back into (27) yields

$$\frac{1 - \left(\frac{(1-\lambda)c}{q_H\zeta}\right)^2}{1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma)}\right)^2} \geq \frac{1-4z}{1-z} \geq \frac{1-\frac{1}{16}}{1-\frac{1}{64}} = \frac{20}{21} > \frac{4}{5}.$$

Equation (26) follows from re-arranging the inequality above. We can now prove $2r^a(\underline{\alpha}, q_H, \bar{\gamma}_s) \le r^c(\underline{\alpha}, q_H, \gamma^{\max})$. Note

$$\begin{aligned}
2r^a(\underline{\alpha}, q_H, \bar{\gamma}_s) &= 2\frac{q_H\bar{\gamma}_s}{4}\left(1-\left(\frac{(1-\lambda)c}{q_H(1-\bar{\gamma}_s)}\right)^2\right) \\
&\le \frac{2q_H(1-\lambda)}{20}\left(1-\left(\frac{(1-\lambda)c}{q_H\,(1-\bar{\gamma}_s)}\right)^2\right) \\
&= \frac{q_H(1-\lambda)\gamma^{\max}}{4}\frac{4}{5}\left(1-\left(\frac{(1-\lambda)c}{q_H\,(1-\bar{\gamma}_s)}\right)^2\right) \\
&\le \frac{q_H(1-\lambda)\gamma^{\max}}{4}\left(1-\left(\frac{(1-\lambda)c}{q_H\zeta}\right)^2\right) \qquad (28) \\
&\le r^c(\underline{\alpha}, q_H, \gamma^{\max}). \qquad (29)
\end{aligned}$$

Equation (28) follows from (26), and (29) follows because $\eta_r \ge 1-\lambda$ for all α.

Step 2. We now choose $\mu_{\min}$ such that $R(\underline{\alpha}, \bar{\gamma}_s) \le R(\underline{\alpha}, \gamma^{\max})$ under $\mu = \mu_{\min}$. Specifically, let $\mu_{\min}$ be the smallest value of μ such that $(1-\mu)r^b(\underline{\alpha}, q_L, \bar{\gamma}_s) = \mu r^a(\underline{\alpha}, q_H, \bar{\gamma}_s)$. Then, for every $\mu \ge \mu_{\min}$, we have

$$R(\underline{\alpha}, \bar{\gamma}_s) = (1-\mu)r^b(\underline{\alpha}, q_L, \bar{\gamma}_s) + \mu r^a(\underline{\alpha}, q_H, \bar{\gamma}_s) \le 2\mu r^a(\underline{\alpha}, q_H, \bar{\gamma}_s) \le \mu r^c(\underline{\alpha}, q_H, \gamma^{\max}) = R(\underline{\alpha}, \gamma^{\max}),$$

where the second inequality follows from the inequality established in Step 1.

Step 3. It follows from Step 2 that $\gamma^* = \bar{\gamma}_s$ cannot hold. Further, because $r^c(\underline{\alpha}, q_H, \gamma)$ is strictly increasing in γ (Lemma 11), we cannot have $\gamma^* \in (\bar{\gamma}_s, \gamma^{\max})$. To establish that $\gamma^* = \gamma^{\max}$, it remains to show $\gamma^* < \bar{\gamma}_s$ cannot hold. We do so by showing $\frac{\partial}{\partial\gamma}R(\alpha,\gamma)|_{\alpha=\underline{\alpha},\gamma=\bar{\gamma}_s} > 0$, or equivalently,

$$\frac{\partial}{\partial\gamma}\left(\mu r^a(\alpha, q_H, \gamma) + (1-\mu)r^b(\alpha, q_L, \gamma)\right)\big|_{\alpha=\underline{\alpha},\gamma=\bar{\gamma}_s} > 0, \qquad (30)$$

for every $\mu \ge \mu_{\min}$. Since the partial derivative above is strictly decreasing in γ (Lemma 11), if (30) holds then $R(\alpha,\gamma)$ strictly increases on $\gamma \in [0, \bar{\gamma}_s]$. We know from Lemma 11 that $r^a(\alpha, q_H, \gamma)$ is strictly increasing in γ so its partial derivative must be positive. In order to prove (30), it is suffices to show that $\frac{\partial}{\partial\gamma}\left(r^b(\alpha, q_L, \gamma)\right)|_{\alpha=\underline{\alpha},\gamma=\bar{\gamma}_s} > 0$. Referring to the expression for the partial derivative in Lemma 11, we have:

$$\begin{aligned}
\frac{\partial}{\partial\gamma}r^b(\alpha, q_L, \gamma)\big|_{\alpha=\underline{\alpha},\gamma=\bar{\gamma}_s} &= \frac{q_L\eta_s}{4}\left(1-\frac{\bar{\gamma}_s^2c^2(1+\bar{\gamma}_s)}{q_L^2(1-\bar{\gamma}_s)^3}\right) \\
&\ge \frac{q_L\eta_s}{4}\left(1-\frac{(1+\bar{\gamma}_s)}{4(1-\bar{\gamma}_s)^3}\right) \qquad (31) \\
&\ge \frac{q_L\eta_s}{4}\left(1-\frac{(\frac{6-\lambda}{5})}{4(\frac{4+\lambda}{5})^3}\right) \qquad (32) \\
&\ge \frac{q_L\eta_s}{4}\left(1-\frac{25(\frac{11}{2})}{4(\frac{9}{2})^3}\right) \qquad (33) \\
&> 0.
\end{aligned}$$

In (31) and (32), we used the facts that $\bar{\gamma}_s \le \frac{q_L}{2c}$ and $\bar{\gamma}_s \le \frac{1-\lambda}{5}$, respectively. In Equation (33), we substituted $\lambda = \frac{1}{2}$ since that is where the expression attains its minimum value. It follows that $\gamma^* = \gamma^{\max}$ for every $\mu \ge \mu_{\min}$.

Step 4. We have thus far established that $\gamma^* = \gamma^{\max}$ for $\alpha = \underline{\alpha}$ and $\mu \geq \mu_{\min}$. We now show that $\gamma^* = \gamma^{\max}$ continues to hold as α is increased from $\underline{\alpha}$, which would imply $\gamma^* \geq \gamma_0$ for all $\alpha \geq \underline{\alpha}$ and $\mu \geq \mu_{\min}$. Note that by the analysis in Step 3, there are only two candidate solutions for the optimal commission rate: $\gamma^* \in \{\bar{\gamma}_s, \gamma^{\max}\}$. Further, because $R(\underline{\alpha}, \bar{\gamma}_s) \leq R(\underline{\alpha}, \gamma^{\max})$ by Step 2, to prove the result it is sufficient to show that $R(\alpha, \bar{\gamma}_s) \leq R(\alpha, \gamma^{\max})$ continues to hold for $\alpha \geq \underline{\alpha}$. By (28), at $\alpha = \underline{\alpha}$ we have

$$R(\alpha, \bar{\gamma}_s) \leq \mu \frac{q_H(1-\lambda)\gamma^{\max}}{4}\left(1 - \frac{(1-\lambda)^2 c^2}{q_H^2 \zeta^2}\right) \leq R(\alpha, \gamma^{\max}).$$

Because ζ increases in α, the middle expression above strictly increases in α. Further, using the expression for $r^c(\alpha, q, \gamma)$ from Lemma 9, the second inequality can be shown to hold for all α. Therefore, it remains to show $R(\alpha, \bar{\gamma}_s)$ strictly decreases on $\alpha \geq \underline{\alpha}$. We do so by showing following two inequalities hold for all $\alpha \geq \underline{\alpha}$:

$$\frac{\partial}{\partial \alpha} r^a(\alpha, q_H, \gamma)\big|_{\gamma = \bar{\gamma}_s} < 0,$$
$$\frac{\partial}{\partial \alpha} r^b(\alpha, q_L, \gamma)\big|_{\gamma = \bar{\gamma}_s} < 0.$$

For the first inequality, note

$$\frac{\partial}{\partial \alpha}\left(r^a(\alpha, q_H, \bar{\gamma}_s)\right) = \frac{\partial}{\partial \bar{\gamma}_s}\left(r^a(\alpha, q_H, \bar{\gamma}_s)\right)\frac{\partial}{\partial \alpha}\bar{\gamma}_s.$$

Further, by Lemma 9, note

$$r^a(\alpha, q_H, \bar{\gamma}_s) = \frac{\bar{\gamma}_s q_H}{4}\left(1 - \frac{(1-\lambda)^2 c^2}{q_H^2 (1-\bar{\gamma}_s)^2}\right).$$

By Lemma 11, $r^a(\alpha, q_H, \gamma)$ is strictly increasing in γ. Because $r^a(\alpha, q_H, \bar{\gamma}_s)$ is strictly increasing in $\bar{\gamma}_s$ and $\bar{\gamma}_s$ is strictly decreasing in α, we conclude $\frac{\partial}{\partial \alpha} r^a(\alpha, q_H, \bar{\gamma}_s) < 0$. For the second inequality, note

$$r^b(\alpha, q_L, \bar{\gamma}_s) = q_L \frac{\bar{\gamma}_s \eta_s}{4}\left(1 - \left(\frac{c^2(1-\eta_{|s})^2}{q_L^2(1-\bar{\gamma}_s)^2}\right)\right) = q_L \frac{(1-\alpha)(1-\lambda)}{4}\left(1 - \left(\frac{c^2 \bar{\gamma}_s^2}{q_L^2 (1-\bar{\gamma}_s)^2}\right)\right).$$

In the second equation above, we used the fact that $\bar{\gamma}_s = 1 - \eta_{|s}$, which implies $\bar{\gamma}_s \eta_s = (1-\alpha)(1-\lambda)$. Differentiating gives us

$$\begin{aligned}
\frac{\partial}{\partial \alpha} r^b(\alpha, q_L, \bar{\gamma}_s) &= \frac{q_L}{4}\left(-(1-\lambda)\left(1 - \frac{c^2\bar{\gamma}_s^2}{q_L^2(1-\bar{\gamma}_s)^2}\right) + 2(1-\alpha)(1-\lambda)\frac{c^2(1-\lambda)\bar{\gamma}_s}{\alpha^2 \lambda q_L^2 (1-\bar{\gamma}_s)}\right) \\
&= \frac{q_L}{4}(1-\lambda)\left(-1 + \frac{c^2\bar{\gamma}_s^2}{q_L^2(1-\bar{\gamma}_s)^2} + 2\frac{c^2\bar{\gamma}_s^2}{\alpha q_L^2 (1-\bar{\gamma}_s)^2}\right) \\
&\leq \frac{q_L}{4}(1-\lambda)\left(-1 + \frac{1}{4(1-\bar{\gamma}_s)^2} + \frac{1}{2\alpha(1-\bar{\gamma}_s)^2}\right) && (34) \\
&\leq \frac{q_L}{4}(1-\lambda)\left(-1 + \frac{1}{4\frac{(4+\lambda)^2}{25}} + \frac{1}{2\frac{(4+\lambda)^3}{25(4+2\lambda)}}\right) && (35) \\
&\leq \frac{q_L}{4}(1-\lambda)\left(-1 + \frac{25}{81} + \frac{500}{729}\right) && (36) \\
&< 0.
\end{aligned}$$

