

Platform Disintermediation: Information Effects and Pricing Remedies

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Abstract

Two-sided platforms, such as labor marketplaces for hiring freelancers, typically generate revenue by matching prospective buyers and sellers and extracting commissions from completed transactions. Disintermediation, where sellers transact off-platform with buyers to bypass commission fees, can undermine the viability of these marketplaces. Although circumventing the platform allows sellers to avoid commission fees, it also leaves them fully exposed to risky buyers (given the absence of the platform’s payment protections) and incurs switching costs (given the absence of the platform’s transaction infrastructure). In this paper, we consider interventions for addressing disintermediation, focusing on the pricing and informational levers available to the platform, where the latter refers to the accuracy of the signal sellers receive about buyers’ riskiness. First, while intuition suggests platforms should counter disintermediation by lowering commission rates, in a high-information environment a platform may be better off raising them. Further, when information quality is high, an increase in sellers’ switching costs may hurt platform revenue. Finally, a platform may strictly benefit from sellers receiving a partially-informative buyer signal (i.e., not perfectly revealing nor concealing a buyer’s riskiness), particularly when switching costs are low. As extensions, we examine the efficacy of two interventions: implementing platform-access fees to capture revenue upfront and banning sellers caught disintermediating. Overall, our results shed light on how disintermediation disrupts platform operations and offer prescriptions for platforms seeking to counteract it.

1 Introduction

Two-sided platforms that generate revenue through commission fees are vulnerable to *disintermediation*, where buyers and sellers transact off-platform to avoid paying the commission. Disintermediation can lead to significant revenue losses – the talent outsourcing platform ZBJ estimates that up to 90% of their service providers’ transactions may occur off-platform (Zhu et al. 2018). In extreme cases, disintermediation can threaten the viability of the platform itself – for example, the demise of home-cleaning platform HomeJoy in 2015 has been partly attributed to disintermediation (Farr 2015). Although disintermediation is difficult to detect, there is growing empirical evidence of it occurring in multiple online marketplaces (Lin et al. 2022, Karacaoglu et al. 2022, Gu 2024,

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Chintagunta et al. 2023). These risks are well-recognized by platforms: Airbnb explicitly warns hosts of buyers attempting to pay through alternative channels (Airbnb 2023b), and the freelance platform Upwork encourages users to report attempts at circumvention (Upwork 2023a). However, platforms’ efforts to clamp down on this behavior have been less than successful (Chintagunta et al. 2023), as also acknowledged by Upwork in a recent 10-K filing:

“Despite our efforts to prevent them from doing so, users circumvent our work marketplace and engage with or take payment through other means to avoid the fees that we charge, and it is difficult or impossible to measure the losses associated with circumvention.” (Upwork 2024b).

Given the limited efficacy of such enforcement measures, effectively addressing disintermediation requires an understanding of what drives sellers to transact off-platform in the first place. Naturally, the platform’s commission rate can play a major role in sellers’ inclination to circumvent the marketplace, as it can amount to a substantial share of their earnings.¹ However, although disintermediating allows sellers to recoup commission fees, it also entails giving up the benefits offered by the platform, including transaction support and policies that protect sellers against risky or even fraudulent behavior by buyers. With respect to the latter, such protections are commonplace: Airbnb insures hosts against property damage by guests (Airbnb 2023a), Upwork holds payments in escrow to safeguard freelancers (Upwork 2023c), and eBay protects sellers against various forms of buyer fraud (eBay 2023). In deciding whether to disintermediate, sellers must therefore weigh the benefits of avoiding commission fees against full exposure to risky buyers, in addition to the *switching costs* induced by forgoing the platform’s infrastructure (Hagiu and Wright 2024).

Whether a seller decides to transact off-platform depends on their assessment of buyer riskiness. In online marketplaces, sellers’ trust in buyers depends on the *quality* of information they obtain via the platform. Thus, to encourage on-platform transactions, many platforms include communication tools and reputation systems for both buyers and sellers. However, high information quality can also improve sellers’ ability to screen risky buyers (Jin et al. 2025), diminishing the value of the platform’s protections. Under the threat of disintermediation, the directional impact of information on platform revenue is therefore unclear – while to some extent necessary to facilitate on-platform transactions, high information quality may also increase the attractiveness of circumventing the platform entirely (Gu and Zhu 2021).

The quality of information that sellers obtain about buyers, whether through reputation systems or direct communication, varies by platform and context. Ratings may be unreliable or prone to

¹For example, Airbnb hosts pay a commission of 14-16%, Upwork charges its freelancers up to 15% of their revenue, and Fiverr charges a commission rate of 20%.

inflation, reducing their usefulness in differentiating users (Nosko and Tadelis 2015), and users may also be imperfect in their ability to interpret ratings (Tadelis 2016). Additionally, communication-related policies differ across platforms: Airbnb algorithmically blocks email addresses and phone numbers in their on-platform chat until bookings are confirmed, while Upwork prohibits sharing contact information, but does not block it. A key question, then, is how the quality of information available to sellers shapes market outcomes in the face of disintermediation. Moreover, the trade-off between trust and disintermediation suggests that platforms may benefit from carefully controlling the information available to sellers, to the extent doing so is practical.

How might platforms alter their pricing strategy to respond to disintermediation? Reducing commission rates may encourage on-platform transactions, but also needlessly sacrifice revenue if some degree of disintermediation is inevitable. Fundamentally, disintermediation poses a challenge to commission-based platforms due to a misalignment between the platform’s value proposition (connecting sellers to buyers) and its pricing strategy (charging for completed transactions). Recognizing this gap, some platforms eschew commission fees and instead charge sellers for *access* to buyers – for example, the home services platform Thumbtack charges sellers for inquiries from potential buyers (“leads”) (Thumbtack 2023), and the caregiver platform Care.com charges service providers for the ability to exchange messages with prospective clients (Care.com 2023). Clearly, charging sellers access fees upfront reduces the incentive to disintermediate, but its revenue implications are less clear, as some sellers may be unwilling to incur the cost of entry.

1.1 Contributions

This paper examines how disintermediation impacts the operations of two-sided platforms and strategies to address this phenomenon, with a focus on the platform’s pricing and informational levers. In our model, heterogeneous sellers set their prices in an *online* (i.e., on-platform) transaction channel prior to being matched to a buyer. The platform charges the seller a fixed fraction of the price if the transaction is completed online. Alternatively, a seller may attempt to bypass the commission by negotiating an off-platform price with the buyer and completing the transaction in an *offline* channel, if doing so is mutually beneficial net of a seller-side switching cost. Buyers’ types are private information; in particular, *risky* buyers impose higher transaction costs on sellers in both channels, and also do not pay sellers when transacting offline.² To capture information quality, we assume the platform has a technology that generates, with varying degrees of accuracy, a noisy signal of the buyer’s type, which the seller observes after selecting their online price and prior to their choice of

²This approximates a variety of payment-related risks that buyers can pose to sellers off-platform. For example, on Upwork, clients may pose risks such as declined payment, delays in payment, chargebacks, client bankruptcy, or fraud, all of which Upwork protects freelancers against only if they transact on-platform (Upwork 2024b).

transaction channel.

Within this setup, we aim to analyze how the threat of disintermediation forces platforms to reconsider their core design decisions to combat sellers' incentives to transact offline. Specifically, we focus on information quality (which facilitates trust) and pricing strategies by examining the following questions:

1. How does a platform's vulnerability to disintermediation impact its choice of commission fee, and consequently, its revenue? (Section 3)
2. What is the platform's optimal information policy in light of disintermediation? (Section 4)
3. In what environment should the platform adopt access-based pricing over commission fees? (Section 5)

Our main results are summarized as follows. First, intuition might suggest that the platform should set a low commission rate to combat disintermediation. Yet, we show that when sellers can easily transact off-platform, the platform's optimal commission rate may be *higher* than when they cannot. This result is a consequence of which transactions the platform chooses to capture value from. Specifically, a platform that is more vulnerable to disintermediation (i.e., low switching costs) must decide whether to prevent circumvention entirely, which requires restricting its commission rate, or to permit disintermediation for some transactions and maximize revenue from those that remain on-platform, which frees the platform to increase its commission rate. We find the latter strategy is optimal when information quality is sufficiently high.

Moreover, when the platform is already prone to disintermediation, an increase in sellers' switching costs can further *decrease* revenue. In short, this occurs because sellers pass on the switching cost to on-platform buyers in the form of higher prices and continue to transact off-platform, which hurts the platform by depressing demand. This suggests that interventions that increase sellers' perceived cost of disintermediation (e.g., adding transaction-related features) without substantively deterring it may undermine platform revenue in equilibrium.

Our findings above suggest that the accuracy of information available to sellers (regarding buyers' riskiness) plays a central role in determining how platforms respond to the threat of disintermediation. Motivated by this, we analyze a setting where the platform can control the information provided to sellers, and characterize when a no-, full-, or partial-information disclosure policy is optimal. In short, we show the optimal policy depends on the competing effects induced by information: higher information quality boosts the volume of on-platform transactions and leads to more revenue-efficient pricing by sellers, but also weakens the platform's pricing power by raising

the threat of disintermediation. We show how the relative strength of these two effects shapes the platform’s optimal information policy.

Lastly, we examine the efficacy of *access*-based pricing, in which sellers are charged upfront to join the platform instead of paying commission fees. By design, access fees immunize the platform against disintermediation by removing the incentive to transact offline. Despite their promise, however, we find that access fees can fall short of the optimal revenue under commissions, depending on the degree of heterogeneity in sellers’ qualities. This result speaks to the prevalence of commission fees in practice, despite their vulnerability to disintermediation.

In addition, we extend our model to consider (i) switching costs for buyers, (ii) banning sellers who are caught disintermediating, and (iii) repeated transactions between buyers and sellers. Overall, our findings build on a recent and growing body of empirical work on quantifying disintermediation (He et al. 2020, Gu and Zhu 2021, Karacaoglu et al. 2022, Astashkina et al. 2022, Gu 2024, Chintagunta et al. 2023) by exploring prescriptions for counteracting it.

1.2 Related Work

Disintermediation. In its most general sense, disintermediation refers to the circumvention of market intermediaries, and has been studied in a number of contexts, including supply chains (Ritchie and Brindley 2000, Federgruen and Hu 2016). Our work bears some similarity to prior work on the cost-benefit trade-off of intermediation in supply chains and other operational settings (e.g., Agrawal and Seshadri (2000), Belavina and Girotra (2012)), although our focus is on platforms that mediate transactions between individual users, rather than firms.

Recently, there is a growing recognition of the threat posed by disintermediation (or *platform leakage*) to a variety of two-sided platforms. For the most part, the extant literature on platform disintermediation is empirical, and uses novel identification approaches to quantify this phenomenon (He et al. 2020, Gu and Zhu 2021, Gu 2024, Astashkina et al. 2022, Karacaoglu et al. 2022, Lin et al. 2022, Chintagunta et al. 2023). For example, Gu and Zhu (2021) use a randomized control trial to find evidence of disintermediation on a large outsourcing platform, Karacaoglu et al. (2022) use data from a home cleaning platform and estimate that the platform loses out on 24% of potential transactions due to disintermediation, and Lin et al. (2022) find the rate of disintermediation on Airbnb to be around 5.4% based on data from Austin, Texas. Our work is especially related to Gu and Zhu (2021), who find that providing more information about freelancer quality positively impacts the volume of transactions, but also increases the likelihood of disintermediation. Our model provides analytical support for the empirical results in Gu and Zhu (2021), and further sheds light

on the impact of information quality on a platform’s revenue and commission rate.

On the modeling side, He et al. (2023) study drivers of disintermediation and investigate the impact of risky sellers under-delivering in two settings: when platforms provide perfect information, or no information at all. In contrast, the risk in our model stems from nonpayment by buyers, and we model information as a continuous parameter. They also propose mechanisms that platforms can implement to avoid disintermediation (e.g., upskilling sellers), whereas we consider pricing and information quality as levers to address disintermediation. Hagiu and Wright (2024) also present a model for platform disintermediation, although their setting differs in a few notable ways: there is no private information on either side of the market, buyers are homogeneous and have zero bargaining power over the off-platform price, and sellers face no risks from transacting offline.

Information disclosure in platforms. Our paper is related to a growing literature on how information influences the decisions of platform users, which has consequences for social welfare or platform revenue. Papanastasiou et al. (2018) show that strategically withholding information from consumers can induce exploration of new or alternative products in a manner that ultimately improves consumer surplus; similarly, Gur et al. (2023) consider how information can be used as a lever to influence sellers’ prices, also with the aim of improving consumer surplus. In a similar vein, Kanoria and Saban (2021) show that matching markets (e.g., dating platforms) can improve welfare by hiding the quality of users. With respect to platform revenue, Bimpikis et al. (2024) and Shi et al. (2023) describe mechanisms through which mislabeling high-quality sellers can benefit the platform, and Jin et al. (2025) and Johari et al. (2019) identify conditions under which it is revenue-maximizing for platforms to filter out low-quality users. More generally, there is a burgeoning literature on information design in a variety of operational contexts (Bimpikis and Papanastasiou 2019, Bimpikis et al. 2019, Lingenbrink and Iyer 2019, Candogan and Drakopoulos 2020, Drakopoulos et al. 2021, Liu et al. 2021, Ma et al. 2024, Anunrojwong et al. 2023, Bimpikis and Mantegazza 2023, Bimpikis et al. 2025). Our paper contributes to this literature by considering a new mechanism through which information shapes platform revenue, namely, disintermediation.