In the above series of expressions, (34) follows from $\bar{\gamma}_s \leq q_L/2c$, and (35) is due to $\bar{\gamma}_s \leq (1-\lambda)/5$, which in turn implies that $1 - \bar{\gamma}_s \geq (4+\lambda)/5$ and $\alpha \geq (4+\lambda)/(4+2\lambda)$. Finally, (36) follows by observing that both $(4+\lambda)$ and $(4+\lambda)^3/(4+2\lambda)$ attain their minimum value over $\lambda \in [\frac{1}{2}, 1]$ at $\lambda = 1/2$. It follows that $\gamma^* = \gamma^{\max}$

for all $\alpha \geq \underline{\alpha}$. □

Proof of Corollary 2. We continue from the the proof of Proposition 2. Additionally, we assume that $q_L \leq c(1-\lambda)/2$, as specified in the statement of the corollary. Note the platform's revenue when only the online channel exists is $R^0(\alpha,\gamma) = \mu r^a(\alpha, q_H, \gamma) + (1-\mu)\max\{r^b(\alpha, q_L, \gamma), 0\}$. Therefore, to show $\gamma_0 < \gamma^{\max}$, it suffices to identify (α, μ) such that $\frac{\partial}{\partial\gamma} R^0(\alpha,\gamma)|_{\gamma=\gamma^{\max}} < 0$. Fix $\alpha = \underline{\alpha}$ and $\mu = \mu_{\min}$. To show $\frac{\partial}{\partial\gamma} R^0(\alpha,\gamma)|_{\gamma=\gamma^{\max}} < 0$ under $(\underline{\alpha}, \mu_{\min})$, it suffices to show that the following holds:

$$r^b(\underline{\alpha}, q_L, \gamma^{\max}) \geq 0, \tag{37}$$

$$\frac{\partial}{\partial\gamma}(\mu_{\min} r^a(\alpha, q_H, \gamma) + (1-\mu_{\min}) r^b(\alpha, q_L, \gamma))|_{\alpha=\underline{\alpha},\gamma=\gamma^{\max}} < 0. \tag{38}$$

We show (37) holds first. Note $r^b(\underline{\alpha}, q_L, \gamma^{\max})$ can be written as

$$r^b(q_L, \underline{\alpha}, \gamma^{\max}) = \frac{q_L \eta_s \gamma^{\max}}{4}\left(1 - \frac{\bar{\gamma}_s^2 c^2}{q_L^2(1-\gamma^{\max})^2}\right) \geq \frac{q_L \eta_s \gamma^{\max}}{4}\left(1 - \frac{\frac{q_L^2}{4c^2} c^2}{q_L^2(\frac{1}{2})^2}\right) \geq 0,$$

where the first inequality follows because $\bar{\gamma}_s \leq \frac{q_L}{2c}$ by our choice of $\underline{\alpha}$. Next, using the expression for $(\partial/\partial) r^a(\alpha, q_H, \gamma)$ from the proof of Lemma 11, we have

$$\frac{\partial}{\partial\gamma} r^a(\alpha, q_H, \gamma)|_{\gamma=\gamma^{\max}} = \frac{q_H}{4}\left(1 - 3\frac{(1-\lambda)^2 c^2}{q_H^2(1-\gamma^{\max})^2}\right) \leq \frac{r^a(\underline{\alpha}, q_H, \bar{\gamma}_s)}{\bar{\gamma}_s},$$

where the final step follows because $\bar{\gamma}_s \leq \gamma^{\max}$ and

$$r^a(\underline{\alpha}, q_H, \bar{\gamma}_s) = \frac{q_H}{4}\bar{\gamma}_s\left(1 - \frac{(1-\lambda)^2 c^2}{q_H^2(1-\bar{\gamma}_s)^2}\right)$$

by Lemma 9. Next, for $\frac{\partial}{\partial\gamma} r^b(\alpha, q, \gamma)$, we have

$$\frac{\partial}{\partial\gamma} r^a(\alpha, q_H, \gamma)|_{\alpha=\underline{\alpha},\gamma=\gamma^{\max}} = \frac{q_L \eta_s}{4}\left(1 - \frac{\bar{\gamma}_s^2 c^2(1+\gamma^{\max})}{q_L^2(1-\gamma^{\max})^3}\right) = \frac{q_L \eta_s}{4}\left(1 - 12\frac{\bar{\gamma}_s^2 c^2}{q_L^2}\right), \tag{39}$$

where the second equality follows because $\gamma^{\max} = 1/2$. Note $\bar{\gamma}_s = \min\left(\frac{1-\lambda}{5}, \frac{q_L}{2c}\right)$. We consider both cases to derive an upper bound for the right hand side of (39). First, suppose $\bar{\gamma}_s = (1-\lambda)/5$. Then because $q_L \leq (1-\lambda)c/2$, we have

$$1 - 12\frac{\bar{\gamma}_s^2 c^2}{q_L^2} \leq 1 - 12\left(\frac{(1-\lambda)^2}{25}c^2\right)\frac{4}{(1-\lambda)^2 c^2} = -\frac{23}{25} < 0.$$

In the case where $\bar{\gamma}_s = q_L/2c$, we have

$$1 - 12\frac{\bar{\gamma}_s^2 c^2}{q_L^2} \leq 1 - 12\frac{q_L^2 c^2}{4q_L^2} = -2 < 0.$$

In both cases, we see the right hand side of (39) is upper bounded by $-23/25$. Plugging this back into (39), we have

$$\frac{\partial}{\partial\gamma} r^a(\alpha, q_H, \gamma)\big|_{\alpha=\underline{\alpha},\gamma=\gamma^{\max}} = \frac{q_L \eta_s}{4}\left(1 - 12\frac{\bar{\gamma}_s^2 c^2}{q_L^2}\right) \leq \frac{q_L \eta_s}{4}\left(-\frac{23}{25}\right) < \frac{q_L \eta_s}{4}\left(\frac{\bar{\gamma}_s^2 c^2}{q_L^2(1-\bar{\gamma}_s)^2} - 1\right) = -\frac{r^b(\underline{\alpha}, q_L, \bar{\gamma}_s)}{\bar{\gamma}_s}$$

where the strict inequality above follows because

$$\left(\frac{\bar{\gamma}_s^2 c^2}{q_L^2(1-\bar{\gamma}_s)^2} - 1\right) \geq \left(\frac{\frac{q_L^2}{4c^2} c^2}{q_L^2(\frac{4+\lambda}{5})^2} - 1\right) \geq \left(\frac{25}{81} - 1\right) \geq -\frac{23}{25}.$$

Using these bounds on $\frac{\partial}{\partial\gamma}r^a(\alpha,q,\gamma)$ and $\frac{\partial}{\partial\gamma}r^b(\alpha,q,\gamma)$, we can now show (38). Note

$$\begin{aligned}\frac{\partial}{\partial\gamma}(\mu_{\min}r^a(\alpha,q_H,\gamma)+(1-\mu_{\min})r^b(\alpha,q_L,\gamma))|_{\alpha=\underline{\alpha},\gamma=\gamma^{\max}} &< \mu_{\min}\frac{r^a(\underline{\alpha},q_H,\bar{\gamma}_s)}{\bar{\gamma}_s}-(1-\mu_{\min})\frac{r^b(\underline{\alpha},q_L,\bar{\gamma}_s)}{\bar{\gamma}_s}\\ &= \frac{1}{\bar{\gamma}_s}\left(\mu_{\min}r^a(\underline{\alpha},q_H,\bar{\gamma}_s)=(1-\mu_{\min})r^b(\underline{\alpha},q_L,\bar{\gamma}_s)\right)\\ &=0,\end{aligned}$$

where the final equation follows due to our choice of $\mu_{\min}$ in Step 2. It follows that (38) also holds. Therefore, we have $\frac{\partial}{\partial\gamma}R^0(\alpha,\gamma)|_{\gamma=\gamma^{\max}}<0$ and thus $\gamma_0<\gamma^{\max}$ at $\alpha=\underline{\alpha}$ and $\mu=\mu_{\min}$.

We conclude by choosing $\bar{\alpha}$ and $\mu_{\max}$. First, it can be shown that both r^b and $\frac{\partial}{\partial\gamma}r^b$ are strictly increasing in α. Fixing $\mu=\mu_{\min}$, let $\bar{\alpha}>\underline{\alpha}$ denote the value of α at which $\frac{\partial}{\partial\gamma}(\mu r^a(\alpha,q_H,\gamma)+(1-\mu)r^b(\alpha,q_L,\gamma))|_{\gamma=\gamma^{\max}}=0$. Therefore, for every $\alpha\in(\underline{\alpha},\bar{\alpha})$, it must be the case that $r^b(\alpha,q_L,\gamma^{\max})>0$ and $\frac{\partial}{\partial\gamma}(\mu_{\min}r^a(\alpha,q_H,\gamma)+(1-\mu_{\min})r^b(\alpha,q_L,\gamma))|_{\gamma=\gamma^{\max}}<0$. As a result, $\gamma_0<\gamma^{\max}$ for $\alpha\in(\underline{\alpha},\bar{\alpha})$. Finally, fixing $\alpha\in(\underline{\alpha},\bar{\alpha})$, as μ increases, there exists a threshold $\mu_{\max}>\mu_{\min}$ at which $\frac{\partial}{\partial\gamma}(\mu_{\max}r^a(\alpha,q_H,\gamma)+(1-\mu_{\max})r^b(\alpha,q_L,\gamma))|_{\gamma=\gamma^{\max}}=0$. The result follows. □

Proof of Lemma 4. The proof proceeds in four steps. First, we show the platform's revenue can be written as

$$R(\alpha,\gamma)=\max\{\mu r^a(\alpha,q_H,\gamma),\ \mu r^a(\alpha,q_H,\gamma)+(1-\mu)r^b(\alpha,q_L,\gamma)\}. \tag{40}$$

Second, we show there exists $\alpha_1\in(\frac{1}{2},1)$ such that $\gamma_0=\gamma^{\max}$ for $\alpha\le\alpha_1$. Third, we show there exists $\alpha_2\in(\alpha_1,1)$ such that γ_0 is increasing on (α_1,α_2) and $\gamma_0=\gamma^{\max}$ for $\alpha\ge\alpha_2$. In the fourth step, we prove that $R^*(\alpha)$ is weakly increasing in α.