Commissions vs. upfront pricing. Platform designers often have a wide range of pricing levers at their disposal, and characterizing the trade-offs between the different mechanisms is an active area of study. For instance, there is a rich literature on advance selling that predates online marketplaces (Xie and Shugan 2001, Randhawa and Kumar 2008, Cachon and Feldman 2011), and it is now well known that a monopolist firm can often extract more revenue from subscriptions than per-use pricing. However, in the case of two-sided platforms with heterogeneous users, the effectiveness of upfront pricing may suffer by excluding users who are uncertainty averse (Edelman and Hu 2016) or

derive low utility from the platform (Birge et al. 2021, Cui and Hamilton 2022). As a consequence, commissions remain the *de facto* pricing strategy in most modern marketplaces.

Naturally, a number of papers have looked at how platforms should set these commissions and whether they should be coupled with other mechanisms, e.g., a fixed fee (Benjaafar et al. 2019, Hu and Zhou 2020, Feldman et al. 2023, Cachon et al. 2025). For instance, Cachon et al. (2025) consider how pricing control (i.e., whether the platform or sellers set prices) impacts the performance of commission and per-unit fees, and show that a two-part tariff that combines them performs well in both centralized and decentralized marketplaces. Birge et al. (2021) identify conditions under which it is optimal for platforms to use subscriptions or commissions, and show that platforms can lose out on revenue by not charging payments from both sides of the market. Our focus is on how the threat of disintermediation influences both the optimal commission rate and the efficacy of upfront pricing, in part motivated by recent interest in the interplay between pricing, information, and distortions in platform operations (Belavina et al. 2020, Mostagir and Siderius 2023, Papanastasiou et al. 2023).

2 Model

Consider a platform with a unit mass of sellers that are heterogeneous in quality. Let $i \in \{L, H\}$ index the sellers' types, where a type- i seller's quality is q_i and where $q_H > q_L > 0$. Let $\mu \in [0, 1]$ be the share of sellers that are type- H . Seller quality is public information and all sellers earn a reservation profit of 0 off the platform. As in many two-sided marketplaces, each seller chooses their own price p for transactions completed on the platform.

Each seller is randomly matched to a buyer. Buyers are heterogeneous in their quality sensitivity, θ , where a buyer with sensitivity θ has valuation $v = \theta q$ for a quality- q seller. We assume θ is distributed uniformly in $[0, 1]$ and that the distribution is common knowledge. Each buyer belongs to one of two types – *risky* or *safe* – where a buyer's type is private information and indexed by $j \in \{r, s\}$. The buyer type influences sellers' costs in two ways. First, a type- s buyer imposes lower transaction costs on sellers both online and offline, $c_s < c_r$; for simplicity, we assume $c_s = 0$ and $c_r = c > 0$. Second, a type- s buyer pays sellers in full in both transaction channels, whereas a type- r buyer does not pay if the transaction occurs off-platform.³ A buyer is type- s with probability λ , which is known to sellers and the platform and is independent of the buyer's quality sensitivity θ . We focus on a setting where a minority of buyers are risky by assuming $\lambda \in [\frac{1}{2}, 1]$.

³Our results extend to a setting where the type- r buyer instead pays offline with probability $\delta \in [0, 1]$.

The platform selects a commission rate $\gamma \in [0, \gamma^m]$, where $\gamma^m \leq \frac{1}{2}$.⁴ For a transaction completed on the platform at price p , the seller and platform receive $(1 - \gamma)p$ and γp , respectively. The platform has a technology that generates a noisy signal σ of the buyer's type, where $\sigma \in \{r, s\}$. We use “type- j ” to refer to a buyer's true and private type, and “ $\sigma = j$ ” to refer to the buyer's label from the platform signal, where $j \in \{r, s\}$. To capture variability in the signal's accuracy, we assume the signal correctly reveals the buyer's type with probability $\alpha \in [\frac{1}{2}, 1]$, where α is known to all parties.⁵ We refer to α as *information quality*.

Each buyer-seller pair chooses between two transaction channels: they may transact *online* (i.e., on-platform) at the seller's posted price p , or transact *offline* (i.e., off-platform) at a different, mutually beneficial price b , if such a price exists. More precisely, we model the offline price b as the solution to a symmetric Nash bargaining game (Nash 1953, Binmore et al. 1986), which in our setting is the price that maximizes the product of the buyer's and seller's surpluses from disintermediating (see Section 2.2). The outcome if price negotiation fails (i.e., the disagreement point of the bargaining game) is to transact online at price p , which is set by the seller prior to accepting the buyer and is thus fixed at the time of negotiation.

Lastly, sellers incur a switching cost of $\phi \geq 0$ if they disintermediate, which represents the relative inconvenience of transacting off-platform.

Timeline. The sequence of events is as follows:

1. The platform sets the commission rate γ .
2. Each seller chooses their online price p .
3. Each seller is randomly matched to a buyer. After observing the seller's price p , the buyer decides whether to contact the seller to initiate a transaction. (If the buyer does not make contact, no transaction occurs.)
4. Each contacted seller observes a noisy signal σ of the buyer's type, updates their belief of the buyer's type, and decides whether to transact with or reject the buyer. (If the seller rejects the buyer, no transaction occurs.)
5. If a seller accepts the buyer, both parties attempt to negotiate an offline price, b . The transaction occurs offline at price b if the negotiation succeeds; otherwise, the transaction occurs online at price p .

⁴We upper-bound the maximum commission rate by $\frac{1}{2}$ for analytical convenience. Our results continue to hold using $\gamma^m \leq 1 - \epsilon$ for any $\epsilon > 0$ if Assumption 1 is modified to include $q_H \geq 2c/\epsilon$.

⁵Note the assumption that $\alpha \geq \frac{1}{2}$ is without loss of generality, because a signal with accuracy $\alpha < \frac{1}{2}$ is equivalent to one with accuracy $1 - \alpha$ with the buyer type flipped.

In the third step, we assume buyers only contact sellers to initiate a transaction if the online price yields non-negative utility for the buyer (i.e., $\theta q \geq p$).⁶

2.1 Model Discussion

Our work is primarily motivated by the prevalence of disintermediation in online labor platforms for hiring freelancers. Our stylized model aims to capture the key drivers of disintermediation for sellers – specifically, trust in buyers and the cost of transacting off-platform – and abstracts away from many nuances in the exact operations of these marketplaces. Below, we briefly comment on how our key modeling choices relate to practice and their limitations.

Information quality. The variability in the platform’s signal accuracy $\alpha \in [\frac{1}{2}, 1]$ reflects the fact that platforms may differ significantly in the informativeness of their reputation systems (Garg and Johari 2021), or in the level of inflated or fraudulent reviews (Filippas et al. 2022, Donaker et al. 2019). For example, the pet-sitting platform Rover does not allow sitters to rate owners, which corresponds to an uninformative signal ($\alpha \approx \frac{1}{2}$), whereas Upwork displays aggregate client information such as the total number of jobs and average rating (intermediate α). In contrast, the freelance platform Fiverr publicly displays the reviews received by a buyer on their profile in addition to awarding “badges” to high-spenders (large α). Moreover, because the accuracy of the “safe buyer” signal $\sigma = s$ increases in α , one can interpret α as a proxy for the level of trust between users (in our case, that sellers have in buyers), which has been theorized to play a role in disintermediation (Gu and Zhu 2021).

Seller switching cost. The sellers’ cost ϕ of transacting off-platform may stem from the platform’s “superior transaction infrastructure” (Hagiu and Wright 2024), including project management tools (Fiverr 2024), AI assistants (Upwork 2024a), or handling cross-currency payments. Similar to Hagiu and Wright (2024), we interpret the switching cost ϕ as parameterizing the platform’s vulnerability to disintermediation. This interpretation aligns with the notion that platforms can limit disintermediation by providing value for its users on-platform, thus making it more costly to disintermediate (Gu and Zhu 2021, He et al. 2023).

Single-period model. We use a single-period model to capture interactions on the platform, meaning buyers and sellers are assumed to have not previously interacted, and sellers rely exclusively on the platform’s signal to assess buyer risk. This assumption allows us to isolate how the quality of information obtained *via the platform* influences disintermediation. Further, in several contexts (e.g., website design, home repair), the vast majority of transactions are between freelancers and employers

⁶Our model precludes outcomes where a buyer contacts a seller whose price exceeds the buyer’s valuation (i.e., $\theta q < p$) with the hope of negotiating a lower price upon disintermediating. This reflects a natural assumption where buyers only contact sellers with acceptable posted prices, and where disintermediation occurs only after both parties have agreed to transact.

who have not engaged previously; empirical evidence suggests that the risk of disintermediation is significant even during first-time transactions (Gu and Zhu 2021, Chintagunta et al. 2023). To capture the possibility that sellers may also obtain information based on previous interactions with a buyer, we consider an extension of our model with repeated transactions in Appendix F.

Matching. Given that disintermediation occurs after buyers and sellers have already been matched, we abstract away details of the platform’s matching algorithm. As a consequence, each seller in our model is (always) matched to a randomly chosen buyer, who is safe with probability λ and risky otherwise.

Buyer riskiness. In freelance and other platforms, unprofessional buyers create friction throughout the engagement, for instance by changing project scope, being unresponsive, or creating payment problems (e.g., delays or non-payment) (Kazi 2020, Shevchuk and Strebkov 2015). Our model treats these different types of buyer risk as correlated by assuming that risky buyers (type- r) expose sellers to both non-payment and higher transaction costs. Empirical evidence suggests that opportunistic client behavior in online labor markets generates both payment-related risks and operational disruptions that are costly to freelancers (Shevchuk and Strebkov 2015). Further, these risks depend on the transaction channel: given that many platforms typically protect sellers (e.g., through escrow or insurance), non-payment is primarily a concern when sellers disintermediate, whereas higher transaction costs may be incurred both on- and off-platform.

Our assumption that safe buyers constitute a majority ($\lambda \geq \frac{1}{2}$) allows us to isolate how disintermediation alters the role of information in platform design, which is the focal question of this paper. When this assumption does not hold (i.e., $\lambda < \frac{1}{2}$), platforms benefit from low information quality (small α) even without disintermediation, as they profit from deceiving sellers into transacting with risky buyers. In contrast, our model aligns with how most platforms function in practice, where higher information quality increases transaction volume (e.g., see Tadelis (2016)), but simultaneously makes disintermediation more attractive for sellers.

Seller qualities. We assume each seller’s quality is both exogenous and observable. Given our focus on freelance marketplaces, quality can be interpreted as the degree to which a seller’s skillset matches the needs of buyers on the platform (as in Bimpikis et al. (2024)), which are typically advertised publicly in sellers’ profiles (e.g., PHP-based web design). This also aligns with the notion that sellers typically engage more frequently with the platform than buyers do, which gives the platform reasonably high accuracy information about sellers, which it can pass on to buyers (e.g., in the form of reviews). Our assumption of public and exogenous seller quality allows us to focus on the risks sellers face in disintermediating, rather than buyers.

2.2 Preliminaries

We conclude this section by discussing the sellers' profit, offline price, and conditions under which disintermediation occurs. We also state two assumptions that are imposed in the paper.

2.2.1 Sellers' Profit

Consider a seller with quality q , and suppose p and b are the prices in the online and offline channels, respectively. Since θ is uniformly distributed over $[0, 1]$ and the buyer has a payoff of $\theta q - p$ for an online transaction, only buyers with $\theta \geq p/q$ contact a quality- q seller. Given an online price $p \leq q$, the quality- q seller's expected profit from an online transaction if matched to a signal- σ buyer is

$$((1 - \gamma)p - (1 - \omega_\sigma)c) \left(1 - \frac{p}{q}\right),$$

where ω_σ is the seller's posterior belief that the buyer is type- s (i.e., safe) conditioned on the signal σ . Note the seller's expected transaction cost for any transaction (online or offline) depends on their belief of the buyer's type, and that only risky buyers impose the cost c .

To determine the seller's offline profit, recall that only type- s buyers pay when transacting offline. Given an offline price of b , the seller's expected payment offline is then $\omega_\sigma b$. The seller's expected profit from an offline transaction with a signal- σ buyer at the price b under a switching cost of ϕ is then

$$(\omega_\sigma b - \phi - (1 - \omega_\sigma)c) \left(1 - \frac{p}{q}\right).$$

If neither transaction channel is profitable for the seller (i.e., both profit expressions above are negative), the seller *rejects* the buyer and collects their reservation profit of 0. Relatedly, we impose an assumption throughout the paper that the two seller types are well-separated with respect to quality:

Assumption 1. *The seller qualities q_L and q_H satisfy $q_L \leq (1 - \lambda)c$ and $q_H \geq 4c$.*

Under Assumption 1, for any commission rate $\gamma \in [0, \gamma^m]$ and information quality $\alpha \in [\frac{1}{2}, 1]$, the type- H seller transacts with both $\sigma = r$ and $\sigma = s$ buyers under their optimal price, and the type- L seller rejects $\sigma = r$ buyers under their optimal price (see Lemma A.5 in Appendix A). This assumption allows us to focus on the interesting case where the type- L seller's quality is low enough that they can transact profitably only with type- s buyers, and thus have an interest in screening out type- r buyers.