Step 1. We begin by showing $r^b(\alpha,q_L,\gamma)\le 0$ if and only if the type-L seller is unprofitable, i.e., $\pi^b(\alpha,q_L,\gamma)\le 0$. For convenience, define

$$\psi=\frac{1}{2}-\frac{(1-\eta_{|s})c}{2q_L(1-\gamma)}.$$

By Lemma 9, we then have $\pi^b(\alpha,q_L,\gamma)=\eta_s(1-\gamma)q_L\psi$ and

$$r^b(\alpha,q_L,\gamma)=\gamma\eta_s p^b\bar{F}\left(\frac{p^b}{q_L}\right)=\gamma\eta_s p^b\left(\frac{1}{2}-\frac{(1-\eta_{|s})c}{2q_L(1-\gamma)}\right)=\gamma\eta_s p^b\psi.$$

Because $\eta_s\ge 0$ and $p^b\ge 0$, we have $r^b(\alpha,q_L,\gamma)\le 0$ if and only if $\pi^b(\alpha,q_L,\gamma)\le 0$. It follows that under the condition $\psi\le 0$, we have

$$\mu r^a(\alpha,q_H,\gamma)\ge\mu r^a(\alpha,q_H,\gamma)+(1-\mu)r^b(\alpha,q_L,\gamma).$$

In other words, $R(\alpha,\gamma)$ can be expressed as (40). Next, by Lemma 11, $r^a(\alpha,q_H,\gamma)$ is strictly increasing in γ and therefore maximized at $\gamma^{\max}$. The platform's optimal revenue can then be written as

$$R^*(\alpha)=\max\begin{cases}\mu r^a(\alpha,q_H,\gamma^{\max}),\\ \max_{\gamma\in[0,\gamma^{\max}]}\left(\mu r^a(\alpha,q_H,\gamma)+(1-\mu)r^b(\alpha,q_L,\gamma)\right).\end{cases}$$

Step 2. For convenience, define $h^a=\mu r^a(\alpha,q_H,\gamma^{\max})$ and $h^b(\alpha)=\max_{\gamma\in[0,\gamma^{\max}]}(\mu r^a(\alpha,q_H,\gamma)+(1-\mu)r^b(\alpha,q_L,\gamma))$, so that $R^*(\alpha)=\max(h^a,h^b(\alpha))$. Note h^a is invariant to α. We now show $h^b(\frac{1}{2})<h^a$ and $h^b(1)>h^a$. For the first inequality, following Step 1 of the proof of Proposition 1, there exists $\alpha_L\in(\frac{1}{2},1)$

such that $\pi^b(\alpha, q_L, \gamma) < 0$ and thus $r^b(\alpha, q_L, \gamma) < 0$ for $\alpha < \alpha_L$. As a consequence, we conclude $h^b(\frac{1}{2}) < h^a$ since the contribution of the r^b term is negative. Next, because $\eta_{|s} = 1$ when $\alpha = 1$, we have $r^b(\alpha, q_L, \gamma) > 0$ at $(\alpha, \gamma) = (1, \gamma^{\max})$, which implies that $h^b(1) > h^a$. Further, we observe that for any choice of the parameters, $r^b(\alpha, q_L, \gamma)$ and therefore $h^b(\alpha)$ is strictly increasing in α. Because $h^b(\frac{1}{2}) < h^a$, $h^b(1) > h^a$ and $h^b(\alpha)$ is increasing in α, it follows there exists α_1 such that $h^b(\alpha) < h^a$ for $\alpha < \alpha_1$ and $h^b(\alpha) > h^a$ for $\alpha > \alpha_1$. Further, by continuity of h^a and h^b we have $h^b(\alpha_1) = h^a$ and $\alpha_1 \in (\frac{1}{2}, 1)$. Therefore, $R^*(\alpha) = h^a$ for $\alpha \leq \alpha_1$. Because by Lemma 11, $r^a(\alpha, q_H, \gamma)$ is strictly increasing in γ, it follows that $\gamma_0 = \gamma^{\max}$ for $\alpha \leq \alpha_1$.

Step 3. Let $\gamma^b = \operatorname{argmax}_{\gamma \geq 0}\{\mu r^a(\alpha, q_H, \gamma) + (1-\mu) r^b(\alpha, q_L, \gamma)\}$. Following the proof of Step 1, $\gamma_0 = \min\{\gamma^b, \gamma^{\max}\}$ for $\alpha \geq \alpha_1$. Next, following an identical argument to the proof of Lemma 15(i), γ^b can be shown to be strictly increasing in α. It follows there exists $\alpha_2 \geq \alpha_1$ such that $\gamma_0 = \gamma^b$, γ_0 strictly increases on (α_1, α_2), and $\gamma_0 = \gamma^{\max}$ for $\alpha \in (\alpha_2, 1]$. Lastly, we show $\alpha_2 < 1$. To see why this holds, note that $r^b(\alpha, q_L, \gamma) > 0$ at $(\alpha, \gamma) = (1, \gamma^{\max})$ as established in Step 2, which implies $R(\alpha, \gamma) = \mu r^a(\alpha, q_H, \gamma) + (1-\mu) r^b(\alpha, q_L, \gamma)$ for $(\alpha, \gamma) = (1, \gamma^{\max})$. By Lemma 11, we also have $\frac{d}{d\gamma} r^a(\alpha, q_H, \gamma) > 0$ for $(\alpha, \gamma) = (1, \gamma^{\max})$, and thus $\frac{d}{d\gamma} R(\alpha, \gamma) > 0$ for $(\alpha, \gamma) = (1, \gamma^{\max})$. Then by continuity of r^a, $\frac{d}{d\gamma} r^a$, r^b, and $\frac{d}{d\gamma} r^b$, there exists $\epsilon > 0$ such that $R(\alpha, \gamma) = \mu r^a(\alpha, q_H, \gamma) + (1-\mu) r^b(\alpha, q_L, \gamma)$ for $(\alpha, \gamma) = (1-\epsilon, \gamma^{\max})$ and $\frac{d}{d\gamma} R(\alpha, \gamma) > 0$ for $(\alpha, \gamma) = (1-\epsilon, \gamma^{\max})$. The result follows.

Step 4. It remains to show $R^*(\alpha)$ weakly increases in $\alpha \in [\frac{1}{2}, 1]$. As an intermediate step, we first show $\mu r^a(\alpha, q_H, \gamma) + (1-\mu) r^b(\alpha, q_L, \gamma)$ is strictly concave in γ.

By Lemma 11, $r^b(\alpha, q, \gamma)$ is strictly concave in γ. For $r^a(\alpha, q, \gamma)$, using the expression in Lemma 9, we have

$$\frac{dr^a}{d\gamma} = \frac{1}{4q}\left(q^2 - \frac{c^2(1+\gamma)(1-\lambda)^2}{(1-\gamma)^3}\right).$$

Note $(1+\gamma)/(1-\gamma)^3$ strictly increases in γ. It follows that $\frac{d}{d\gamma} r^a$ strictly decreases in γ and thus $r^a(\alpha, q, \gamma)$ is strictly concave in γ. Therefore, $\mu r^a(\alpha, q_H, \gamma) + (1-\mu) r^b(\alpha, q_L, \gamma)$ is strictly concave in γ.

We now show $R^*(\alpha)$ weakly increases in $\alpha \in [\frac{1}{2}, 1]$. By Proposition 4, there exists $\alpha_1 \in (\frac{1}{2}, 1)$ and $\alpha_2 \in [\alpha_1, 1)$ such that $\gamma_0 = \gamma^{\max}$ for $\alpha \leq \alpha_1$ and $\alpha \geq \alpha_2$, and $\gamma_0 < \gamma^{\max}$ for $\alpha \in (\alpha_1, \alpha_2)$. Note that if $\gamma_0 = \gamma^{\max}$, then $\frac{d}{d\alpha}\gamma_0 = 0$, which implies

$$\frac{dR^*}{d\alpha} = \left(\frac{\partial R}{\partial \gamma}\frac{d\gamma_0}{d\alpha} + \frac{\partial R}{\partial \alpha}\right)\bigg|_{\gamma = \gamma^{\max}} = \frac{\partial R}{\partial \alpha}\bigg|_{\gamma = \gamma^{\max}} \geq 0,$$

where the inequality follows from Proposition 3. It follows that if $\alpha < \alpha_1$ or $\alpha > \alpha_2$, then $R^*(\alpha)$ is weakly increasing at α. Next, pick $\alpha \in (\alpha_1, \alpha_2)$. Following the proof of Proposition 4, this implies $\gamma_0 < \gamma^{\max}$ and that the platform's revenue at γ_0 given by $\mu r^a(\alpha, q_H, \gamma_0) + (1-\mu) r^b(\alpha, q_L, \gamma_0)$. By Step 1, this function is strictly concave in γ, which implies $R(\alpha, \gamma)$ is differentiable in γ at $\gamma = \gamma_0$. It follows from the envelope theorem that

$$\frac{dR^*}{d\alpha}\bigg|_{\gamma = \gamma_0} = \frac{\partial R}{\partial \alpha}\bigg|_{\gamma = \gamma_0} \geq 0,$$

where the inequality again follows from Proposition 3. We have therefore shown that $R^*(\alpha)$ is increasing at α for $\alpha < \alpha_1$, $\alpha \in (\alpha_1, \alpha_2)$, and $\alpha > \alpha_2$. It remains to address $\alpha = \alpha_1$ and $\alpha = \alpha_2$. Following the proof of Proposition 4, γ_0 is continuous at $\alpha = \alpha_2$. It follows that $R(\alpha, \gamma)$ is also continuous at $\alpha = \alpha_2$, which implies $R^*(\alpha)$ weakly increases on $\alpha \geq \alpha_1$. It remains to show that $\lim_{\alpha \to \alpha_1^-} R^*(\alpha) \leq \lim_{\alpha \to \alpha_1^+} R^*(\alpha)$. Following the proof of Lemma 4, $R^*(\alpha) = \mu r^a(\alpha, q_H, \gamma_0)$ for $\alpha \leq \alpha_1$ and $R^*(\alpha) = \mu r^a(\alpha, q_H, \gamma_0) + (1-\mu) r^b(\alpha, q_L, \gamma_0)$ for $\alpha > \alpha_1$, where $r^b(\alpha, q_L, \gamma_0) > 0$ if and only if $\alpha > \alpha_1$. The result follows. $\square$

Corollary 3. *(Corollary of Lemma 4). Suppose transactions can occur online-only. For every* $q_H \geq 4c$, *there exists* $\hat{\mu} \in [0,1]$ *such that for all* $\mu \leq \hat{\mu}$, $\gamma_0 < \gamma^{\max}$ *at* $\alpha = \alpha_1$ *and* γ^0 *strictly increases on* $\alpha \in (\alpha_1, \alpha_2)$.

Proof. Following the proof of Proposition 4, note α_1 is the largest value of α such that $\mu r^a(\alpha, q_H, \gamma^{\max}) \geq \max_\gamma \left(\mu r^a(\alpha, q_H, \gamma) + (1-\mu) r^b(\alpha, q_L, \gamma)\right)$. Analogously, α_2 is the smallest value of α on $\alpha \geq \alpha_1$ such that $\gamma^{\max} = \operatorname{argmax}_{\gamma \in [0,\gamma^{\max}]} \left(\mu r^a(\alpha, q_H, \gamma) + (1-\mu) r^b(\alpha, q_L, \gamma)\right)$. To prove the corollary, suppose by way of contradiction that $\gamma_0 < \gamma^{\max}$ at α_1 for all $\mu \in [0,1]$. This implies the following two inequalities must hold:

$$r^b(\alpha_1, q_L, \gamma^{\max}) \leq 0, \tag{41}$$

$$\frac{d}{d\gamma}\left(\mu r^a(\alpha, q_H, \gamma) + (1-\mu) r^b(\alpha, q_L, \gamma)\right)\Big|_{\alpha=\alpha_1, \gamma=\gamma^{\max}} \geq 0. \tag{42}$$

Note by Proposition 4 that $\alpha_2 < 1$. Next, (41) can be written equivalently as

$$\gamma^{\max} \frac{q_L \eta_s}{4}\left(1 - \left(\frac{(1-\eta_{|s})c}{q_L(1-\gamma^{\max})}\right)^2\right) \leq 0$$

which implies

$$\frac{(1-\eta_{|s})c}{q_L(1-\gamma^{\max})} \geq 1. \tag{43}$$

Using the expressions from the proof of Lemma 11, (42) is equivalent to

$$\mu \frac{q_H}{4}\left(1 - \frac{c^2(1-\lambda)^2(1+\gamma^{\max})}{q_H(1-\gamma^{\max})^3}\right) + (1-\mu)\frac{q_L \eta_s}{4}\left(1 - \frac{c^2(1-\eta_{|s})^2(1+\gamma^{\max})}{q_L(1-\gamma^{\max})^3}\right) \geq 0$$

By applying (43) and using the fact that $\gamma^{\max} = 1/2$ and $\eta_s \geq 1/2$, it follows that the inequality above implies

$$\mu \frac{q_H}{4}\left(1 - 12\frac{c^2(1-\lambda)^2}{q_H}\right) - (1-\mu)\frac{q_L}{4} \geq 0. \tag{44}$$

We conclude that if $\alpha_1 = \alpha_2$ for all $\mu \in [0,1]$, then (44) must hold for all μ. However, note that (44) fails to hold as $\mu \to 0$, which yields a contradiction. We conclude there exists $\hat{\mu}$ such that $\gamma_0 < \gamma^{\max}$ at $\alpha = \alpha_1$ for $\mu \leq \hat{\mu}$. That γ_0 strictly increases on (α_1, α_2) follows from Proposition 4. □

The following two lemmas are useful towards proving Proposition 3.

Lemma 14. *Suppose Assumption 1 holds. There exists* $\bar{\mu}$ *such that if* $\mu \geq \bar{\mu}$, *the following two statements hold.* (i). *There exists* $\underline{\alpha} \in (\frac{1}{2}, 1)$ *and* $\bar{\alpha} \in (\underline{\alpha}, 1)$ *such that* $\gamma^* = \bar{\gamma}_s$ *if* $\alpha \leq \underline{\alpha}$ *and* $\gamma^* = \gamma^{\max}$ *if* $\alpha \geq \bar{\alpha}$. (ii). *There exists* $\bar{\lambda}$ *such that if* $\lambda \geq \bar{\lambda}$, *there exists* $\tilde{\alpha} \in (\underline{\alpha}, \bar{\alpha})$ *such that* $\gamma^* = \bar{\gamma}_r$ *for* $\lambda \geq \bar{\lambda}$ *and* $\alpha \in (\tilde{\alpha}, \bar{\alpha})$.

Proof. We prove the statements in order. (i). For convenience, define

$$R_1(\gamma) = \begin{cases} \mu r^a(\alpha, q_H, \gamma), & \text{if } \alpha \in [\frac{1}{2}, \alpha_L), \\ \mu r^a(\alpha, q_H, \gamma) + (1-\mu) r^b(\alpha, q_L, \gamma), & \text{if } \alpha \in [\alpha_L, 1], \end{cases}$$

and $R_2(\gamma) = \mu r^c(\alpha, q_H, \gamma)$. Following parallel arguments to the proof of Lemma 13, the platform's revenue function in the presence of the offline channel can be written as

$$R(\gamma) = \begin{cases} R_1(\gamma) & \text{if } \gamma < \bar{\gamma}_s, \\ R_2(\gamma) & \text{if } \bar{\gamma}_s \leq \gamma < \bar{\gamma}_r, \\ 0 & \text{if } \bar{\gamma}_r \leq \gamma, \end{cases}$$

where dependence of R_1 and R_2 on α is suppressed for clarity. By Lemma 11, $r^a(\alpha, q_H, \gamma)$ and $r^c(\alpha, q_H, \gamma)$ are both strictly increasing in γ on $\gamma \in [0, \gamma^{\max}]$. It follows that there exists $\bar{\mu}$ such that $R_1(\gamma)$ and $R_2(\gamma)$ strictly increase in γ on $[0, \gamma^{\max}]$ if $\mu \geq \bar{\mu}$. Because $R_1(\gamma)$ and $R_2(\gamma)$ are both strictly increasing in γ for $\mu \geq \bar{\mu}$, we have $\gamma^* \in \{\bar{\gamma}_s, \bar{\gamma}_r, \bar{\gamma}^{\max}\}$ for $\mu \geq \bar{\mu}$. Next, by definition of $\bar{\gamma}_s$ (Lemma 7), we have $\lim_{\alpha \to 1} \bar{\gamma}_s = 0$, which by definition of R_1 implies $\lim_{\alpha \to 1} R_1(\bar{\gamma}_s) = 0$. With some effort, it can be verified that $\lim_{\alpha \to 1} R_2(\bar{\gamma}_r) > 0$ and $\lim_{\alpha \to 1} R_2(\gamma^{\max}) > 0$, which implies $\gamma^* \in \{\bar{\gamma}_r, \gamma^{\max}\}$. Further, because $\lim_{\alpha \to 1} \bar{\gamma}_r = 1 > \gamma^{\max}$, it follows that there exists $\bar{\alpha} \in (\frac{1}{2}, 1)$ such that $\gamma^* = \gamma^{\max}$ for $\alpha \geq \bar{\alpha}$.

It remains to show there exists $\underline{\alpha} < \bar{\alpha}$ such that $\gamma^* = \bar{\gamma}_s$ for $\alpha \leq \underline{\alpha}$. We again start with $\gamma^* \in \{\bar{\gamma}_s, \bar{\gamma}_r, \gamma^{\max}\}$, and show $\gamma^* = \bar{\gamma}_r$ and $\gamma^* = \gamma^{\max}$ cannot hold for sufficiently small α. Note for $\alpha \leq \alpha_L$, $R_1(\gamma) = \mu r^a(\alpha, q_H, \gamma)$. Further, using the expressions in Lemma 6 we have $\lim_{\alpha \to 1/2} \bar{\gamma}_s = \lim_{\alpha \to 1/2} \bar{\gamma}_r = 1 - \lambda$. Using this fact and the expressions for $r^a(\alpha, q, \gamma)$ and $r^c(\alpha, q, \gamma)$ in Lemma 9, it can be shown algebraically that

$$\lim_{\alpha \to 1/2} (R_1(\bar{\gamma}_s) - R_2(\bar{\gamma}_r)) = \frac{(1-\lambda)(c(1-\lambda) - q_H \lambda)^2}{8 q_H \lambda^2} > 0.$$

Therefore, we cannot have $\gamma^* = \bar{\gamma}_r$. Lastly, because $\lim_{\alpha \to 1/2} \bar{\gamma}_r = 1 - \lambda$ and $\lambda \in (\frac{1}{2}, 1)$, for sufficiently small α we have $\bar{\gamma}_r > \gamma^{\max}$. Because $R_2(\gamma)$ is strictly increasing, we also have $R_2(\bar{\gamma}_r) > R_2(\gamma^{\max})$, which implies $\gamma^* = \gamma^{\max}$ cannot hold. The result follows by picking $\underline{\alpha}$ appropriately.

(ii). Using the expression for $\bar{\gamma}_r$ in Lemma 6 and the fact that $\bar{\gamma}_r$ is increasing in α (Lemmas 2 and 6), it can be shown that $\bar{\gamma}_r \leq \frac{1}{2} = \gamma^{\max}$ if and only if $\alpha \leq \lambda$. Set $\bar{\lambda} = \bar{\alpha}$. Because $\bar{\alpha} > \underline{\alpha}$, it follows that there exists $\tilde{\alpha} \in (\underline{\alpha}, \bar{\alpha})$ such that $\bar{\gamma}_r < \gamma^{\max}$ and $R_2(\bar{\gamma}_r) > R_1(\bar{\gamma}_s)$ for $\alpha \in (\tilde{\alpha}, \bar{\alpha})$. The result follows. □

LEMMA 15. *Suppose $\gamma^* < \gamma^{\max}$. Then the following results hold:*

(i) If $\gamma^* < \bar{\gamma}_s$, then either $\frac{d}{d\alpha}\gamma^* = 0$ or $\frac{d}{d\alpha}\gamma^* > 0$.

(ii) If $\gamma^* = \bar{\gamma}_s$, then $\frac{d}{d\alpha}\gamma^* < 0$.

(iii) If $\gamma^* \in (\bar{\gamma}_s, \bar{\gamma}_r)$, then there exists $\underline{\lambda} \in (\frac{1}{2}, 1)$ and $\bar{\lambda} \in (\underline{\lambda}, 1)$ such that $\frac{d}{d\alpha}\gamma^* < 0$ if $\lambda \leq \underline{\lambda}$ and $\frac{d}{d\alpha}\gamma^* > 0$ if $\lambda \geq \bar{\lambda}$.

(iv) If $\gamma^* = \bar{\gamma}_r$, then $\frac{d}{d\alpha}\gamma^* > 0$.

Proof. We prove the statements in order. (i). Suppose $\gamma^* < \bar{\gamma}_s$. It follows from Lemmas 1 and 7 that there are two cases for the revenue function depending on whether the type-L seller accepts or rejects the $\sigma = s$ buyer. By Lemma 10, there exists $\underline{q}$ such that

$$R(\alpha, \gamma) = \begin{cases} \mu r^a(\alpha, q_H, \gamma) & \text{if } q_L < \underline{q}, \\ \mu r^a(\alpha, q_H, \gamma) + (1-\mu) r^b(\alpha, q_L, \gamma) & \text{if } q_L \geq \underline{q}. \end{cases}$$

Further, because $\gamma^* < \bar{\gamma}_s$, we must have $\frac{d}{d\gamma} R(\alpha, \gamma)|_{\gamma = \gamma^*} = 0$. By an application of the implicit function theorem,

$$\frac{d\gamma^*}{d\alpha} = -\left(\frac{\partial^2 R}{\partial \gamma \partial \alpha}\right)\left(\frac{\partial^2 R}{\partial \gamma^2}\right)^{-1}\Bigg|_{\gamma = \gamma^*}.$$

Because γ^* is a local maximizer of $R(\alpha,\gamma)$, we must have $\frac{d^2}{d\gamma^2}R|_{\gamma=\gamma^*}<0$. It follows that $\frac{d}{d\alpha}\gamma^*$ has the same sign as $\frac{d^2}{d\gamma d\alpha}R|_{\gamma=\gamma^*}$. Next, by Lemma 9, $\frac{\partial}{\partial\alpha}p^a=0$, which implies $\frac{d}{d\alpha}r^a=0$. Therefore $q_L<\underline{q}$ implies $\frac{d}{d\alpha}\gamma^*=0$. In the case where $q_L\geq\underline{q}$, it follows that $\frac{d^2}{d\gamma d\alpha}R$, and therefore $\frac{d}{d\alpha}\gamma^*$, has the same sign as $\frac{d^2}{d\gamma d\alpha}r^b$. We then have

$$\frac{d^2r^b}{d\gamma d\alpha}=\frac{\partial}{\partial\alpha}\left(\frac{\partial r^b}{\partial p}\frac{\partial p^b}{\partial\gamma}+\frac{\partial r^b}{\partial\gamma}\right)=\frac{\partial^2r^b}{\partial p\partial\alpha}\frac{\partial p^b}{\partial\gamma}+\frac{\partial r^b}{\partial p}\frac{\partial^2p^b}{\partial\gamma\partial\alpha}+\frac{\partial^2r^b}{\partial\gamma\partial\alpha}. \tag{45}$$

Next, with some effort it can be shown that

$$\frac{\partial^2r^b}{\partial p\partial\alpha}=-\frac{\gamma(2p-q)(2\lambda-1)}{q},\quad \frac{\partial r^b}{\partial p}=\frac{\gamma(2p-q)\eta_s}{q},\quad \frac{\partial^2r^b}{\partial\gamma\partial\alpha}=p\left(1-\frac{p}{q}\right)(2\lambda-1),$$

and

$$\frac{\partial p^b}{\partial\gamma}=\frac{c(1-\eta_{|s})}{2(1-\gamma)^2},\quad \frac{\partial^2p^b}{\partial\gamma\partial\alpha}=-\frac{c\lambda(1-\lambda)}{2\eta_s^2(1-\gamma)^2}.$$