2.2.2 Offline Price Bargaining

Next, to specify the offline price, we model the buyer's surplus from disintermediating as simply the price reduction⁷ $p - b$, whereas the seller's surplus is the expected increase in payment from disintermediating $\omega_\sigma b - (1 - \gamma)p$ less the switching cost ϕ . The product of the buyer and seller's surpluses is then $(p - b)(\omega_\sigma b - (1 - \gamma)p - \phi)$. Note that the Nash product is strictly concave and quadratic in b . Solving for the unique maximizer yields the offline price $b_\sigma(p)$, where

$$b_\sigma(p) := \frac{p(1 - \gamma + \omega_\sigma) + \phi}{2\omega_\sigma}.$$

The expression for $b_\sigma(p)$ satisfies intuition: for a *fixed* price p , a higher commission rate strengthens the buyer's bargaining position and produces a lower offline price. Conversely, a large switching cost reduces sellers' incentive to disintermediate, leading to a higher offline price. Further, because $\omega_s > \omega_r$, the offline price is higher for $\sigma = r$ buyers than $\sigma = s$, reflecting the increased risk assumed by the seller.

2.2.3 Commission Thresholds and Platform Revenue

Given a fixed online price p , the buyer and seller choose to disintermediate if and only if both prefer transacting offline at price $b_\sigma(p)$ over transacting online at price p . The following remark describes when this occurs.

Remark 1. *Let p be a seller's online price. Then both the buyer and seller prefer transacting offline at price $b_\sigma(p)$ over transacting online at price p if and only if $\gamma > \hat{\gamma}_\sigma(p)$, where*

$$\hat{\gamma}_\sigma(p) := 1 - \omega_\sigma + \frac{\phi}{p}.$$

The offline channel is preferred by both the buyer and seller if and only if the commission rate is sufficiently high. Note ω_σ is the probability a signal- σ buyer pays the seller offline, which implies the commission threshold for disintermediation $\hat{\gamma}_\sigma(p)$ is decreasing in the seller's posterior belief that the buyer is safe (ω_σ). It can be shown that $\hat{\gamma}_s(p) < \hat{\gamma}_r(p)$ for each $p > 0$, which implies that at a given price p and commission rate γ , one of three outcomes is possible: a seller transacts offline with no buyers, with only $\sigma = s$ buyers, or with all buyers.

Finally, since the seller's online price p depends on the commission rate γ , one can endogenize this dependence in the expression for the disintermediation thresholds in Remark 1. As a consequence, for each seller-buyer pair, we can derive a unique threshold γ_σ^i that is independent of price, such that type- i sellers disintermediate with signal- σ buyers if and only if $\gamma > \gamma_\sigma^i$ (see Appendix A.3

⁷This assumption corresponds to an online and offline buyer utility of $\theta q - p$ and $\theta q - b$, respectively.

for details). Further, after endogenizing the seller's price p , the platform's revenue is a piecewise function of γ with breakpoints defined by the thresholds γ_σ^i (see Lemma A.11).

Figure 1 depicts how the commission thresholds γ_σ^i evolve with information quality α . Note sellers disintermediate at lower (higher) commission rates with $\sigma = s$ ($\sigma = r$) buyers as α increases, due to the improved accuracy of the platform signal σ . Note γ_r^L does not appear in Figure 1, because type- L sellers do not transact at all with $\sigma = r$ buyers under Assumption 1.

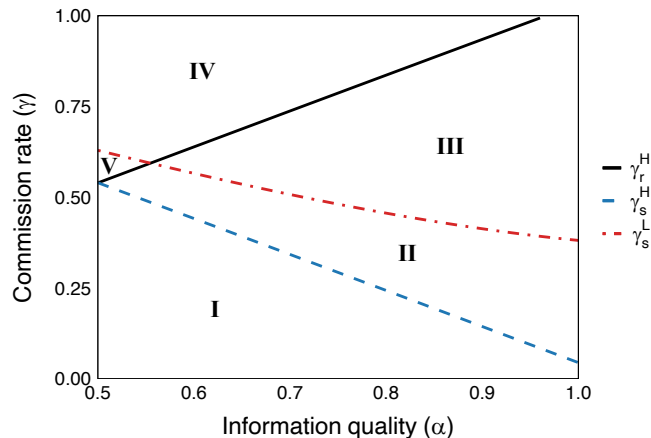


FIGURE 1. Commission thresholds for disintermediation at different levels of information quality α ($\lambda = 0.5$, $c = 1$, $\phi = 0.1$). Type- i sellers disintermediate with signal- σ buyers if and only if $\gamma > \gamma_\sigma^i$. Regions I–V each correspond to a different set of seller-buyer pairs (i, σ) that disintermediate: I = $\{\emptyset\}$, II = $\{(H, s)\}$, III = $\{(H, s), (L, s)\}$, IV = $\{(H, s), (L, s), (H, r)\}$, and V = $\{(H, s), (H, r)\}$.

To exclude the uninteresting case where all transactions occur offline and the platform earns zero revenue, we focus on a setting where no seller disintermediates with a $\sigma = r$ buyer, which holds under the following assumption:

Assumption 2. *The maximum commission rate γ^m and the probability that the $\sigma = r$ buyer pays the seller offline ω_r satisfy the inequality $\gamma^m + \omega_r < 1$ for all $\alpha \in [\frac{1}{2}, 1]$, where ω_r depends on the share of safe buyers λ and information quality α .*

In simple terms, the above assumption guarantees that for all $\phi \geq 0$ and $\gamma \leq \gamma^m$, at least one type of transaction occurs on the platform. Specifically, under Assumption 2, sellers always transact online with the $\sigma = r$ buyers. Lastly, in choosing a revenue-maximizing commission rate, we assume the platform breaks ties by choosing the smallest rate.

3 Platform Commission and Revenue under Disintermediation

We begin by considering how disintermediation influences the platform's optimal commission rate (Section 3.1) and revenue (Section 3.2) under different information quality levels. In particular, we

focus on comparative statics with respect to the switching cost ϕ , which parametrizes the ease of disintermediation for sellers. In what follows, we assume information quality α is exogenous, which allows us to compare across platforms that vary in their informational environments (we consider the setting where the platform can jointly optimize over both information quality α and the commission rate γ in Section 4).

3.1 Impact of Disintermediation on the Optimal Commission Rate

A platform's commission rate is arguably its most natural lever for responding to disintermediation. Given that a high commission rate incentivizes sellers to transact offline (e.g., Figure 1), intuition might prescribe setting a lower commission rate when the threat of disintermediation is strong. Our first result shows that this prescription does not hold universally.

Proposition 1. *Let $\gamma^*(\phi)$ be the platform's optimal commission rate under switching cost ϕ . There exist thresholds $\underline{\alpha} \in (\frac{1}{2}, 1]$ and $\bar{\alpha} \in [\underline{\alpha}, 1)$ such that the following statements hold.*

- (i) *Suppose information quality is low, i.e., $\alpha \leq \underline{\alpha}$. Then the optimal commission rate $\gamma^*(\phi)$ weakly increases in the switching cost ϕ for all $\phi \geq 0$.*
- (ii) *Suppose information quality is high, i.e., $\alpha > \bar{\alpha}$. Then there exists $\bar{\phi} > 0$ such that for each $\phi \geq \bar{\phi}$, the optimal commission rate is higher in the absence of switching costs, $\gamma^*(0) \geq \gamma^*(\phi)$, where the inequality is strict if $\gamma^*(\phi) < \gamma^m$. Further, there exists $\underline{\phi} \in (0, \bar{\phi}]$ such that $\gamma^*(\phi)$ strictly decreases in ϕ on $\phi \in [0, \underline{\phi}]$ wherever $\gamma^*(\phi) < \gamma^m$.*

All proofs for this section are in Appendix B. Proposition 1 suggests that platforms that are vulnerable to disintermediation (i.e., $\phi = 0$) may operate under a *higher* commission rate than platforms that are not ($\phi \geq \underline{\phi}$), provided information quality is also high ($\alpha \geq \bar{\alpha}$). To unpack the intuition, consider that the platform can adopt one of two strategies to respond to disintermediation via the commission rate γ : a “back down” strategy, in which the platform chooses a low commission rate to prevent disintermediation entirely; or a “double down” strategy, in which the platform chooses a high commission rate that forfeits some revenue to disintermediation, but extracts maximal revenue from on-platform transactions. When information quality is low (Proposition 1(i)), the back down strategy is viable, because sellers' trust in $\sigma = s$ buyers is low enough that the platform can set a high commission rate γ without triggering disintermediation. Then, as the switching cost ϕ increases, the platform's pricing power (i.e., with respect to setting the commission rate γ) is only further strengthened, leading the optimal commission rate γ^* to increase in ϕ .

Proposition 1(ii) shows that the prescription from part (i) is reversed when information quality is high. In this setting, the signal σ is highly reliable, and thus sellers face minimal risk in transacting

offline with $\sigma = s$ buyers. In this high-trust environment, disintermediation with $\sigma = s$ buyers is inevitable at all but the very lowest commission rates, which makes the back down strategy sacrifice substantial revenue. As a consequence, the platform is better off adopting the double down strategy – that is, absorbing the revenue losses from sellers disintermediating with $\sigma = s$ buyers, and using a high commission rate to maximize revenue from the on-platform transactions with $\sigma = r$ buyers. Alternatively, one may interpret Proposition 1(ii) as providing conditions under which it is optimal for the platform to not compete with sellers’ “outside option” of disintermediating, and instead specialize in handling risky transactions on-platform.

What does Proposition 1 imply for platforms facing potential disintermediation by sellers? For platforms that operate with a high degree of trust between participants, attempting to thwart disintermediation by lowering commission rates may be ill-advised due to excessive revenue losses. Moreover, many platforms have been criticized for their high commission rates (e.g., see Gurley (2013)), and some have naturally argued that high commission rates cause disintermediation (Edelman and Hu 2016). Our finding here lends support to the alternative view that a high commission rate may in fact be the correct response to disintermediation, for a revenue-maximizing platform. Indeed, Proposition 1 may help reconcile the apparent tension behind the freelance platform Upwork’s decision to raise its commission rate twice in the last two years despite its ongoing concerns over disintermediation (Upwork 2024b).

3.2 Impact of Disintermediation on the Platform’s Optimal Revenue

It is also natural to ask how the maximal revenue attainable under commissions is impacted by a platform’s vulnerability to disintermediation. Analogous to Proposition 1, we find that the directional impact of switching costs on the platform’s optimal revenue depends on information quality. Strikingly, the platform may be *worse* off as disintermediation becomes more costly for sellers:

Proposition 2. *Let $R(\gamma^*)$ be the platform’s revenue under the optimal commission rate γ^* . There exist thresholds $\underline{\alpha} \in (\frac{1}{2}, 1]$ and $\bar{\alpha} \in [\underline{\alpha}, 1)$ such that the following statements hold.*

- (i) *Suppose information quality is low, $\alpha \leq \underline{\alpha}$. Then the platform’s optimal revenue $R(\gamma^*)$ weakly increases in the switching cost ϕ on $\phi \in [0, \infty)$.*
- (ii) *Suppose information quality is high, $\alpha \geq \bar{\alpha}$. Then there exists $\underline{\phi}$ such that the platform’s optimal revenue $R(\gamma^*)$ strictly decreases in the switching cost ϕ on $\phi \in [0, \underline{\phi}]$.*

To understand Proposition 2, it is helpful to consider the effects induced by an increase in the switching cost ϕ . The first is a “pricing power effect”: an increase in ϕ raises the cost of disintermediating for sellers, which the platform can exploit by setting a higher commission rate and extracting

more revenue. However, an increase in ϕ can also induce a “cost pass-on effect”: in anticipation of disintermediation, sellers set *higher* online prices to defray the switching cost. In other words, sellers partially pass on the switching cost ϕ to on-platform buyers, which depresses on-platform demand, and thus platform revenue, as visualized in the left panel of Figure 2. As a consequence, the difference in behavior in parts (i) and (ii) of Proposition 2 depends on which of the two effects described above dominates.

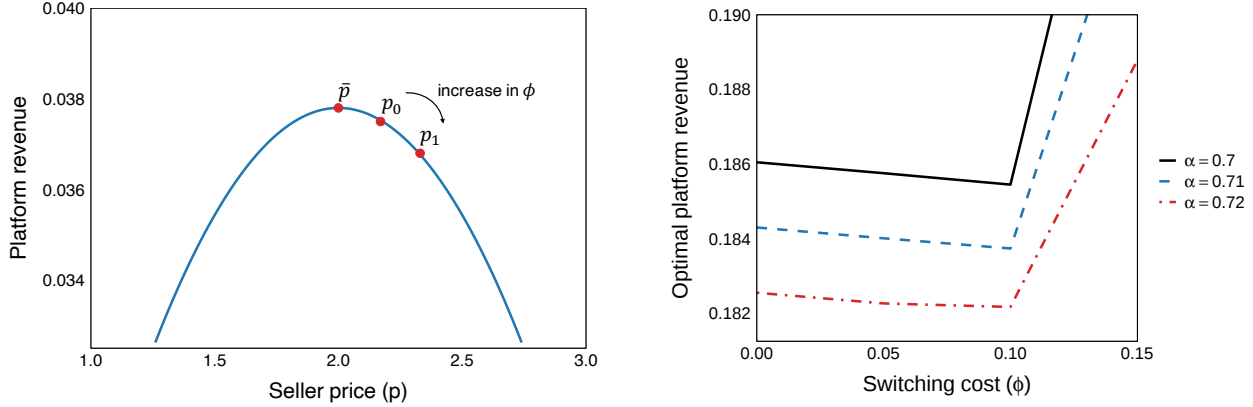


FIGURE 2. **Left:** Platform revenue as a function of the type-H seller’s price when transacting on-platform with $\sigma = r$ buyers and disintermediating with $\sigma = s$ buyers ($\gamma = 0.1$, $\mu = 0.9$, $\lambda = 0.7$, $c = 1$, $q_H = 4$, $\alpha = 0.7$). Price $\bar{p}$ maximizes the sellers’ contribution to platform revenue via commissions. Here, p_0 and p_1 represent the seller’s optimal price under switching costs $\phi = 0$ and $\phi = 1$, respectively. **Right:** Impact of switching cost on optimal platform revenue for varying information quality levels, depicting Proposition 2(ii).