Next, for convenience let $\psi=c(1-\eta_{|s})/(1-\gamma)$. Then by Lemma 9, we have $p^b=\frac{1}{2}(q+\psi)$ and $2p^b-q=\psi$. Substituting for ψ and using the definitions of $\eta_{|s}$ and η_s, we can now re-write (45) equivalently as

$$\frac{\partial^2r^b}{\partial\gamma\partial\alpha}=\frac{1}{2}(q+\psi)\left(1-\frac{1}{2q}(q+\psi)\right)-\frac{\gamma\psi^2}{2q(1-\gamma)}\left(1+\frac{\lambda}{1-\alpha}\right).$$

Next, using the expression above, with some effort it can be shown that

$$\begin{aligned}
&\lim_{\alpha\to1/2}\frac{\partial^2r^b}{\partial\gamma\partial\alpha}=(1-\mu)\left(\frac{1}{4}q(2\lambda+1)+\frac{c(1+\gamma)(1-\lambda)^2(1+2\lambda)}{4q(1-\gamma)^3}\right)>0,\\
&\lim_{\alpha\to1}\frac{\partial^2r^b}{\partial\gamma\partial\alpha}=\frac{1}{4}q(2\lambda+1)>0,\\
&\frac{\partial^3r^b}{\partial\gamma\partial\alpha^2}=-\frac{(1+\gamma)(c\lambda(1-\lambda))^2}{2q\eta_s^3(1-\gamma)^3}<0.
\end{aligned}$$

Because $(\partial^2r^b/\partial\gamma\partial\alpha)>0$ at both $\alpha=\frac{1}{2}$ and $\alpha=1$ and is strictly decreasing for all α, we must have $(\partial^2/\partial\gamma\partial\alpha)r^b>0$ for all $\alpha\in[\frac{1}{2},1]$. Therefore, $\frac{d}{d\alpha}\gamma^*>0$ for all $\alpha\in[\frac{1}{2},1]$.

(ii). If $\gamma^*=\bar{\gamma}_s$, then by definition of $\bar{\gamma}_s$ in Lemma 7 we have

$$\frac{d\gamma^*}{d\alpha}=\frac{d}{d\alpha}\left(\frac{(1-\alpha)(1-\lambda)}{\alpha\lambda+(1-\alpha)(1-\lambda)}\right)=-\frac{(1-\lambda)\lambda}{\eta_s^2}<0.$$

Therefore, if $\gamma^*=\bar{\gamma}_s$, then $\frac{d}{d\alpha}\gamma^*<0$.

(iii). Suppose $\gamma^*\in(\bar{\gamma}_s,\bar{\gamma}_r)$. The proof proceeds by parallel argument to Case 1. Similar to Case 1, it follows from Lemmas 1 and 7 that there are two cases for the revenue function depending on whether the type-L seller accepts or rejects the $\sigma=s$ buyer. By Lemma 10, there exists $\underline{q}$ such that

$$R(\alpha,\gamma)=\begin{cases}\mu r^c(\alpha,q_H,\gamma) & \text{if } q_L<\underline{q},\\ \mu r^c(\alpha,q_H,\gamma)+(1-\mu)r^d(\alpha,q_L,\gamma) & \text{if } q_L\geq\underline{q}.\end{cases}$$

Note that $r^d(\alpha,q,\gamma)=0$ by Lemma 9, meaning the platform's revenue is simply $R(\alpha,\gamma)=\mu r^c(\alpha,q_H,\gamma)$. Because $\gamma^*\in(\bar{\gamma}_s,\bar{\gamma}_r)$, we must have $\frac{d}{d\gamma}R(\alpha,\gamma)|_{\gamma=\gamma^*}=0$. By the implicit function theorem and the definition of $R(\alpha,\gamma)$ above, the sign of $\frac{d}{d\alpha}\gamma^*$ is given by the sign of $\frac{d^2}{d\gamma d\alpha}r^c$. First, using the price and revenue expressions in Lemma 9, we have

$$\begin{aligned}
\frac{dr^c}{d\gamma}&=\frac{\partial r^c}{\partial p}\frac{\partial p^c}{\partial\gamma}+\frac{\partial r^c}{\partial\gamma},\\
&=\gamma\eta_s\left(1-\frac{2p^c}{q}\right)\left(1-\frac{\eta_r}{2}\right)\left(\frac{c(1-\lambda)}{2\zeta^2}\right)+p^c\left(1-\frac{p^c}{q}\right)\eta_r,\\
&=\underbrace{(1-\eta_s)\frac{p^c(p^c-q)}{q}+\eta_s(1+\eta_s)\frac{\gamma c(1-\lambda)\zeta^2(2p^c-q)}{4q}}_{\omega},
\end{aligned}\tag{47a}$$

where the third line follows because $\eta_s+\eta_r=1$ by Lemma 6. For convenience, we define ω as the right hand side of (47a). Next, using the expression for p^c in Lemma 9, it can be shown that

$$\begin{aligned}\frac{\partial\omega}{\partial\alpha}&=\frac{\partial\omega}{\partial\eta_s}\frac{\partial\eta_s}{\partial\alpha}+\frac{\partial\omega}{\partial\zeta}\frac{\partial\zeta}{\partial\alpha}\\&=\left(\frac{q}{4}-\frac{c^2(1+\zeta^3\gamma(1+2\eta_s))(1-\lambda)^2}{4q\zeta^2}\right)\frac{\partial\eta_s}{\partial\alpha}-\frac{c^2(2+\eta_s^2\zeta^3\gamma-\eta_s(2-\zeta^3\gamma))(1-\lambda)^2}{4q\zeta^3}\frac{\partial\zeta}{\partial\alpha},\end{aligned}$$

where $\frac{\partial}{\partial\alpha}\eta_s=1-2\lambda$ and $\frac{\partial}{\partial\alpha}\zeta=(\frac{1}{2})(\lambda\gamma+(1-\lambda)(1-\gamma))$. It is straightforward to verify from the expression above that $\lim_{\lambda\to1}\frac{\partial}{\partial\alpha}\omega=q/4>0$. Similarly, using the definitions of η_s and ζ from Lemmas 6 and 9 respectively, it can be shown that

$$\lim_{\lambda\to1/2}\frac{\partial\omega}{\partial\alpha}=-\frac{c^2}{q}\left(\frac{1}{(\alpha+3(1-\gamma))^3}+\frac{3\gamma}{256}\right)<0.$$

By continuity of $\frac{\partial}{\partial\alpha}\omega$, it follows that there exists $\bar{\lambda}\in(\frac{1}{2},1)$ and $\underline{\lambda}\in(\frac{1}{2},\bar{\lambda})$ such that if $\gamma^*\in(\bar{\gamma}_s,\bar{\gamma}_r)$, then $\frac{d}{d\alpha}\gamma^*>0$ for $\lambda\geq\bar{\lambda}$ and $\frac{d}{d\alpha}\gamma^*<0$ for $\lambda\leq\underline{\lambda}$.

(iv). If $\gamma^*=\bar{\gamma}_r$, then by definition of $\bar{\gamma}_r$ in Lemma 7 we have

$$\frac{d\gamma^*}{d\alpha}=\frac{d}{d\alpha}\left(\frac{\alpha(1-\lambda)}{\alpha(1-\lambda)+(1-\alpha)\lambda}\right)=\frac{\lambda(1-\lambda)}{\eta_r^2}>0.$$

The result follows. □

Proof of Proposition 3. We show γ^* is decreasing first. By Lemma 14, there exists $\bar{\mu}$ and $\underline{\alpha}$ such that if $\mu\geq\mu$, then $\gamma^*=\bar{\gamma}_s$ for $\alpha\leq\underline{\alpha}$. By Lemma 15, $\bar{\gamma}_s$ is strictly decreasing in α. It follows that γ^* strictly decreases on $\alpha\in[\frac{1}{2},\underline{\alpha}]$. Next, we show $R^*(\alpha)$ is decreasing on some interval $\alpha\in[\frac{1}{2},\bar{\bar{\alpha}}]$, differentiating $R^*(\alpha)$ in α yields

$$\frac{dR^*}{d\alpha}=\left(\frac{\partial R}{\partial\gamma}\frac{d\gamma^*}{d\alpha}+\frac{\partial R}{\partial\alpha}\right)\bigg|_{\gamma=\gamma^*}.$$

We prove the result by showing there exists $\bar{\bar{\alpha}}$ and $\bar{\mu}$ such that for $\mu\geq\bar{\mu}$ and $\alpha\leq\bar{\bar{\alpha}}$ we have $\frac{\partial}{\partial\gamma}R>0$, $\frac{d}{d\alpha}\gamma^*<0$, and $(\partial/\partial\alpha)R=0$ at $\gamma=\gamma^*$. For $\frac{\partial}{\partial\gamma}R$, consider the following three statements: By Lemma 14 there exists $\bar{\mu}\in(\frac{1}{2},1)$ and $\alpha'\in(\frac{1}{2},1)$ such that $\gamma^*=\bar{\gamma}_s$ if $\mu\geq\bar{\mu}$ and $\alpha\leq\alpha'$; by Step 3 of the proof of Lemma 13 there exists $\alpha''\in(\frac{1}{2},1)$ such that $R(\alpha,\gamma)=\mu r^a(\alpha,q_H,\gamma)$ for $\alpha\leq\alpha''$; and by Lemma 11 $r^a(\alpha,q_H,\gamma)$ strictly increases on $\gamma\in[0,\bar{\gamma}_s]$. Combining these three statements and setting $\bar{\bar{\alpha}}=\min\{\alpha',\alpha''\}$ implies $\frac{\partial}{\partial\gamma}R>0$ at $\gamma=\gamma^*$ for $\alpha\leq\bar{\bar{\alpha}}$ and $\mu\geq\bar{\mu}$. Next, for $\frac{d}{d\alpha}\gamma^*$, because $\gamma^*=\bar{\gamma}_s$ for $\mu\geq\bar{\mu}$ and $\alpha\leq\alpha'$, by Lemma 15 we have $\frac{d}{d\alpha}\gamma^*<0$. Lastly, for $\frac{\partial}{\partial\alpha}R$, because $\frac{\partial}{\partial\alpha}p^a=0$ by Lemma 9, we have $\frac{d}{d\alpha}r^a(\alpha,q,\gamma)=0$. Further, because $R(\alpha,\gamma)=\mu r^a(\alpha,q_H,\gamma)$ for $\alpha\leq\alpha''$ as established above, it follows that $\frac{\partial}{\partial\alpha}R(\alpha,\gamma)=0$ for $\alpha\leq\bar{\bar{\alpha}}$. The proposition statement follows by setting $\bar{\alpha}=\min\{\underline{\alpha},\bar{\bar{\alpha}}\}$. □

F. Proofs for Section 5

LEMMA 16. *Let $\phi_L(\alpha)=\Pi_0^L$ and $\phi_H(\alpha)=\Pi_0^H$. (i). Then for each seller type $j\in\{H,L\}$, access fee $\phi>0$ and accuracy $\alpha\in[\frac{1}{2},1]$, a type-j seller joins the platform if and only if $\phi\leq\phi_j(\alpha)$. Further, $\phi_H(\alpha)$ is invariant to α, $\phi_L(\alpha)$ is strictly increasing in α, and $0<\phi_L(\alpha)<\phi_H(\alpha)$ for all $\alpha\in[\frac{1}{2},1]$. (ii). The platform's optimal revenue under access fees is given by $R_\phi^*=\max\{\mu\Pi_0^H,\Pi_0^L\}$.*

Proof. We first address the type-H seller, followed by the type-L seller. By Lemmas 1 and 9, the type-H seller joins if and only if $\phi \leq \phi_H(\alpha)$. Further, because $\frac{d}{d\alpha}\Pi_0^H = 0$ by Lemma 9, we have $\frac{d}{d\alpha}\phi_H(\alpha) = 0$. By parallel argument to the type-H case, by Lemmas 1 and 9 the type-L seller joins if and only if $\phi \leq \phi_L(\alpha)$. Further, by Lemma 6 we have $\frac{\partial}{\partial\alpha}\eta_s > 0$ and $\frac{\partial}{\partial\alpha}\eta_{|s} > 0$, which implies $\frac{d}{d\alpha}\Pi_0^L > 0$. It follows that $\frac{d}{d\alpha}\phi_L(\alpha) > 0$. Finally, to see that $0 < \phi_L(\alpha) < \phi_H(\alpha)$ for all $\alpha \in [\frac{1}{2}, 1]$, note

$$\phi_L(\alpha) \leq \lim_{\alpha\to 1}\phi_L(\alpha) = \lim_{\alpha\to 1}\Pi_0^L < \lim_{\alpha\to 1}\pi^b(\alpha, q_H, 0) \leq \Pi_0^H = \phi_H(\alpha),$$

where above we have used the fact that $\phi_L(\alpha)$ increases in α (Lemma 16), the definition of $\phi_L(\alpha)$, that $\pi^b(\alpha, q, 0)$ increases in q, and that $\pi^b(\alpha, q_H, 0) < \Pi_0^H$ for all $\alpha \in [\frac{1}{2}, 1]$ by the expressions in Lemma 9. Lastly, $\phi_L(\alpha) > 0$ can be verified by definition of $\phi_L(\alpha)$ and Lemma 9.

(ii). By part (i), if $\phi \leq \pi^b(\alpha, q_H, 0)$ then both seller types join, which generates a revenue of ϕ. If $\phi \in [\Pi_0^L, \Pi_0^H]$, then only the type-$H$ seller joins the platform, which generates revenue $\mu\phi$. It follows that $\phi^* \in \{\Pi_0^H, \Pi_0^L\}$ and thus $R_\phi^* = \max\{\mu\Pi_0^H, \Pi_0^L\}$. $\square$

LEMMA 17. *Suppose for some $\gamma \in [0, \gamma^{\max}]$ that only one type of seller transacts online. Then the platform's commission revenue under γ is strictly smaller than the optimal revenue from access fees:*

$$R_\gamma(\alpha, \gamma) < R_\phi^*.$$

Proof of Lemma 17. The proof proceeds in two steps. First, we show $r^a(\alpha, q_H, \gamma) < \Pi_0^H$ for all $\gamma \in [0, \gamma^{\max}]$. Second, we use this inequality to prove the main result.

Step 1. Note for $\gamma \in [0, \gamma^{\max}]$, we have

$$\begin{aligned} r^a(\alpha, q_H, \gamma) &\leq r^a(\alpha, q_H, \gamma^{\max}) \\ &= \gamma^{\max} q_H \left(1 - \frac{(1-\lambda)^2 c^2}{q_H^2 (1-\gamma^{\max})^2}\right) \\ &\leq \gamma^{\max} q_H \cdot \frac{9}{7}\left(1 - \frac{(1-\lambda)c}{q_H}\right)^2 \\ &= \gamma^{\max}\frac{9}{7}\Pi_0^H \\ &< \Pi_0^H, \end{aligned} \tag{49}$$

where the first line follows from Lemma 11, the second and fourth lines from Lemma 9, and the final line because $\gamma^{\max} = 1/2$. To show that (49) holds, we have

$$\frac{1 - \frac{(1-\lambda)^2 c^2}{q_H^2(1-\gamma^{\max})^2}}{\left(1 - \frac{(1-\lambda)c}{q_H}\right)^2} \leq \frac{1 - \frac{(1-\lambda)^2 c^2}{q_H^2}}{\left(1 - \frac{(1-\lambda)c}{q_H}\right)^2} \leq \frac{1 - \frac{(1-\frac{1}{2})^2 c^2}{16c^2}}{\left(1 - \frac{(1-\frac{1}{2})c}{4c}\right)^2} = \frac{9}{7},$$

where the second inequality above follows from maximizing the ratio over $\lambda \in [\frac{1}{2}, 1]$ and $q_H \geq 4c$, which occurs at $\lambda = \frac{1}{2}$ and $q_H = 4c$. Therefore, $r^a(\alpha, q_H, \gamma) < \Pi_0^H$ for all $\gamma \in [0, \gamma^{\max}]$.

Step 2. We now show $R_\gamma(\alpha, \gamma) < R_\phi^*$ if only one seller type transacts online under γ. Following Lemma 2, there are only two cases in which only one seller type transacts online: (1) if $r^b(\alpha, q_L, \gamma) = 0$ and $\gamma \leq \bar{\gamma}_s$, so the type-$H$ seller transacts with both $\sigma = r$ and $\sigma = s$ buyers online and the type-L seller does not participate, or (2) if $\gamma > \bar{\gamma}_s$, so the type-H seller transacts only with the $\sigma = r$ buyer online and the type-L seller either does not participate or transacts only offline with the $\sigma = s$ buyer.

For case (1), let $\phi = \Pi_0^H$. Then

$$R_\gamma(\alpha,\gamma) = \mu r^a(\alpha, q_H, \gamma) < \mu \Pi_0^H = \mu\phi \leq R_\phi^*,$$

where the first equality follows because $r^b(\alpha, q_L, \gamma) = 0$, the strict inequality follows from Step 1, the second equality follows because $\phi = \Pi_0^H$, and the final inequality follows by definition of $R_\phi(\alpha,\phi) = \mu\phi$. For case (2), by Lemma 9 the platform's revenue is given by $r^c(\alpha, q_H, \gamma)$. Using the expressions in Lemma 9, it is straightforward to show algebraically that $r^c(\alpha, q_H, \gamma) < r^a(\alpha, q_H, \gamma)$ for any $\gamma \in [0, \gamma^{\max}]$. The remainder of the proof follows by parallel argument to case (1). $\square$

Lemma 18. *Define $\hat{\mu} = \Pi_0^L/\Pi_0^H$. Suppose for some $\mu \neq \hat{\mu}$ and γ, platform revenue under commission fees is weakly larger than the optimal revenue under access fees: $R_\gamma(\alpha,\gamma) \geq R_\phi^*$. Then $R_\gamma(\alpha,\gamma) \geq R_\phi^*$ also holds for $\mu = \hat{\mu}$.*

Proof. To make dependence on μ explicit, we use $R_\gamma(\alpha,\gamma,\mu)$ to denote the platform's revenue under commission rate γ and $R_\phi^*(\mu)$ to denote the platform's optimal revenue under access fees. First, suppose for some $\mu \neq \hat{\mu}$ there exists γ such that $R_\gamma(\alpha,\gamma,\mu) \geq R_\phi^*(\mu)$. We show $R_\gamma(\alpha,\gamma,\hat{\mu}) \geq R_\phi^*(\hat{\mu})$ must also hold. We consider two cases: $\mu > \hat{\mu}$ and $\mu < \hat{\mu}$.

Case 1. Suppose $\mu > \hat{\mu}$. Then it is straightforward to verify that $R_\phi^*(\mu') = \mu'\Pi_0^H$ for all $\mu' \geq \hat{\mu}$. Now suppose μ is decreased to $\hat{\mu}$. Then

$$\begin{aligned}
R_\gamma(\alpha,\gamma,\hat{\mu}) &= R_\gamma(\alpha,\gamma,\mu) + [R_\gamma(\alpha,\gamma,\hat{\mu}) - R_\gamma(\alpha,\gamma,\mu)] \\
&= R_\gamma(\alpha,\gamma,\mu) + [(\hat{\mu}-\mu) r^a(\alpha,q_H,\gamma) + (\hat{\mu}-\mu) r^b(\alpha,q_H,\gamma)] \\
&\geq R_\gamma(\alpha,\gamma,\mu) + [(\hat{\mu}-\mu)\pi^a(\alpha,q_H,\gamma)] \\
&= R_\gamma(\alpha,\gamma,\mu) + [R_\phi^*(\hat{\mu}) - R_\phi^*(\mu)] \\
&\geq R_\phi^*(\mu) + [R_\phi^*(\hat{\mu}) - R_\phi^*(\mu)] \\
&= R_\phi^*(\hat{\mu}).
\end{aligned}$$

The first inequality follows because $r^a(\alpha,q_H,\gamma) < \pi^a(\alpha,q_H,\gamma)$ for any $\gamma \leq \gamma^{\max}$ by Lemma 17. The second inequality follows because $R_\gamma(\alpha,\gamma,\mu) \geq R_\phi^*(\mu)$ by assumption. Therefore, if $R_\gamma(\alpha,\gamma) \geq R_\phi(\alpha,\gamma)$ for some $\mu > \hat{\mu}$, $R_\gamma(\alpha,\gamma,\hat{\mu}) \geq R_\phi(\alpha,\gamma,\hat{\mu})$ as well.

Case 2. Suppose $\mu < \hat{\mu}$. Then $R_\phi^*(\mu') = \pi^b(\alpha,q_H,0)$ for all $\mu' \leq \hat{\mu}$, which implies $R_\phi^*(\hat{\mu}) = R_\phi^*(\mu)$. Then

$$R_\gamma(\alpha,\gamma,\hat{\mu}) = R_\gamma(\alpha,\gamma,\mu) + [(\hat{\mu}-\mu) r^a(\alpha,q_H,\gamma) + (\hat{\mu}-\mu) r^b(\alpha,q_H,\gamma)] \geq R_\gamma(\alpha,\gamma,\mu) \geq R_\phi^*(\mu) = R_\phi^*(\hat{\mu}),$$

where the first inequality follows because $r^a(\alpha,q_H,\gamma) > r^b(\alpha,q_L,\gamma)$ by Lemma 9. The result follows. $\square$

Proof of Lemma 5. We prove $R_\phi^* \geq R_\gamma^*$ first. By Lemma 17, if only one seller type transacts online in equilibrium — which corresponds to either $r^b(\alpha,q_L,\gamma^*) = 0$ (by Lemma 9) or $\gamma^* > \bar{\gamma}_s$ (by Lemma 2) — then access fees always generate higher revenue. As a result, it suffices to restrict attention to the case where $r^b(\alpha, q_L, \gamma^*) > 0$ and $\gamma \leq \bar{\gamma}_s$. In this setting, the platform's revenue under commission rate γ can be written as

$$\tilde{R}(\alpha, \gamma) = \mu r^a(\alpha, q_H, \gamma) + (1-\mu) r^b(\alpha, q_L, \gamma).$$

Therefore, to prove the result, it suffices to show $\tilde{R}(\alpha, \gamma) < R^*_\phi$ for all $\gamma \leq \bar{\gamma}_s$. Further, as a consequence of Lemma 18, if $\tilde{R}(\alpha, \gamma) < R^*_\phi$ holds for all $\gamma \leq \bar{\gamma}_s$ under $\mu = \hat{\mu}$, then $\tilde{R}(\alpha, \gamma) < R^*_\phi$ holds for all $\gamma \leq \bar{\gamma}_s$ for any $\mu \in [0, 1]$, where $\hat{\mu}$ is defined in Lemma 18. Moreover, under $\mu = \hat{\mu}$, the platform's optimal revenue under access fees is

$$\hat{R}_\phi = \mu \Pi_0^H = \Pi_0^L,$$

and the optimal access fee is also $\phi^* = \Pi_0^L$. Therefore, it remains to show $\tilde{R}(\alpha, \gamma) < \hat{R}_\phi$ for all $\gamma \leq \bar{\gamma}_s$ at $\mu = \hat{\mu}$. The remainder of the proof follows in three steps. First, for type-H sellers, we bound the ratio between platform revenue under a commission rate γ and seller profit under the optimal access fee. Second, we do the same for type-L sellers. Third, we prove the main result.