When information quality is sufficiently low, the platform is immune to disintermediation due to the high risk sellers face offline. In this case, the cost pass-on effect is absent because sellers do not factor the switching cost ϕ into their on-platform price. As a consequence, an increase in ϕ acts only to improve the platform’s pricing power, which lifts revenue (Proposition 2(i)). In contrast, when information quality is high and switching costs are low, sellers are undeterred from transacting offline, and the (revenue-decreasing) cost pass-on effect is at play. Further, in this regime the pricing power effect is weak, because sellers simply abandon the platform at higher commission rates. The net result is that platform revenue decreases in ϕ (Proposition 2(ii)). Figure 2 (right panel) depicts a numerical example. Note the threshold in Figure 2 ($\phi \approx 0.1$) is the point at which sellers cease transacting off-platform; beyond this point, further increases in the switching cost improve platform revenue, as expected.

Proposition 2 adds an important caveat to the intuitive prescription from the literature (Gu and Zhu 2021, Hagi and Wright 2024) that recommends combating disintermediation by building value for users on-platform (e.g., through improved transaction support or other features). In particular, this result suggests that if sellers are committed to transacting off-platform, adding incremental value to

the platform may simply be internalized by sellers as the cost of disintermediating, which ultimately distorts their on-platform prices and undermines the platform’s revenue. Nonetheless, platforms can still benefit by *substantially* increasing sellers’ switching costs, which brings sellers back on-platform and restores the platform’s pricing power.

4 Optimal Information Design

Our results in Section 3 indicate that the quality of information available to sellers plays a central role in shaping the platform’s pricing response to disintermediation. Although we assumed information quality to be exogenous in the previous section, in practice marketplace designers may have some limited ability to influence the information available to sellers (Fradkin et al. 2021, Garg and Johari 2021); for example, by altering the platform’s reputation system or moderating sellers’ ability to communicate with buyers (Astashkina et al. 2022). In our setting, because the quality of information α influences sellers’ decisions to disintermediate, the platform could plausibly boost revenue by judiciously choosing the information quality α alongside the commission rate γ . This informational advantage may arise from the platform’s proprietary data on buyer payment history, chat logs, and private reviews from past sellers. To that end, this section considers the conditions under which it is optimal for the platform to withhold information about buyers’ riskiness in light of potential disintermediation.

As a warmup, we first consider how the platform’s optimal revenue changes when α varies exogenously. This exercise allows us to isolate the potentially competing effects of information quality on platform revenue before considering endogenous information quality. Moreover, in situations where the platform cannot optimize information quality, Lemma 1 provides insight into whether small improvements in the signal’s accuracy are in the platform’s best interest.

Lemma 1. *There exist thresholds $\underline{\phi} > 0$ and $\bar{\phi} \geq \underline{\phi}$ such that the following statements hold.*

- (i) *Suppose the switching cost is high, $\phi > \bar{\phi}$. Then the platform’s optimal revenue $R(\gamma^*)$ weakly increases in information quality α on $\alpha \in [\frac{1}{2}, 1]$.*
- (ii) *Suppose the switching cost is low, $\phi \leq \underline{\phi}$. Then there exists $\underline{\alpha} \in (\frac{1}{2}, 1]$, $\bar{\alpha} \in [\underline{\alpha}, 1)$ and $\bar{\lambda} \in [\frac{1}{2}, 1)$ such that the platform’s optimal revenue $R(\gamma^*)$ weakly increases in α on $\alpha \in [\frac{1}{2}, \underline{\alpha}]$ for all $\lambda \in [\frac{1}{2}, 1]$ and strictly decreases in α on $\alpha \in [\bar{\alpha}, 1]$ if $\lambda \geq \bar{\lambda}$.*

All proofs for this section are in Appendix C. Lemma 1 presents conditions under which the platform benefits from – or is hurt by – an increase in information quality α , and is depicted in Figure 3. To see why it holds, note that an increase in α generates two effects on platform revenue. First, as α

increases, sellers set more efficient prices, which we call the “price effect”. Second, as α increases, a greater share of type- s buyers are correctly identified as such to the sellers, which we call the “trust effect”. The net impact of an increase in α on platform revenue depends on the sum of these two effects.

To understand the price effect, it is helpful to examine the optimal price of type- L sellers (see Lemma A.3 in the Appendix):

$$p^* = \frac{1}{2} \left(q_L + \underbrace{\frac{(1 - \omega_s)c}{1 - \gamma}}_{\kappa} \right).$$

In the optimal price p^* , the first term $\frac{1}{2}q_L$ reflects that buyers’ valuations increase in a seller’s quality. The second term $\frac{1}{2}\kappa$ is a “premium” charged by sellers due to the risk of transacting with a type- r buyer mislabeled with the signal $\sigma = s$. The premium $\frac{1}{2}\kappa$ is a pricing inefficiency that stems from the information asymmetry faced by the seller. The premium hurts platform revenue because it raises the price strictly higher than $\frac{1}{2}q_L$, which is the price that maximizes the type- L seller’s commissions to the platform. As information quality α increases, the signal σ becomes more informative to sellers, compelling them to reduce the premium, which benefits the platform.

In both parts of Lemma 1, the price effect acts to lift platform revenue. However, the direction of the trust effect is ambiguous, which drives the divergence between parts (i) and (ii) of Lemma 1. When switching costs are high enough such that disintermediating is prohibitive for sellers, the trust effect only further increases the on-platform transaction volume, which boosts platform revenue (Lemma 1(i)). However, when switching costs are low and information quality is high, the platform is prone to disintermediation, and the direction of the trust effect is reversed – higher information quality amplifies disintermediation by pulling a greater share of transactions off the platform. Lemma 1(ii) states that in the high-information setting where sellers disintermediate (with $\sigma = s$ buyers), the harmful trust effect dominates the helpful price effect, provided the share of buyers that are non-risky is not excessively low ($\lambda \geq \bar{\lambda}$).

The behavior in Lemma 1 suggests that sellers having perfect information about buyers’ types (i.e., $\alpha = 1$) may in some cases lead to sub-optimal platform revenue. Therefore, the platform faces a clear trade-off: a highly accurate signal σ may lead to large revenue losses from disintermediation, but an inaccurate signal may throttle on-platform transactions and lead to inefficient pricing by sellers. This naturally raises the question of when the platform should adopt a no-, partial-, or full-information policy, which we address in the next result.

Proposition 3. *Let α^* be the platform’s revenue-maximizing information quality when jointly op-*

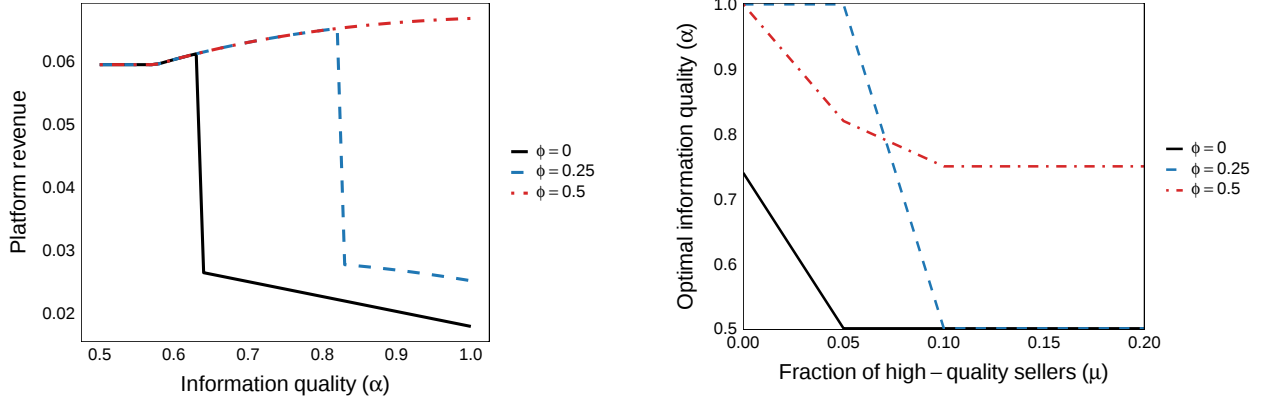


FIGURE 3. **Left:** Platform revenue against varying information quality (α) for fixed commission rate γ and different switching costs ϕ ($\lambda = 0.7$, $c = 1$, $\gamma = 0.2$), depicting Lemma 1. **Right:** Optimal information quality α^* for varying shares of type- H sellers μ , at different switching costs ϕ ($\lambda = 0.5$, $c = 1$, $\gamma^m = 0.35$), depicting Proposition 3. The profit margin of on-platform sellers increases in the share of type- H sellers μ . This leads the platform to set a higher commission rate (not shown), and compensate for the increased risk of disintermediation by lowering information quality.

timized with the commission rate. There exist thresholds $\bar{\mu} \in [0, 1]$, $\bar{\phi} > 0$, $\underline{\alpha} \in (\frac{1}{2}, 1)$ and $\bar{\alpha} \in (\underline{\alpha}, 1)$ such that the following statements hold.

- (i) A no-information policy is optimal $\alpha^* = \frac{1}{2}$ if the share of type- H sellers is large $\mu > \bar{\mu}$ and there is no switching cost $\phi = 0$.
- (ii) A partial-information policy is optimal $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$ if the share of type- H sellers is small $\mu \leq \bar{\mu}$ and there is no switching cost $\phi = 0$. Further, α^* strictly decreases in μ for all $\mu \in [0, \bar{\mu}]$.
- (iii) A full-information policy is optimal $\alpha^* = 1$ for all $\mu \in [0, 1]$ if the switching cost is high $\phi \geq \bar{\phi}$.

Proposition 3 shows how the optimal information quality α^* depends on the share μ of sellers that are type- H (i.e., high-quality) and the sellers' switching cost ϕ . Part (iii) captures the baseline setting where the switching cost is high enough such that no disintermediation occurs – in this case, full-information is optimal because it maximizes the beneficial trust and price effects of the platform's signal, as discussed following Lemma 1. Parts (i) and (ii) focus on the more interesting case where the platform is vulnerable to disintermediation, which is exemplified in the case with no switching costs ($\phi = 0$). The main results are that the optimal information quality α^* is a strictly interior solution $\alpha^* \in (\frac{1}{2}, 1)$ when the share of type- H sellers is not too large ($\mu \leq \bar{\mu}$), where additionally, α^* strictly decreases in μ .

At a high level, the behavior described in parts (i) and (ii) is driven by the different ways in which information quality and the commission rate generate revenue for the platform. In short, the commission rate γ extracts value predominantly from type- H sellers, who command high prices on the platform and thus pay high commission fees. In contrast, the information quality lever α

generates value through type- L sellers, because these sellers only transact with $\sigma = s$ buyers, and thus are the ones for whom the beneficial price and trust effects described above are at play. The tension arises because choosing a large value of α – which increases revenue from type- L sellers – also lowers the commission threshold beyond which type- H sellers transact offline, limiting the largest commission rate the platform can impose without triggering disintermediation.

As a consequence of these trade-offs, when the share of type- H sellers μ is sufficiently large, it is optimal for the platform to disclose no information at all ($\alpha = \frac{1}{2}$). While this policy sacrifices all revenue from type- L sellers, it allows the platform to compensate by choosing a large commission rate γ without risking disintermediation (part (i)). At smaller values of μ , the potential revenue from type- L sellers is too large for the platform to ignore entirely, so the platform should adopt a partial-information policy $\alpha^* \in (\frac{1}{2}, 1)$ to balance the revenue from both seller types. The intuition for why α^* strictly decreases in μ follows similarly. Figure 3 visualizes the behavior described in Proposition 3 for different values of the switching cost ϕ .

Proposition 3(ii) describes a setting where partial-information is optimal when there is no switching cost $\phi = 0$. The following corollary provides an alternative characterization that shows a partial-information policy can also be optimal when switching costs are strictly positive.

Corollary 1. *There exist thresholds $\bar{\mu} \in [0, 1)$, $\underline{\phi} \geq 0$, and $\bar{\phi} > \underline{\phi}$ such that partial-information is optimal $\alpha^* \in (\frac{1}{2}, 1)$ if the switching cost is moderate $\phi \in [\underline{\phi}, \bar{\phi}]$ and the share of type- H sellers is large $\mu > \bar{\mu}$.*

High-information environments have been classically viewed as strictly beneficial for two-sided marketplaces (e.g., Resnick and Zeckhauser (2002)). Lemma 1 adds some nuance to this belief by showing that excessive trust between buyers and sellers can undermine revenue by promoting disintermediation, supporting the empirical finding in Gu and Zhu (2021). Further, Proposition 3 and Corollary 1 indicate that platforms can benefit from withholding information about buyers when sellers set high prices on-platform (i.e., are high-quality), essentially by exploiting the off-platform risks faced by sellers to set higher commission fees. Note this is true even when the platform features offer “moderate” value to its sellers (Corollary 1). In general, our results highlight how for some sellers (type- L), information about buyers helps them judiciously filter out costly buyers before transacting on-platform, but for others (type- H), it only amplifies disintermediation. Therefore, in designing the informational environment, platforms should consider the mix of sellers operating in the marketplace, and the different impacts of buyer information on sellers’ pricing and transaction decisions.