Step 1. Note $r^a(\alpha, q_H, \gamma)$ is the type-H seller's contribution to platform revenue under the commission rate γ, and Π_0^H is the seller's profit under the access fee ϕ^*. Then we have

$$\frac{r^a(\alpha, q_H, \gamma)}{\Pi_0^H} = \gamma \frac{1 - \frac{(1-\lambda)^2 c^2}{q_H^2 (1-\gamma)^2}}{\left(1 - \frac{(1-\lambda)c}{q_H}\right)^2} \leq \gamma \frac{1 - \frac{(1-\lambda)^2 c^2}{q_H^2}}{\left(1 - \frac{(1-\lambda)c}{q_H}\right)^2} \leq \gamma \frac{1 - \frac{(1-\frac{1}{2})^2 c^2}{16c^2}}{\left(1 - \frac{(1-\frac{1}{2})c}{4c}\right)^2} = \gamma \frac{9}{7}.$$

The second inequality follows from the observation that the ratio is maximized when $(1-\lambda)/q_H$ is maximized over $\lambda \in [\frac{1}{2}, 1]$ and $q_H \geq 4c$, which occurs at $\lambda = 1/2$ and $q_H = 4c$.

Step 2. Similarly, consider the ratio $r^b(\alpha, q_L, \gamma)/\Pi_0^L$ for fixed $\gamma \leq \bar{\gamma}_s$. Further, note $r^b(\alpha, q_L, \gamma) > 0$ implies $q_L^2(1-\gamma)^2 > \bar{\gamma}_s^2 c^2$ (Lemma 9), or equivalently,

$$\bar{\gamma}_s < \frac{q_L(1-\gamma)}{c} \leq \frac{(1-\lambda)c(1-\gamma)}{c} = \frac{1-\gamma}{2}.$$

Therefore, we must have $\bar{\gamma}_s \in [\gamma, \frac{(1-\gamma)}{2}]$. This immediately rules out cases where $\gamma \geq (1-\gamma)/2$, i.e., $\gamma \geq 1/3$, because $r^b(\alpha, q_L, \gamma) \leq 0$ if $\gamma \geq 1/3$. Next, we prove that for any $\gamma \leq 1/3$, the following holds:

$$\frac{r^b(\alpha, q_L, \gamma)}{\Pi_0^L} \leq \begin{cases} \frac{1}{2-\gamma} & 0 \leq \gamma \leq 2 - \sqrt{3}, \\ \gamma \frac{(1+\gamma)(1-3\gamma)}{(1-\gamma)^2(1-2\gamma)^2} & 2 - \sqrt{3} \leq \gamma \leq \frac{1}{3}. \end{cases} \tag{50}$$

To see why this holds, first note that

$$\frac{r^b(\alpha, q_L, \gamma)}{\Pi_0^L} = \frac{\gamma q_L^2}{(q_L - \bar{\gamma}_s c)^2} \left(1 - \frac{\bar{\gamma}_s^2 c^2}{q_L^2 (1-\gamma)^2}\right).$$

For simplicity, let $z = \bar{\gamma}_s c / q_L$. Then the above expression can be rewritten as

$$\frac{r^b(\alpha, q_L, \gamma)}{\Pi_0^L} = \frac{\gamma}{(1-z)^2} \left(1 - \frac{z^2}{(1-\gamma)^2}\right).$$

Fixing γ, let us differentiate the above ratio with respect to z to identify where it is maximized. Specifically, we have

$$\frac{\partial}{\partial z} \left(\frac{r^b(\alpha, q_L, \gamma)}{\Pi_0^L}\right) = \frac{\gamma}{(1-z)^4} \left((1-z)^2 \left(-2\frac{z}{(1-\gamma)^2}\right) + 2\left(1 - \frac{z^2}{(1-\gamma)^2}\right)(1-z)\right) = \frac{2\gamma}{(1-z)^3} \left(1 - \frac{z}{(1-\gamma)^2}\right)$$

To determine the sign of the derivative, consider the range of possible values for z. For fixed γ, we know that $\bar{\gamma}_s \in [\gamma, \frac{1-\gamma}{2}]$. Further, we know that $c/q_L \geq 1/(1-\lambda) \geq 2$ by Assumption 1. As a result, we have $z \in [2\gamma, \infty)$. Therefore, we can conclude that:

$$\arg\max_z \left(\frac{r^b(\alpha, q_L, \gamma)}{\Pi_0^L}\right) = \begin{cases} 2\gamma & \text{if} \quad 2\gamma \geq (1-\gamma)^2 \implies \gamma \in [2-\sqrt{3}, \frac{1}{3}] \\ (1-\gamma)^2 & \text{if} \quad 2\gamma \leq (1-\gamma)^2 \implies \gamma \in [0, 2-\sqrt{3}]. \end{cases}$$

Substituting $z = \bar{\gamma}_s c/q_L$ yields (50), as desired.

Step 3. We now show $\tilde{R}(\alpha, \gamma) < \hat{R}_\phi$ for all $\gamma \leq \bar{\gamma}_s$ at $\mu = \hat{\mu}$. Following the two cases established in Step 2, first suppose $\gamma \leq 2 - \sqrt{3}$. In this case, we have

$$\begin{aligned} \tilde{R}(\alpha, \gamma) &= \hat{\mu} r^a(\alpha, q_H, \gamma) + (1-\hat{\mu}) r^b(\alpha, q_L, \gamma) \\ &\leq \gamma \frac{9}{7} \hat{\mu} \Pi_0^H + \gamma \frac{1}{2\gamma - \gamma^2}(1-\hat{\mu})\Pi_0^L \\ &< \gamma \frac{9}{7} \hat{\mu} \Pi_0^H + \frac{1}{2-\gamma} \Pi_0^L \\ &= \hat{\mu} \pi^a(\alpha, q_H, 0) \left(\gamma \frac{9}{7} + \frac{1}{2-\gamma}\right) \\ &\leq \hat{\mu} \Pi_0^H \left((2-\sqrt{3})\frac{9}{7} + \frac{1}{\sqrt{3}}\right) \\ &< \hat{\mu} \Pi_0^H \\ &= R_\phi^*. \end{aligned}$$

The sequence above uses the results from Steps 1 and 2, the definition of $\hat{\mu}$, and the observation that $\left(\gamma \frac{9}{7} + \frac{1}{2-\gamma}\right)$ strictly increases in γ on $\gamma \in [0, 2-\sqrt{3}]$. Next, suppose $\gamma \in [2-\sqrt{3}, \frac{1}{3}]$. Again using the results from Steps 1 and 2, we have

$$\begin{aligned} \tilde{R}(\alpha, \gamma) &= \mu r^a(\alpha, q_H, \gamma) + (1-\mu) r^b(\alpha, q_L, \gamma) \\ &< \mu r^a(\alpha, q_H, \gamma) + r^b(\alpha, q_L, \gamma) \\ &\leq \gamma \frac{9}{7} \mu \Pi_0^H + \gamma \frac{(1+\gamma)(1-3\gamma)}{(1-\gamma)^2(1-2\gamma)^2} \Pi_0^L \\ &= \mu \Pi_0^H \underbrace{\left(\gamma \frac{9}{7} + \gamma \frac{(1+\gamma)(1-3\gamma)}{(1-\gamma)^2(1-2\gamma)^2}\right)}_{\omega} \\ &< \mu \Pi_0^H \\ &= R_\phi^*. \end{aligned}$$

The final inequality follows because $\omega \leq 1$ in the interval $\gamma \in [2-\sqrt{3}, \frac{1}{3}]$. It follows that $R_\phi^* \geq R_\gamma^*$.

We now show $R_\phi(\alpha, \phi)$ weakly increases in α. Define $\phi_u^m = \lim_{\alpha \to 1} \phi_L(\alpha)$. Note $\phi_L(\alpha)$ strictly increases in α and $\phi_u^m < \phi_H$ by Lemma 16. We therefore consider three cases: $\phi \leq \phi_u^m$, $\phi \in (\phi_u^m, \phi_H]$, and $\phi > \phi_H$. First, if $\phi \leq \phi_u^m$, the type-H seller always joins the platform, and platform revenue depends on whether the type-L seller joins. Because $\phi_L(\alpha)$ strictly increases in α, there exists $\alpha_L(\phi) \in [\frac{1}{2}, 1]$ such that revenue under access fees can be written as

$$R_\phi(\alpha, \phi) = \begin{cases} \mu\phi, & \text{if } \alpha < \alpha_L(\phi), \\ \phi, & \text{if } \alpha \geq \alpha_L(\phi). \end{cases}$$

Because $\mu \leq 1$, $R_\phi(\alpha,\phi)$ weakly increases in α. Next, if $\phi \in (\phi_L^m, \phi_H]$, then the type-$L$ seller never joins, which implies $R_\phi(\alpha,\phi) = \mu\phi$ for all $\alpha \in [\frac{1}{2}, 1]$, in which case $R_\phi(\alpha,\phi)$ is invariant to α. Finally, if $\phi > \phi_H$, then neither seller joins, which implies $R_\phi(\alpha,\phi) = 0$ for all $\alpha \in [\frac{1}{2}, 1]$. Therefore, in all three cases, platform revenue is weakly increasing in α. $\square$

Proof of Proposition 4. The proof proceeds in two steps. First, we construct thresholds $\underline{\alpha}$, $\mu_{\min}$, and $\mu_{\max}$ such that when $\alpha \geq \underline{\alpha}$ and $\mu \in (\mu_{\min}, \mu_{\max})$, commissions outperform access fees: $R_\gamma^* > R_\phi^*$. Second, we then apply a simple transformation from the μ-space to the relative earnings parameter β, and prove the bounds on β_1 and β_2.