5 Extensions

This section considers two extensions of our main model. In Section 5.1, we examine an alternative to commission pricing where the platform charges sellers an access fee before matching them to a buyer. Section 5.2 examines the punitive measure of banning sellers from the platform if they are caught transacting off-platform.

5.1 Platform-Access Fees

Beyond adjusting commission rates and information quality, a platform may consider a variety of interventions for building resilience to disintermediation. In this section, we examine an alternative pricing strategy observed in practice: charging sellers an upfront *access fee* to join the platform, instead of extracting commissions from on-platform transactions.

Intuitively, eliminating commissions in favor of access fees can mitigate revenue losses from disintermediation by reducing sellers' incentives to transact off-platform. What is less clear is their efficiency: do access fees generate higher revenue than commissions, even when the threat of disintermediation is weak? Proposition 4 below answers this question in the negative – while access fees can dominate commissions when the platform is disintermediation-prone (i.e., switching costs are low), they can fall short of the maximum possible commission revenue when sellers transact exclusively online and sellers are sufficiently heterogeneous in quality.

Our model is a straightforward extension of the main model from Section 2. Under access fees, each seller pays the platform a fixed fee of $\psi > 0$ to join the platform. The use of a common fee ψ for all sellers aligns with practice, as platforms typically do not discriminate among sellers based on attributes such as quality. Sellers set their online prices after joining, and the remainder of the game proceeds as described in Section 2 (with $\gamma = 0$). Similar to Section 4, we assume the platform jointly optimizes information quality α along with the access fee ψ . For consistency with the commission-based model in Section 2, we assume that the access fee allows the seller to complete at most one transaction on the platform. Additionally, we relax Assumption 2 in this section and set the maximum commission rate to $\gamma^m = \frac{1}{2}$, which ensures that the platform's revenue under commissions is not artificially capped when comparing against access fees.

For $i \in \{L, H\}$, let Π_0^i be the profit of a type- i seller on the platform in the absence of any commission or access fees. Because sellers' outside options are normalized to 0, a type- i seller pays the access fee ψ if and only if $\Pi_0^i \geq \psi$. Further, $\Pi_0^H > \Pi_0^L$ because $q_H > q_L$. Therefore, for any access fee $\psi \leq \Pi_0^H$, either both seller types join the platform, or only the type- H sellers join. The platform's

revenue under an access fee of ψ is then

$$R_A(\alpha, \psi) := \begin{cases} \psi, & \text{if } \psi \in [0, \Pi_0^L], \\ \mu\psi, & \text{if } \psi \in (\Pi_0^L, \Pi_0^H], \\ 0, & \text{if } \psi > \Pi_0^H. \end{cases}$$

It follows that the platform's optimal access fee satisfies $\psi^* \in \{\Pi_0^L, \Pi_0^H\}$, and the corresponding optimal revenue is $R_A^* := \max\{\Pi_0^L, \mu\Pi_0^H\}$. We now present our main result of this section:

Proposition 4. *Let R_A^* and R_C^* be the platform's revenue under the optimal pricing and information policies for access and commission fees, respectively. Let Π_0^i be the on-platform profit of a type- i seller under a commission rate of $\gamma = 0$. There exists $\bar{\phi} > 0$ such that the following statements hold.*

- (i) *Suppose the switching cost is low, $\phi \leq \bar{\phi}$. Then access fees generate higher revenue than commission fees $R_A^* \geq R_C^*$ for all $\mu \in [0, 1]$.*
- (ii) *Suppose the switching cost is high, $\phi > \bar{\phi}$. Then there exists $\underline{\mu} \in \left(0, \frac{\Pi_0^L}{\Pi_0^H}\right)$ and $\bar{\mu} \in \left(\frac{\Pi_0^L}{\Pi_0^H}, 1\right)$ such that access fees generate lower revenue than commission fees $R_A^* < R_C^*$ if and only if the share of type- H sellers is moderate $\mu \in [\underline{\mu}, \bar{\mu}]$.*

All proofs for this section are in Appendix D. Naturally, the performance of both pricing strategies depends on the extent to which they can extract sellers' on-platform profit. For the intuition behind Proposition 4(i), note that under the optimal information and commission policy (α^*, γ^*) , either both seller types transact online, or only type- H sellers do (which occurs when the type- L seller either disintermediates or is unprofitable). If only type- H sellers transact online under (α^*, γ^*) , the result is straightforward – the platform can extract the entirety of type- H sellers' on-platform profit Π_0^H by simply setting ψ to that amount, which commission fees cannot match. The result in part (i) is less immediate when both seller types transact online; essentially, if both seller types transact online under (α^*, γ^*) , then the commission rate γ^* must be small enough such that the type- L seller is profitable and neither seller type disintermediates. This yields an upper bound on the optimal commission revenue, which we show is exceeded by the revenue attained under the optimal access fee.

Next, to understand Proposition 4(ii) (depicted in Figure 4), note that when the switching cost is high ($\phi > \bar{\phi}$), the threat of disintermediation is weak, and both seller types transact online even at high commission rates. Then, it is helpful to consider the sellers' "uncaptured profit": the portion of all sellers' potential on-platform profit (under $\gamma = 0$) that is not extracted by the access fee. When sellers are highly heterogeneous (i.e., the share of type- H sellers μ is moderate), the uncaptured

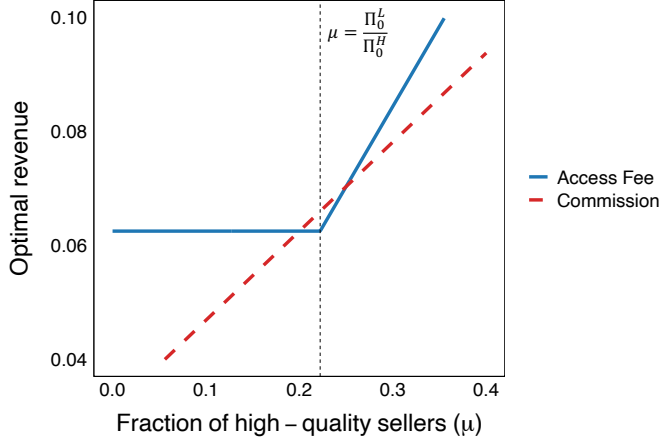


FIGURE 4. Optimal revenue under platform-access fees and commissions ($\lambda = 0.5$, $c = 1$, $q_H = 2$, $q_L = 0.5$, $\phi = 1$), depicting Proposition 4(ii).

profit is large regardless of how the platform chooses ψ , making access fees inefficient. In other words, platform revenue under access fees suffers when the aggregate profits of type- L and type- H sellers are comparable. In contrast, because commissions resemble personalized pricing, they can efficiently extract surplus from transactions occurring at different price points, regardless of the composition of sellers. We note that while Proposition 4 assumes information quality α is controlled by the platform in both pricing mechanisms, the same result can be shown to hold for exogenous α , provided it is not too low.

Upfront pricing mechanisms have drawn significant attention in recent years, and pose a variety of advantages over commission fees (Hu and Zhou 2020, Feldman et al. 2023, Cachon et al. 2025, Cui and Hamilton 2022). Our result complements this line of work by showing that commission fees – despite creating clear incentives for sellers to disintermediate – can nevertheless outperform access fees when sellers are heterogeneous in quality and transacting offline is sufficiently costly. This finding may help explain the prevalence of commission fees in practice, despite their vulnerability to disintermediation. Moreover, Proposition 4 suggests that platforms that charge sellers for access to buyers (e.g., Thumbtack and Care.com) may perceive disintermediation to be more of a threat than those that continue to rely on commissions.

Our result may also help explain why the pricing strategy of a platform may vary across markets. For example, Uber charges drivers commission fees in North America, but in 2022 unveiled access-based pricing with 0% commission for drivers in South Asia (Uber 2023). This may be due to variability in the perceived costs of disintermediation and the level of social trust across markets; indeed, anecdotal evidence suggests platforms may witness higher levels of disintermediation in emerging economies (Rampal 2023).

Our model abstracts away from additional practical considerations that may influence a seller’s willingness to pay upfront for platform access. In particular, we assume sellers engage in a single transaction at most, know their own quality, and face no uncertainty about the demand state on the platform. We conjecture that our main insight – that access fees may underperform commission fees despite immunizing the platform from disintermediation – is likely to persist when accounting for these additional characteristics of two-sided platforms.

5.2 Detecting Disintermediation: Should Platforms Ban Sellers?

We have mainly focused on two costs sellers face when transacting offline: non-payment from risky buyers and the switching cost (ϕ). However, platforms may also have an (imperfect) ability to detect when a buyer and seller matched on the platform transact offline – for example, by scanning messages (Chintagunta et al. 2023) or encouraging platform users to report others’ attempts at circumvention (Upwork 2023a). For several platforms, disintermediation violates user policy, and can result in account suspension or outright bans from future use of the platform (Upwork 2023a, Taskrabbbit 2024). In this regard, the risk of punishment for disintermediating introduces an additional off-platform cost that can influence sellers’ willingness to disintermediate. Here we consider the question of whether platforms should adopt a policy of banning sellers who are caught disintermediating.

We consider a simple extension of our model to a two-period setting. Sellers are long-lived and participate in both periods; buyers are short-lived and arrive independently in each period. Each seller selects an online price p_t for period $t \in \{1, 2\}$. In each period, price bargaining for offline transactions proceeds as outlined in Section 2.2. We assume the platform selects a single commission rate $\gamma \in [0, \gamma^m]$ for the entire horizon. To isolate how the detection mechanism impacts platform revenue, we assume information quality $\alpha \in [\frac{1}{2}, 1]$ is exogenous, as in Section 3.

At the start of the horizon, the platform announces and commits to one of two policies (in addition to the commission rate γ): a “banning” policy in which detected sellers are removed from the platform prior to period 2, and a “blind eye” policy in which disintermediation goes unpunished. Under the banning policy, the platform detects disintermediation by sellers in period 1 with probability $d \in [0, 1]$, where d is known to sellers. If the platform chooses to ban, all sellers who are detected in period 1 earn zero profit in period 2. For simplicity, we focus on a setting with no switching costs ($\phi = 0$), where the threat of disintermediation is strongest.

When should platforms adopt a policy of banning sellers for disintermediating? The following result sheds light on the answer in terms of the informational environment and strength of the detection

mechanism.

Proposition 5. *Let $R^0(\gamma^*)$ and $R^d(\gamma^*)$ be the platform’s optimal revenue under the blind eye and banning policies, respectively. Then, there exist thresholds $\underline{\alpha} \in (\frac{1}{2}, 1]$ and $\bar{\alpha} \in [\underline{\alpha}, 1)$ such that the following statements hold.*

- (i) *Suppose information quality is low $\alpha \leq \underline{\alpha}$. Then for all detection probabilities $d \in [0, 1]$, the banning policy generates weakly higher revenue than the blind eye policy, $R^0(\gamma^*) \leq R^d(\gamma^*)$.*
- (ii) *Suppose information quality is high $\alpha \geq \bar{\alpha}$. Then there exists $\bar{d} \in (0, 1)$ such that if the detection probability is low $d \in (0, \bar{d}]$, the banning policy generates strictly lower revenue than the blind eye policy, i.e., $R^0(\gamma^*) > R^d(\gamma^*)$.*

The proof is in Appendix E. Notably, Proposition 5 shows that a policy of banning sellers who are caught disintermediating can “backfire” and hurt platform revenue if the detection mechanism is weak ($d \leq \bar{d}$). To see why, note that the banning policy raises sellers’ cost of transacting offline in period 1 due to the risk of being detected and banned from period 2. This deterrence effect strengthens the platform’s pricing power, allowing it to set a higher commission rate γ and increase revenue (Proposition 5(i)). However, the banning policy can also hurt revenue, because it commits the platform to forgo the commission fees that detected sellers would have otherwise paid in period 2. Proposition 5(ii) shows that in a high-information environment ($\alpha \geq \bar{\alpha}$), a low detection probability d makes the deterrence effect of the ban too weak to overcome the revenue losses from having fewer sellers on the platform. Thus, a policy of banning sellers can be counterproductive precisely in cases where the platform is most vulnerable to disintermediation.

Naturally, as the detection probability d approaches one, sellers never disintermediate due to the nearly guaranteed loss of future profit. However, in practice, the algorithmic filters employed by platforms to detect disintermediation have very low success probabilities - for example, Chintagunta et al. (2023) state it is “technically challenging and resource-costly” to detect disintermediation. Therefore, the regime where d is small is particularly relevant for platforms considering punitive measures for disintermediation. Our result suggests that unless the detection mechanism is powerful enough to create a strong deterrent, the platform may be better off turning a blind eye to disintermediation entirely.

Our stylized model does not capture two other aspects of detection that may arise in practice: (1) *false positives*, wherein the platform wrongly bans sellers who did not engage in disintermediation; and (2) *price signals*, where the platform spots the intent to disintermediate from sellers’ prices. Naturally, incorporating these features would yield a richer model in which banning sellers has more

nuanced impacts on platform revenue. For example, a detection mechanism with false positives would lead to innocent (i.e., non-disintermediating) sellers being punished, and detection based on sellers’ on-platform prices may result in lower prices, both of which may hurt the platform’s commission revenue. Our model also does not capture buyers substituting among sellers, which would alleviate the revenue losses associated with banning.

6 Discussion

Disintermediation poses a major challenge to commission-based platforms, with efforts to counteract it being met with limited success.⁸ Motivated by this, we offer guidelines on when commonly prescribed strategies to combat disintermediation may either succeed or further hurt platform revenue. Our findings suggest that how platforms respond to disintermediation is shaped by two key characteristics: the ease of disintermediation for sellers (switching cost ϕ) and the accuracy of information sellers receive about buyers (information quality α). We summarize our main results and their implications for platform operators in Table 1.

Table 1: Summary of Findings and Managerial Implications.

Dimension	Main Finding	Implication for Platforms
Commission Rate (γ)	When information quality is high, the optimal commission rate can be <i>higher</i> in the absence of switching costs than when such costs are present.	When disintermediation is inevitable, platforms should increase their commission rates to extract more revenue per transaction.
Switching Cost (ϕ)	When information quality is high, platform revenue <i>decreases</i> in switching cost (due to higher on-platform seller prices).	If sellers are going to disintermediate regardless, making it harder by investing in platform features may backfire.
Information Quality (α)	Partial-information disclosure is optimal when switching costs are moderate and the share of high-quality sellers is large.	Platforms may benefit by deliberately keeping reputation systems noisy to make disintermediation riskier.
Access Fees (ψ)	Commissions outperform access fees when high- and low-quality sellers contribute equally to revenue and switching costs are high.	Despite eliminating sellers’ incentive to disintermediate, an upfront fee may be worse for the platform than charging commission fees.
Banning Sellers (d)	A policy of banning sellers hurts revenue when the platform’s probability of detecting disintermediation is low and information quality is high.	Unless banning creates a sufficiently strong deterrent, platforms may be better off tolerating disintermediation to retain more sellers.

When faced with disintermediation, it is natural to expect platforms to lower their commission rates

⁸Upwork’s 2024 10-K filing states: “Our efforts to reduce circumvention may be costly or disruptive to implement, have results that are difficult or impossible to measure, fail to have the intended effect or have an adverse effect on our brand or user experience, reduce the attractiveness of our work marketplace, divert the attention of management, or otherwise harm our business.” (Upwork 2024b).

to maintain transaction volume. Our results challenge this conventional prescription by showing that – in some scenarios – platforms may be better off setting higher commission rates to collect more revenue from those transactions that remain on-platform. Moreover, increasing the switching cost of sellers by investing in better platform features (higher ϕ) improves platform revenue only when sellers are less informed about which buyers are risky.

In order to curb disintermediation, platforms may utilize information or pricing levers, or even resort to banning sellers outright. First, withholding information about risky buyers can reduce off-platform transactions, but one must balance this benefit against the risk of deterring seller participation. Our results also suggest that platforms have strong incentives to control sellers’ perception of risk, e.g., by warning them about off-platform threats (Upwork 2023b) or exaggerating the prevalence of fraudulent buyers. Alternatively, one could eliminate the need to disintermediate by charging upfront access fees. However, access fees discourage marginal sellers from joining (Better Business Bureau 2023) and can underperform commissions when switching costs are high. This helps explain why commission models persist in practice despite creating incentives for circumvention. Finally, a policy of banning sellers caught bypassing the platform can deter “bad actors” to an extent, but ultimately hurt revenue when detection rates are low. Given users’ sophistication in evading detection, this provides a rationale for why platforms have favored softer interventions over de-platforming sellers.

Our results also provide theoretical grounding for recent empirical findings, most notably Gu and Zhu (2021) by formalizing the mechanism by which enhanced trust increases disintermediation. The behavior of our model also provides support for patterns documented in the empirical literature – specifically, that (i) platforms can successfully curb disintermediation by limiting communication between buyers and sellers (Astashkina et al. 2022, Lin et al. 2022), thereby reducing trust, (ii) higher quality sellers exhibit greater propensity to transact off-platform (Gu 2024, Chintagunta et al. 2023), and (iii) targeted commission reductions for experienced sellers can reduce disintermediation (Astashkina et al. 2022).

Naturally, platforms could also utilize other – more sophisticated – design levers in the face of disintermediation beyond those considered here. In particular, two-part tariffs, which combine access fees with commissions, may be effective across a wide range of environments, as previously shown in the literature (Cachon et al. 2025). However, the optimal implementation of these two-part tariffs is non-trivial, since upfront pricing excludes low-demand sellers and commissions reduce seller revenues, limiting the access fee the platform can charge. Further analysis is needed to determine whether the revenue gains from these pricing mechanisms justify their added complexity. It may

also be informative to understand when the optimal two-part tariff consists of only the access fee or commission.

Beyond pricing strategies, platforms could curb disintermediation through strategic matching policies – for example, pairing safe buyers with low-quality sellers who are less likely to disintermediate. While theoretically appealing, such centralized matching approaches could face practical hurdles, since steering buyers away from high-quality sellers could harm both user satisfaction and long-term platform viability. Lastly, while our analysis assumes random matching and no seller competition, relaxing this assumption to allow buyers to choose among (price-competing) sellers would introduce richer dynamics. For instance, promoting seller competition could lead to lower equilibrium prices and thus a reduced incentive to disintermediate, although this could also erode commission revenue and harm low-quality sellers. These extensions represent promising avenues for future work that may further our understanding of how platforms can best respond to disintermediation.

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Appendix

The appendix is organized as follows: Appendix A presents preliminary results that are used throughout the main proofs. Appendices B, C, D, and E contain the proofs for Sections 3, 4, 5.1 and 5.2 respectively. In Appendix F, we study an extension of our main model where buyers and sellers may engage in multiple transactions; the proofs from this section are present in Appendix F.4.

A Preliminary Results

Section A.1 characterizes relevant probabilities, offline prices, and possible cases for a seller's profit and optimal price; Section A.2 characterizes the sellers' profit functions and optimal prices; and Section A.3 provides conditions under which disintermediation occurs and defines the platform's revenue.

A.1 Key Definitions

The main result in this section is Lemma A.3, which describes possible cases for a seller's profit and optimal price, and is referred to extensively throughout the remainder of the appendix.

Lemma A.1 (Signal probabilities and sellers' beliefs). *The following statements hold for $\sigma \in \{r, s\}$.*

- (i) *The share of all buyers assigned the signal σ is η_σ , where $\eta_r := (1 - \alpha)\lambda + \alpha(1 - \lambda)$ and $\eta_s := \alpha\lambda + (1 - \alpha)(1 - \lambda)$. Further, η_r and η_s strictly decrease and strictly increase in α on $\alpha \in [\frac{1}{2}, 1]$, respectively.*
- (ii) *A seller's posterior belief that a buyer with signal σ is type- s is ω_σ , where $\omega_r := \frac{(1-\alpha)\lambda}{\eta_r}$ and $\omega_s := \frac{\alpha\lambda}{\eta_s}$. Further, ω_r and ω_s strictly decrease and increase in α on $\alpha \in [\frac{1}{2}, 1]$, respectively.*

Proof. For statement (i), note $\lambda = \Pr(j = s)$ and $\alpha = \Pr(j = s | \sigma = s) = \Pr(j = r | \sigma = r)$ by definition. The expressions for η_r and η_s then follow by the total probability rule. Further, note $\frac{\partial}{\partial \alpha} \eta_r = 1 - 2\lambda < 0$ and $\frac{\partial}{\partial \alpha} \eta_s = 2\lambda - 1 > 0$. For statement (ii), by Bayes' rule we have

$$\begin{aligned}\omega_r &= \Pr(j = s | \sigma = r) = \frac{(1 - \alpha)\lambda}{(1 - \alpha)\lambda + \alpha(1 - \lambda)} = (1 - \alpha)\lambda / \eta_r, \\ \omega_s &= \Pr(j = s | \sigma = s) = \frac{\alpha\lambda}{\alpha\lambda + (1 - \alpha)(1 - \lambda)} = \alpha\lambda / \eta_s.\end{aligned}$$

Further, note

$$\begin{aligned}\frac{\partial \omega_r}{\partial \alpha} &= -\frac{(1 - \lambda)\lambda}{(\alpha(1 - \lambda) + (1 - \alpha)\lambda)^2} < 0, \\ \frac{\partial \omega_s}{\partial \alpha} &= \frac{(1 - \lambda)\lambda}{((1 - \alpha)(1 - \lambda) + \alpha\lambda)^2} > 0,\end{aligned}$$

where the strict inequalities follow because $\lambda \in [\frac{1}{2}, 1]$. □

Lemma A.2 (Offline price for fixed commission rate γ and online price p). *Consider the transaction between a seller with online price p and a buyer with signal σ at a fixed commission rate $\gamma > 0$. The price in the offline channel is then given by*

$$b_\sigma(p) := \frac{1}{2} \left(\frac{p(1 - \gamma + \omega_\sigma)}{\omega_\sigma} + \frac{\phi}{\omega_\sigma} \right). \quad (1)$$

Proof. Under an offline price of b , the buyer and seller's expected surpluses from transacting offline are $p - b$ and $\omega_\sigma b - (1 - \gamma)p - \phi$, respectively. The Nash bargaining function is thus $N(b) := (p - b)(\omega_\sigma b - (1 - \gamma)p - \phi)$, which can be verified to be strictly concave in b . Solving $\frac{\partial}{\partial b} N(b) = 0$ yields the expression (1). $\square$

Lemma A.3 (Sellers' profit and price cases). *Fix the commission rate γ and consider a unit mass of sellers with quality q and online price $p \leq q$. Let $\Pi(p)$ be the sellers' expected profit (over buyer signals), let $\tilde{p}$ be the maximizer of $\Pi(p)$, and let $\tilde{\Pi}$ be the sellers' expected profit under price $\tilde{p}$.*

(a) *If the sellers transact online with both $\sigma = r$ and $\sigma = s$ buyers,*

$$\begin{aligned}\Pi(p) &= \pi^a(p) := ((1 - \gamma)p - (1 - \lambda)c) \left(1 - \frac{p}{q}\right), \\ \tilde{p} &= p^a := \frac{1}{2} \left(q + \frac{(1 - \lambda)c}{1 - \gamma}\right), \\ \tilde{\Pi} &= \pi^a(p^a) = (1 - \gamma)q \left(\frac{1}{2} - \frac{(1 - \lambda)c}{2q(1 - \gamma)}\right)^2.\end{aligned}$$

(b) *If the sellers reject $\sigma = r$ and transact online with $\sigma = s$,*

$$\begin{aligned}\Pi(p) &= \pi^b(p) := \eta_s ((1 - \gamma)p - (1 - \omega_s)c) \left(1 - \frac{p}{q}\right), \\ \tilde{p} &= p^b := \frac{1}{2} \left(q + \frac{(1 - \omega_s)c}{1 - \gamma}\right), \\ \tilde{\Pi} &= \pi^b(p^b) = \eta_s (1 - \gamma)q \left(\frac{1}{2} - \frac{(1 - \omega_s)c}{2q(1 - \gamma)}\right)^2.\end{aligned}$$

(c) *If the sellers transact online with $\sigma = r$ and offline with $\sigma = s$,*

$$\begin{aligned}\Pi(p) &= \pi^c(p) := \left(\eta_r (1 - \gamma)p + \eta_s (\omega_s b_s(p) - \phi) - (1 - \lambda)c\right) \left(1 - \frac{p}{q}\right), \\ \tilde{p} &= p^c := \frac{1}{2} \left(q + \frac{2(1 - \lambda)c + \eta_s \phi}{2\zeta}\right), \\ \tilde{\Pi} &= \pi^c(p^c) = q\zeta \left(\frac{1}{2} - \frac{2c(1 - \lambda) + \eta_s \phi}{4q\zeta}\right)^2.\end{aligned}$$

where $\zeta = \eta_r (1 - \gamma) + \frac{1}{2} \eta_s (1 - \gamma + \omega_s)$.

(d) *If the sellers reject $\sigma = r$ and transact offline with $\sigma = s$,*

$$\begin{aligned}\Pi(p) &= \pi^d(p) := \eta_s \left(1 - \frac{p}{q}\right) \left(\omega_s b_s(p) - \phi - (1 - \omega_s)c\right), \\ \tilde{p} &= p^d := \frac{1}{2} \left(q + \frac{2c(1 - \omega_s) + \phi}{1 - \gamma + \omega_s}\right), \\ \tilde{\Pi} &= \pi^d(p^d) = \frac{\eta_s q}{2} (1 - \gamma + \omega_s) \left(\frac{1}{2} - \frac{2c(1 - \omega_s) + \phi}{2q(1 - \gamma + \omega_s)}\right)^2.\end{aligned}$$

(e) *If the sellers accept $\sigma = r$ and $\sigma = s$ offline,*

$$\Pi(p) = \pi^e(p) := (\eta_s \omega_s b_s(p) + \eta_r \omega_r b_r(p) - \phi - (1 - \lambda)c) \left(1 - \frac{p}{q}\right),$$

$$\tilde{p} = p^e := \frac{1}{2} \left(q + \frac{2c(1-\lambda) + \phi}{1-\gamma + \eta_r \omega_r + \eta_s \omega_s} \right),$$

$$\tilde{\Pi} = \pi^e(p^e) := \frac{q\zeta'}{2} \left(\frac{1}{2} - \frac{2c(1-\lambda) + \phi}{2q\zeta'} \right)^2,$$

where $\zeta' = 1 - \gamma + \omega_r \eta_r + \omega_s \eta_s$.