Step 1. Define $\underline{\alpha}$ to be the solution to $\bar{\gamma}_s = 2q_L/5c$, where $\bar{\gamma}_s$ is defined in Lemma 6. Because $q_L \leq (1-\lambda)c$ by Assumption 1 and $\bar{\gamma}_s = 1-\lambda$ when $\alpha = 1/2$, we have $\underline{\alpha} > 1/2$. We show that for any $\alpha \geq \underline{\alpha}$, $R_\gamma^* > R_\phi^*$ over an interval $(\mu_{\min}, \mu_{\max})$. It suffices to show $R(\alpha, \gamma^{\max}) > R_\phi^*$, or equivalently,

$$\mu r^a(\alpha, q_H, \gamma^{\max}) + (1-\mu) r^b(\alpha, q_L, \gamma^{\max}) > \max\left(\mu \Pi_0^H, \Pi_0^L\right).$$

In what follows, we fix $\alpha \geq \underline{\alpha}$ and let $\hat{\mu} = \Pi_0^L / \Pi_0^H$. First, note that at this value of α, we have

$$2r^b(\alpha, q_L, \gamma^{\max}) = \frac{q_H \eta_s}{4}\left(1 - \frac{\bar{\gamma}_s^2 c^2}{q_L^2(1-\gamma^{\max})^2}\right) \geq \frac{q_L \eta_s}{4}\left(1 - \frac{\bar{\gamma}_s c}{q_L}\right)^2 = \Pi_0^L. \tag{51}$$

The inequality above follows because $(1-x)^2 \leq 1 - 4x^2$ for $x \in [0, \frac{2}{5}]$, which can be verified algebraically, and because $\bar{\gamma}_s c / q_L \leq 2/5$ by our choice of $\underline{\alpha}$. Moreover, we claim (and prove below) that

$$\frac{2r^a(\alpha, q_H, \gamma^{\max})}{\Pi_0^H} > 1 + \hat{\mu}. \tag{52}$$

We now show (51) and (52) imply $R(\alpha, \gamma^{\max}) > R_\phi^*$ over some interval $(\mu_{\min}, \mu_{\max})$. To see why, note that at $\mu = \hat{\mu}$,

$$\begin{aligned}
\hat{\mu} r^a(\alpha, q_H, \gamma^{\max}) + (1-\hat{\mu}) r^b(\alpha, q_L, \gamma^{\max}) &> \frac{\hat{\mu}}{2}\Pi_0^H(1+\hat{\mu}) + \frac{(1-\hat{\mu})}{2}\Pi_0^L \\
&= \frac{\hat{\mu}}{2}\Pi_0^H + \frac{\hat{\mu}}{2}\Pi_0^L + \frac{(1-\hat{\mu})}{2}\Pi_0^L \\
&= \frac{\Pi_0^L}{2} + \frac{\Pi_0^L}{2} \\
&= R_\phi^*.
\end{aligned}$$

The first inequality follows from (51) and (52), and the second and third lines follow by definition of $\hat{\mu}$. Therefore, for the given value of α, $R(\alpha, \gamma^{\max}) > R_\phi^*$ under $\mu = \hat{\mu}$. Further, because the inequality is strict, it follows from the continuity of $R(\alpha, \gamma^{\max})$ and R_ϕ^* that there exists $\mu_{\min} < \hat{\mu}$ and $\mu_{\max} > \hat{\mu}$ such that $R(\alpha, \gamma^{\max}) > R_\phi^*$ for all $\mu \in (\mu_{\min}, \mu_{\max})$, as desired.

It remains to show (52) holds. Note that by definition of $\hat{\mu}$, (52) holds if and only if

$$2r^a(\alpha, q_H, \gamma^{\max}) - \Pi_0^H > \Pi_0^L. \tag{53}$$

Therefore, to prove (52) holds, it suffices to show (53). We can now write

$$\begin{aligned}
2r^a(\alpha, q_H, \gamma^{\max}) - \Pi_0^H &= \frac{q_H}{4}\left(\left(1 - \frac{(1-\lambda)^2c^2}{q_H^2(1-\gamma^{\max})^2}\right) - \left(1 - \frac{(1-\lambda)c}{q_H}\right)^2\right) \\
&= \frac{q_H}{4}\left(2\frac{(1-\lambda)c}{q_H} - 5\frac{(1-\lambda)^2c^2}{q_H^2}\right) \\
&= \frac{q_H}{4}\frac{2(1-\lambda)c}{q_H}\left(1 - \frac{5(1-\lambda)c}{2q_H}\right) \\
&\geq \frac{1}{4}\cdot 2q_L\left(1 - \frac{5(1-\lambda)c}{2q_H}\right) \qquad (54) \\
&> \frac{1}{4}q_L\lambda \qquad (55) \\
&\geq \frac{1}{4}q_L\eta_s\left(1 - \frac{(1-\eta_{|s})c}{q_L}\right)^2. \\
&= \Pi_0^L.
\end{aligned}$$

The first four lines follow algebraically and because $\gamma^{\max} = \frac{1}{2}$. Equation (54) follows because $q_L \leq (1-\lambda)c$ by Assumption 1 and (55) follows because $2\left(1 - 5(1-\lambda)c/2q_H\right) > \lambda$. To see why the latter inequality holds, note that that $q_H \geq 4c$ by Assumption 1, which implies

$$2\left(1 - \frac{5(1-\lambda)c}{2q_H}\right) \geq 2\left(1 - \frac{5(1-\lambda)}{8}\right) = 2\left(\frac{3}{8} + \frac{5\lambda}{8}\right) > \lambda.$$

Step 2. We now re-state the result from Step 1 in terms of the parameter β and establish the bounds on β_1 and β_2. Note that by Definition 1, β strictly decreases in μ. It immediately follows that for each $\alpha \geq \underline{\alpha}$ there exists β_1 and β_2 such that $R_\gamma^* > R_\phi^*$ for all $\beta \in (\beta_1, \beta_2)$. Next, we derive the bounds on β_1 and β_2. We do so in three steps. First, we derive a bound on $r^a(\alpha, q_H, \gamma^*)/r^b(\alpha, q_L, \gamma^{\max})$. Second, we derive bounds on the ratios $r^a(\alpha, q_H, \gamma^{\max})/\Pi_0^H$ and $r^b(\alpha, q_L, \gamma^{\max})/\Pi_0^L$. Third, we use the results from the first two steps to obtain our final bounds on β_1 and β_2.

Step 2.1. We show $r^b(\alpha, q_L, \gamma)/r^b(\alpha, q_L, \gamma^{\max}) \leq 5/4$ for any $\gamma \in [0, \gamma^{\max}]$. Pick any γ. Then we have

$$\frac{r^b(\alpha, q_L, \gamma)}{r^b(\alpha, q_L, \gamma^{\max})} = \underbrace{\frac{2\gamma q_L^2}{q_L^2 - 4\bar{\gamma}_s^2c^2}\left(1 - \frac{\bar{\gamma}_s^2c^2}{q_L^2(1-\gamma)^2}\right)}_{\omega_1} \leq \underbrace{\frac{50}{9}\gamma\left(1 - \frac{4}{25(1-\gamma)^2}\right)}_{\omega_2} < \frac{5}{4}. \qquad (56)$$

The first inequality above follows from the observation that the expression ω_1 is maximized when $\bar{\gamma}_s$ takes on its minimum value over $\alpha \geq \underline{\alpha} = 2q_L/5c$, which is at $2q_L/5c$. The second inequality follows from maximizing the expression ω_2 over $\gamma \in [0, \gamma^{\max}]$, where $\gamma^{\max} = 1/2$.

Step 2.2. We now prove the following two sets of inequalities:

$$\frac{1}{2} \leq \frac{r^a(\alpha, q_H, \gamma^{\max})}{\Pi_0^H} \leq \frac{30}{49} \qquad (57)$$

$$\frac{1}{2} \leq \frac{r^b(\alpha, q_L, \gamma^{\max})}{\Pi_0^L} \leq \frac{2}{3}. \qquad (58)$$

To see why these hold, note that using the expressions in Lemma 9 we can write

$$\begin{aligned}
\frac{r^a(\alpha, q_H, \gamma^{\max})}{\Pi_0^H} &= \frac{q_H^2\gamma^{\max}}{(q_H - (1-\lambda)c)^2}\left(1 - \frac{(1-\lambda)^2c^2}{q_H^2(1-\gamma^{\max})^2}\right) \\
\frac{r^b(\alpha, q_L, \gamma^{\max})}{\Pi_0^L} &= \frac{q_L^2\gamma^{\max}}{(q_L - \bar{\gamma}_s c)^2}\left(1 - \frac{\bar{\gamma}_s^2c^2}{q_L^2(1-\gamma^{\max})^2}\right).
\end{aligned}$$

The lower bound of 1/2 in (57) follows from (52) and the lower bound of 1/2 in (58) follows from (51). Next, to derive the upper bound of 30/49 in (57), note that the ratio r^a/π^a attains its maximum value over $\lambda \in [\frac{1}{2}, 1]$ and $q_H \geq 4_c$ at $\lambda = 1/2$ and $q_H = 4c$. Substituting these values produces the bound. Lastly, for the upper bound of 2/3 in (58), note that

$$\frac{r^b(\alpha, q_L, \gamma^{\max})}{\Pi_0^L} = \frac{1 - 4z^2}{2(1-z)^2}, \tag{59}$$

where $z = \bar{\gamma}_s c/q_L$. Because $\alpha \geq \underline{\alpha}$ and $\underline{\alpha}$ was chosen to be the solution to $\bar{\gamma}_s = 2q_L/5c$ in Step 1, we have $z \in [0, \frac{2}{5}]$. The maximizer of the right hand side of (59) on $z \in [0, \frac{2}{5}]$ is $z = 1/4$, which yields the bound 2/3.

Step 2.3. We now prove there cannot be any instance where $\beta < 1/4$ or $\beta > 5$ such that $R_\gamma^* > R_\phi^*$. First, suppose $\beta < 1/4$. Then we have

$$\begin{aligned}
R_\gamma^* &= \mu r^a(\alpha, q_H, \gamma^*) + (1-\mu) r^b(\alpha, q_L, \gamma^*) \\
&\leq \frac{5}{4}\left(\mu r^a(\alpha, q_H, \gamma^{\max}) + (1-\mu) r^b(\alpha, q_L, \gamma^{\max})\right) \\
&\leq \frac{5}{4}\left(\frac{30}{49}\mu \Pi_0^H + \frac{2}{3}(1-\mu)\pi^b(\alpha, q_H, 0)\right) \\
&= \frac{5}{4}\left(\frac{30}{49} + \frac{2\beta}{3}\right)\mu \Pi_0^H \\
&< \mu \Pi_0^H \\
&\leq R_\phi^*.
\end{aligned}$$

The second line above follows from (56), the third line follows from (57) and (58), the fourth line follows by definition of β, the fifth line follows because $\beta < 1/4$, and the final line follows because $R_\phi^* = \max\{\mu \Pi_0^H, \Pi_0^L\}$ by Lemma 16. Therefore, $\beta < 1/4$ implies $R_\gamma^* < R_\phi^*$. Next, suppose $\beta > 5$. Following a parallel argument to above, we have

$$\begin{aligned}
R_\gamma^* &= \mu r^a(\alpha, q_H, \gamma^*) + (1-\mu) r^b(\alpha, q_L, \gamma^*) \\
&\leq \frac{5}{4}\left(\mu r^a(\alpha, q_H, \gamma^{\max}) + (1-\mu) r^b(\alpha, q_L, \gamma^{\max})\right) \\
&\leq \frac{5}{4}\left(\frac{30}{49}\mu \Pi_0^H + \frac{2}{3}(1-\mu)\pi^b(\alpha, q_H, 0)\right) \\
&< \frac{5}{4}\left(\frac{30}{49\beta} + \frac{2}{3}\right)(1-\mu)\Pi_0^L \\
&< \Pi_0^L \\
&\leq R_\phi^*.
\end{aligned}$$

The second line above follows from (56), the third line follows from (57) and (58), the fourth line follows by definition of β, and the fifth line follows because $\beta > 5$ and $\mu \leq 1$. Therefore, $\beta > 5$ implies $R_\gamma^* < R_\phi^*$, which is also a contradiction. The result follows. □