Proof. For each case $x \in \{a, b, c, d, e\}$, the profit expression $\pi^x(p)$ follows from the definitions of η_σ , ω_σ , and ω_σ given in Lemma A.1. The profit maximizing price p^x follow by substituting the expression for the offline price $b_\sigma(p)$ (Lemma A.2) into $\pi^x(p)$, noting that $\pi^x(p)$ is then strictly concave in p , and solving the first order condition $\frac{\partial}{\partial p} \pi^x(p) = 0$; the algebraic details are straightforward and omitted. $\square$

A.2 Sellers' Profit Functions and Optimal Prices

Lemma A.4 (Disintermediation under fixed online price p). *Consider a transaction between a seller with online price p and a buyer with signal σ . Both the buyer and seller prefer to transact offline if and only if $\gamma > \hat{\gamma}_\sigma(p)$, where*

$$\hat{\gamma}_\sigma(p) := 1 - \omega_\sigma + \frac{\phi}{p}.$$

Further, suppose $\gamma > 1 - \omega_\sigma$. Then $\gamma > \hat{\gamma}_\sigma(p)$ holds if and only if $p > \hat{p}_\sigma$, where

$$\hat{p}_\sigma := \frac{\phi}{\gamma - (1 - \omega_\sigma)}.$$

Proof. Under the offline price $b_\sigma(p)$ given in (1), the seller has positive surplus from disintermediating if and only if $\omega_\sigma b_\sigma(p) - (1 - \gamma)p - \phi > 0$, or equivalently, $\frac{1}{2}(-p(1 - \gamma - \omega_\sigma) - \phi) > 0$. Re-arranging for γ , the seller's surplus is strictly positive at $b_\sigma(p)$ if and only if $\gamma > 1 - \omega_\sigma + \frac{\phi}{p} = \hat{\gamma}_\sigma(p)$. Similarly, the buyer's surplus is strictly positive if and only if $p - b_\sigma(p) > 0$, or equivalently, $\frac{1}{2\omega_\sigma}(-p(1 - \gamma - \omega_\sigma) - \phi) > 0$. Re-arranging for γ , the buyer's surplus is also positive if and only if $\gamma > \hat{\gamma}_\sigma(p)$. Lastly, in the case where $\gamma > 1 - \omega_\sigma$, the definition of $\hat{p}_\sigma$ follows by re-arranging the inequality $\gamma > \hat{\gamma}_\sigma(p)$. $\square$

Lemma A.5 (Sellers' transaction decisions). *Let Assumption 1 hold. Then the following statements hold for all $\gamma \in [0, \gamma^m]$ and $\alpha \in [\frac{1}{2}, 1]$.*

- (i) *At their optimal price, type-L sellers reject $\sigma = r$ buyers.*
- (ii) *At their optimal price, type-H sellers transact with both $\sigma = r$ and $\sigma = s$ buyers.*

Proof. This proof uses the profit and price expressions from Lemma A.3. **(i).** We show that the type-L seller's profit if they accept the $\sigma = r$ buyer is non-positive in both transaction channels for all $p \geq 0$ and $\gamma \geq 0$. Note the type-L seller's demand is $(1 - \frac{p}{q_L})^+$, which implies their profit is zero for all $p \geq q_L$. For $p < q_L$, the seller's expected payment from a $\sigma = r$ buyer is at most p in either channel. It follows that the seller's profit from transacting with the $\sigma = r$ buyer is at most $p - (1 - \omega_r)c$. Next, note $p - (1 - \omega_r)c \leq q_L - (1 - \omega_r)c \leq (1 - \lambda)c - (1 - \omega_r)c \leq 0$, where the third and fourth inequalities follow from Assumption 1 and because $\lambda \geq \omega_r$ for all $\alpha \in [\frac{1}{2}, 1]$, respectively.

(ii). It is straightforward to verify that $\omega_s \geq \omega_r$ (Lemma A.1), which implies a seller accepts the $\sigma = r$ buyer only if they also accept the $\sigma = s$ buyer. Therefore, to show statement (ii), it suffices to show the

type- H seller accepts the $\sigma = r$ buyer for all $\gamma \in [0, \gamma^m]$ and $\alpha \in [\frac{1}{2}, 1]$. Note there are two cases to consider depending on whether the $\sigma = s$ buyer transacts online or offline; thus, following Lemma A.3, it suffices to show $\pi^a(p^a) \geq \pi^b(p^b) > 0$ and $\pi^c(p^c) \geq \pi^d(p^d) > 0$ both hold. We show $\pi^a(p^a) \geq \pi^b(p^b) > 0$ first. It is straightforward to verify that $\pi^b(p^b) > 0$ using the fact that $\pi^b(p^b)$ strictly decreases in q_H and $q_H \geq 4c$. Because p^a is the maximizer of $\pi^a(p)$, it suffices to show $\pi^a(p^b) - \pi^b(p^b) \geq 0$. Note

$$\pi^a(p^b) - \pi^b(p^b) = \left(1 - \frac{p^b}{q_H}\right) (\alpha c(1 - \lambda) + (1 - \gamma)p^b(\lambda - \alpha(2\lambda - 1)))$$

and that $\pi^b(p^b) > 0$ implies $p^b < q_H$. It remains to show $p^b \geq \frac{c\alpha(1-\lambda)}{(1-\gamma)(\lambda-\alpha(2\lambda-1))}$, or equivalently,

$$\frac{1}{2} \left(q_H + \frac{c(1 - \omega_s)}{1 - \gamma} \right) \geq \frac{c\alpha(1 - \lambda)}{(1 - \gamma)(\lambda - \alpha(2\lambda - 1))}. \quad (2)$$

Note the left hand side of (2) decreases in α because ω_s and ω_s both increase in α (Lemma A.1), and the right hand side of (2) increases in α . Plugging in $\alpha = 1$, it follows that (2) holds if $\frac{q_H}{2} \geq \frac{c}{1-\gamma}$ holds. The preceding inequality holds because $q_H \geq 4c$ by Assumption 1 and $\gamma \leq \frac{1}{2}$. We now show $\pi^c(p^c) \geq \pi^d(p^d) > 0$ using a similar argument. It is straightforward to verify that $\pi^d(p^d) > 0$ using the fact that $\pi^d(p^d)$ strictly decreases in q_H and $q_H \geq 4c$. Because p^c is the maximizer of $\pi^c(p)$, it suffices to show $\pi^c(p^d) - \pi^d(p^d) \geq 0$. Note

$$\pi^c(p^d) - \pi^d(p^d) = \left(1 - \frac{p^d}{q_H}\right) (\alpha c(1 - \lambda) + (1 - \gamma)p^d(\alpha(2\lambda - 1) - \lambda)),$$

and that $\pi^d(p^d) > 0$ implies $p^d < q_H$. It remains to show $p^d \geq \frac{c\alpha(1-\lambda)}{(1-\gamma)(\lambda-\alpha(2\lambda-1))}$. Because p^d increases in ϕ , letting $\phi = 0$ yields the lower bound $p^d \geq \frac{1}{2} \left(q_H + \frac{2c(1-\omega_s)}{1-\gamma+\omega_s} \right)$. Therefore, it remains to show

$$\frac{1}{2} \left(q_H + \frac{c(1 - \omega_s)}{1 - \gamma + \omega_s} \right) \geq \frac{c\alpha(1 - \lambda)}{(1 - \gamma)(\lambda - \alpha(2\lambda - 1))}. \quad (3)$$

Note the left hand side of (3) decreases in α because ω_s and ω_s both increase in α (Lemma A.1), and the right hand side of (3) increases in α . The result follows by plugging in $\alpha = 1$ and noting $q_H \geq 4c$ and $\gamma \leq \frac{1}{2}$. $\square$

Lemma A.6 (Sellers' profit functions). *For $i \in \{L, H\}$, the profit function for the type- i seller is given by $\Pi^i(p)$, defined as follows.*

- (i) *If $\gamma \leq 1 - \omega_s$, then $\Pi^L(p) := \pi^b(p)$ and $\Pi^H(p) := \pi^a(p)$ for all $p \geq 0$.*
- (ii) *If $\gamma > 1 - \omega_s$, then*

$$\Pi^L(p) := \begin{cases} \pi^b(p), & \text{if } p \leq \hat{p}_s, \\ \pi^d(p), & \text{if } \hat{p}_s < p. \end{cases}$$

and

$$\Pi^H(p) := \begin{cases} \pi^a(p), & \text{if } p \leq \hat{p}_s, \\ \pi^c(p), & \text{if } \hat{p}_s < p < \hat{p}_r, \\ \pi^e(p), & \text{if } \hat{p}_r < p. \end{cases}$$

Proof. Note Lemma A.3 defines a seller's profit based on the platform signal σ and the transaction channel, Lemma A.4 provides conditions under which the transaction occurs offline, and Lemma A.5 defines which

signals $\sigma \in \{r, s\}$ are accepted by the type- L and type- H sellers. Combining Lemmas A.3, A.4, and A.5 yields the profit expressions $\Pi^L(p)$ and $\Pi^H(p)$. $\square$

Lemma A.7 (No disintermediation with $\sigma = r$ buyers). *Let Assumption 2 hold. Then neither seller type transacts offline with the $\sigma = r$ buyer for all $\gamma \leq \gamma^m$.*

Proof. Note by Lemma A.5, the type- L seller never transacts with the $\sigma = r$ buyer, so it remains to prove the result for the type- H seller. First, if for some online price p the type- H seller transacts offline with the $\sigma = r$ buyer, then by Lemma A.4 we must have $p > \hat{p}_r$, and by Lemma A.6 the seller's profit is given by $\pi^e(p)$. Further, because $\pi^e(p)$ is maximized at p^e , it follows that $p^e \leq \hat{p}_r$ is a sufficient condition for the transaction with the $\sigma = r$ buyer to be online. By definition of $\hat{p}_r$, the condition $p^e \leq \hat{p}_r$ can be written equivalently as $\gamma \leq \underline{\gamma}_r^H$, where $\underline{\gamma}_r^H$ is defined as the solution to

$$\gamma = 1 - \omega_r + \frac{\phi}{p^e(\gamma)}.$$

It remains to show $\underline{\gamma}_r^H > \gamma^m$ holds for all $\alpha \in [\frac{1}{2}, 1]$. Note that for any ϕ , we have the following lower bound on $\underline{\gamma}_r^H$:

$$\underline{\gamma}_r^H = 1 - \omega_r + \frac{\phi}{p^e(\underline{\gamma}_r^H)} \geq 1 - \omega_r. \quad (4)$$

Next, note

$$\omega_r = \frac{(1 - \alpha)\lambda}{\alpha(1 - \lambda) + (1 - \alpha)\lambda}. \quad (5)$$

It is straightforward to show that the right hand side of (5) strictly decreases in α . Plugging $\alpha = \frac{1}{2}$ into the right hand side of (5) and combining with (4) then yields $\underline{\gamma}_r^H \geq 1 - \lambda$ for all $\alpha \in [\frac{1}{2}, 1]$. It follows that $\lim_{\lambda \rightarrow \frac{1}{2}} \underline{\gamma}_r^H \geq \frac{1}{2} > \gamma^m$ for all $\alpha \in [\frac{1}{2}, 1]$. $\square$

Lemma A.8 (Sellers' optimal prices). *Suppose for some fixed $\gamma \in (0, \gamma^m]$, neither seller type transacts offline with the $\sigma = r$ buyer at their optimal price. Then the following statements hold.*

- (i) *The type- H seller's optimal price satisfies $p^* \in \{p^a, p^c\}$, where $p^* = p^a$ if and only if the type- H seller transacts online with the $\sigma = s$ buyer.*
- (ii) *The type- L seller's optimal price satisfies $p^* \in \{p^b, p^d\}$, where $p^* = p^b$ if and only if the type- L seller transacts online with the $\sigma = s$ buyer.*

Proof. We focus on proving statement (i); the proof of (ii) follows by a similar argument and is briefly addressed afterward.

(i). We proceed in two steps. First, we show $p^* \in \{p^a, p^c\}$. Second, we show $p^* = p^a$ if and only if the type- H seller transacts online with the $\sigma = s$ buyer.

Step 1. Note the type- H seller's profit function $\Pi^H(p)$ is given in Lemma A.6. It is straightforward to verify that $\pi^a(p)$, $\pi^c(p)$, and $\pi^e(p)$ are each strictly concave. Therefore, $\Pi^H(p)$ has five possible local maxima that are candidates for the optimal price p^* : p^a , p^c , and p^e , and the breakpoints $\hat{p}_s$ and $\hat{p}_r$. Note $p^* \neq p^e$ because no seller transacts offline with the $\sigma = r$ buyer by Lemma A.7. Thus, we $p^* \in \{p^a, p^c\}$ by showing $p^* \notin \{\hat{p}_r, \hat{p}_s\}$. Consider $\hat{p}_s$ first. Note that if $\gamma < 1 - \omega_s$, then $\hat{p}_s < 0$, and the result holds trivially. Next,

suppose $\gamma > 1 - \omega_s$ and assume by way of contradiction that $p^* = \hat{p}_s$. Then we must have

$$\lim_{p \rightarrow \hat{p}_s^-} \frac{\partial \Pi^H}{\partial p} \geq \lim_{p \rightarrow \hat{p}_s^+} \frac{\partial \Pi^H}{\partial p}, \quad (6)$$

i.e., $\hat{p}_s$ must be a local maximum of $\Pi^H(p)$. Note that the piecewise profit function $\Pi^H(p)$ switches from $\pi^a(p)$ to $\pi^c(p)$ at $\hat{p}_s$. The inequality (6) is therefore equivalent to

$$\left. \frac{\partial \pi^a}{\partial p} \right|_{p=\hat{p}_s} \geq \left. \frac{\partial \pi^c}{\partial p} \right|_{p=\hat{p}_s}.$$

Using the profit expressions from Lemma A.3, it is straightforward to show

$$\left(\frac{\partial \pi^a}{\partial p} - \frac{\partial \pi^c}{\partial p} \right) \Big|_{p=\hat{p}_s} = \frac{\eta_s(\phi - q_H(\gamma - (1 - \omega_s)))}{2q_H}. \quad (7)$$

Because $\gamma > 1 - \omega_s$, the expression in (7) is non-negative if and only if

$$q_H \leq \frac{\phi}{\gamma - (1 - \omega_s)}. \quad (8)$$

Note that the right hand side (8) is precisely $\hat{p}_s$ (Lemma A.4). Therefore, $p^* = \hat{p}_s$ implies $\hat{p}_s \geq q_H$. However, the type- H seller generates zero demand for all prices $p \geq q_H$, which contradicts $p^* = \hat{p}_s$. We conclude $p^* \neq \hat{p}_s$. By a parallel argument, $p^* = \hat{p}_r$ implies $\gamma > 1 - \omega_r$ and

$$\left(\frac{\partial \pi^c}{\partial p} - \frac{\partial \pi^e}{\partial p} \right) \Big|_{p=\hat{p}_s} = \frac{(1 - \eta_s)(\phi - q_H(\gamma - (1 - \omega_r)))}{2q_H} \geq 0,$$

which implies

$$q_H \leq \frac{\phi}{\gamma - (1 - \omega_r)} = \hat{p}_r.$$

We again obtain a contradiction to $p^* = \hat{p}_r$ because the seller generates zero demand for all $p \geq q_H$. Therefore, $p^* \in \{p^a, p^c\}$.

Step 2. We now show $p^* = p^a$ if and only if the seller transacts online with the $\sigma = s$ buyer. Consider two cases: $\gamma \leq 1 - \omega_s$ and $\gamma > 1 - \omega_s$. Note if $\gamma \leq 1 - \omega_s$, by Lemma A.4 the transaction is never offline because $\gamma \leq \hat{\gamma}_s(p)$ for all $p \geq 0$. In this case, the profit function is simply $\Pi^H(p) = \pi^a(p)$, and the result follows. Now suppose $\gamma > 1 - \omega_s$, which implies $\hat{p}_s > 0$. From Step 1, we know $\hat{p}_s$ cannot be a local maximum, and thus by concavity of $\pi^a(p)$ and $\pi^c(p)$, $p^c < \hat{p}_s < p^a$ cannot hold. It follows that if $\gamma > 1 - \omega_s$, one of three cases must hold: $\hat{p}_s \leq \min\{p^a, p^c\}$, $\hat{p}_s \geq \max\{p^a, p^c\}$, or $p^a < \hat{p}_s < p^c$. In the first case, $\hat{p}_s \leq p^a$ implies $\Pi^H(p)$ has a single local maximum at p^c , which implies $p^* = p^c > \hat{p}_s$. Because the transaction is online at p if $p \leq \hat{p}_s$ (Lemma A.4), we conclude $\hat{p}_s \leq p^a$ implies both $p^* = p^c$ and that the transaction is offline. In the second case, $p^c \leq \hat{p}_s$ implies $\Pi^H(p)$ has a single local maximum at p^a , which implies $p^* = p^a < \hat{p}_s$, which implies the transaction is online. Lastly, if $p^a < \hat{p}_s < p^c$, $\Pi^H(p)$ has two local maxima at p^a and p^c . In this case, either $p^* = p^a < \hat{p}_s$ or $p^* = p^c > \hat{p}_s$ must hold. Thus, in all three cases, $p^* = p^a$ if and only if the transaction is online.

(ii). By Lemma A.6, if $\gamma \leq 1 - \omega_s$ the type- L seller's profit function is simply $\Pi^L(p) = \pi^b(p)$ for all $p \geq 0$.

If $\gamma > 1 - \omega_s$, the q_L seller's profit function is then

$$\Pi^L(p) = \begin{cases} \pi^b(p), & \text{if } p \leq \hat{p}_s, \\ \pi^d(p), & \text{if } \hat{p}_s < p. \end{cases}$$

The result follows by parallel argument to the proof of statement (i), with $\Pi^L(p)$, p^b and p^d in place of $\Pi^H(p)$, p^a and p^c , respectively. $\square$

A.3 Commission Thresholds for Disintermediation and Platform Revenue

The main results in this section are Lemma A.10, which defines thresholds on the commission rate γ that trigger disintermediation, and Lemma A.11, which defines the platform's revenue function. Lemmas A.12 and A.13 provide useful properties of the platform revenue function that are used in later proofs.

Lemma A.9 (Sufficient and necessary conditions for disintermediation). *Suppose a type- i seller transacts with a $\sigma = s$ buyer, where $i \in \{L, H\}$.*

- (i) *For the type- H seller, $\gamma \leq \hat{\gamma}_s(p^a)$ and $\gamma < \hat{\gamma}_s(p^c)$ are necessary and sufficient for the transaction to occur online, respectively.*
- (ii) *For the type- L seller, $\gamma \leq \hat{\gamma}_s(p^b)$ and $\gamma < \hat{\gamma}_s(p^d)$ are necessary and sufficient for the transaction to occur online, respectively.*

Proof. The results largely follow from Lemma A.4, which provides the definitions of $\hat{\gamma}_s(p)$ and $\hat{p}_s$. We briefly address statement (i); the proof of (ii) follows by parallel argument and is omitted. First, suppose the transaction is online, and consider two cases: $\gamma \leq 1 - \omega_s$ and $\gamma > 1 - \omega_s$. If $\gamma \leq 1 - \omega_s$, then $\gamma \leq \hat{\gamma}_s(p^a)$ must hold by definition of $\hat{\gamma}_s$. Now suppose $\gamma > 1 - \omega_s$. Because the transaction is online, by Lemma A.8 we have $p^* = p^a < \hat{p}_s$. By Lemma A.4, $p^a < \hat{p}_s$ implies $\gamma < \hat{\gamma}_s(p^a)$, as desired. Now suppose $\gamma < \hat{\gamma}_s(p^c)$, and consider two cases: $\gamma \leq 1 - \omega_s$ and $\gamma > 1 - \omega_s$. If $\gamma \leq 1 - \omega_s$, then $\gamma \leq \hat{\gamma}(p)$ for all $p \geq 0$, which implies the transaction is online by Lemma A.4. If $\gamma > 1 - \omega_s$, then $\gamma < \hat{\gamma}_s(p^c)$ implies $p^c < \hat{p}_s$ by Lemma A.4. Because p^c is the maximizer of $\pi^c(p)$ and $\pi^c(p)$ is strictly concave, $p^c < \hat{p}_s$ implies the seller's profit function $\Pi^H(p)$ strictly decreases in p on $[\hat{p}_s, \hat{p}_r]$. Further, $p^* \leq p_r$ by Lemma A.7, which implies $p^* \leq \hat{p}_s$. It follows from Lemma A.4 that the transaction occurs online. $\square$

Lemma A.10 (Commission thresholds for disintermediation). *Let $\underline{\gamma}_s^H$, $\bar{\gamma}_s^H$, $\underline{\gamma}_s^L$, and $\bar{\gamma}_s^L$ be the solutions to (9a), (9b), (9c), and (9d) respectively:*

$$\gamma = 1 - \omega_s + \frac{\phi}{p^c(\gamma)}, \tag{9a}$$

$$\gamma = 1 - \omega_s + \frac{\phi}{p^a(\gamma)}, \tag{9b}$$

$$\gamma = 1 - \omega_s + \frac{\phi}{p^d(\gamma)}, \tag{9c}$$

$$\gamma = 1 - \omega_s + \frac{\phi}{p^b(\gamma)}. \tag{9d}$$

Suppose both type- L and type- H sellers accept the $\sigma = s$ buyer for some $\gamma \in (0, \gamma^m]$. Then for each $i \in \{L, H\}$, there exists a unique threshold $\gamma_s^i \in [\underline{\gamma}_s^i, \bar{\gamma}_s^i]$ such that the type- i seller transacts offline with the $\sigma = s$ buyer

under their optimal price if and only if $\gamma > \gamma_s^i$. Further, $\gamma_s^H \leq \gamma_s^L$ holds for all $\phi \geq 0$, and $\gamma_s^i = \underline{\gamma}_s^i = \bar{\gamma}_s^i = 1 - \omega_s$ for $i \in \{L, H\}$ if $\phi = 0$.

Proof. The proof proceeds in four steps. First, we show the type- H seller's transaction is online if $\gamma \leq \underline{\gamma}_s^H$ and offline if $\gamma > \bar{\gamma}_s^H$, which also establishes that $\underline{\gamma}_s^H < \bar{\gamma}_s^H$. Second, we show the existence of the threshold $\gamma_s^H \in [\underline{\gamma}_s^H, \bar{\gamma}_s^H]$. Third, we show the analogous result for the type- L seller. Fourth, we show the final sentence of the lemma statement.

Step 1. By definition of $\underline{\gamma}_s^H$, we have

$$\underline{\gamma}_s^H = 1 - \omega_s + \frac{\phi}{p^c(\underline{\gamma}_s^H)}.$$

By inspecting the expression for $p^c(\gamma)$ (Lemma A.3), it is straightforward to verify that $p^c(\gamma)$ strictly increases in γ . Therefore, $\gamma \leq \underline{\gamma}_s^H$ implies

$$\gamma \leq 1 - \omega_s + \frac{\phi}{p^c(\gamma)}.$$

By Lemma A.9, the inequality above is a sufficient condition for the transaction to be online. Similarly, if $\gamma > \bar{\gamma}_s^H$, then by definition of $\bar{\gamma}_s^H$ and because $p^a(\gamma)$ increases in γ , we must have

$$\gamma > 1 - \omega_s + \frac{\phi}{p^a(\gamma)}.$$

By Lemma A.9, the inequality above is a sufficient condition for the transaction to be offline. This completes the first step.

Step 2. We now show the existence of the threshold $\gamma_s^H \in [\underline{\gamma}_s^H, \bar{\gamma}_s^H]$. To avoid the trivial case where transactions occur online for all $\gamma \in [0, \gamma^m]$, we assume $\gamma_s^H < \gamma^m$. Note that the first step of the proof and the definition of the seller's profit function $\Pi^H(p)$ (Lemma A.6) implies $\pi^a(p^a) < \pi^c(p^c)$ for $\gamma > \bar{\gamma}_s^H$ and $\pi^a(p^a) > \pi^c(p^c)$ for $\gamma < \bar{\gamma}_s^H$. Therefore, because $\pi^a(p^a)$ and $\pi^c(p^c)$ are both continuous in γ , to prove statement (i) it suffices to show $\pi^a(p^a) - \pi^c(p^c)$ strictly decreases in γ on $[\underline{\gamma}_s^H, \bar{\gamma}_s^H]$. Differentiating the seller profit functions, for any γ on $[\underline{\gamma}_s^H, \bar{\gamma}_s^H]$ we have

$$\begin{aligned} \frac{d}{d\gamma}(\pi^a(p^a) - \pi^c(p^c)) &= \left(\frac{\partial \pi^a}{\partial p} \cdot \frac{dp^a}{d\gamma} + \frac{\partial \pi^a}{\partial \gamma} \right) \Big|_{p=p^a} - \left(\frac{\partial \pi^c}{\partial p} \cdot \frac{dp^c}{d\gamma} + \frac{\partial \pi^c}{\partial \gamma} \right) \Big|_{p=p^c} \\ &= \frac{\partial \pi^a}{\partial \gamma} \Big|_{p=p^a} - \frac{\partial \pi^c}{\partial \gamma} \Big|_{p=p^c} \\ &= -p^a \left(1 - \frac{p^a}{q_H} \right) + \left(1 - \frac{\eta_s}{2} \right) p^c \left(1 - \frac{p^c}{q_H} \right) \\ &\leq \left(\left(1 - \frac{\eta_s}{2} \right) p^c - p^a \right) \left(1 - \frac{p^a}{q_H} \right). \end{aligned}$$

In the relations above, the second line follows from the envelope theorem, the third from evaluating the derivative algebraically, and the fourth because $p^c \geq p^a$ when $\gamma \in [\underline{\gamma}_s^H, \bar{\gamma}_s^H]$, which follows from the definitions of $\underline{\gamma}_s^H$ and $\bar{\gamma}_s^H$. Because $\left(1 - \frac{p^a}{q_H} \right) > 0$ and $\eta_s \in (0, 1)$, to show $\frac{d}{d\gamma}(\pi^a(p^a) - \pi^c(p^c)) < 0$ it suffices to show that $2p^a - p^c > 0$. Note we must have $p^c < q_H$ because the type- H seller earns zero profit for all prices above q_H . Therefore, it remains to show $2p^a > q_H$. This follows immediately from inspecting the expression for p^a (Lemma A.3), which shows $p^a > q_H/2$. Therefore, $\pi^a(p^a) - \pi^c(p^c)$ strictly decreases on $\gamma \in [\underline{\gamma}_s^H, \bar{\gamma}_s^H]$, as desired. Lastly, the result that $\gamma_s^H = \underline{\gamma}_s^H = \bar{\gamma}_s^H = 1 - \omega_s$ when $\phi = 0$ follows from the definition of $\underline{\gamma}_s^H$ and $\bar{\gamma}_s^H$.

Step 3. The proof follows similarly to the result for the type- H seller. Using the definitions of $\underline{\gamma}_s^L$ and $\bar{\gamma}_s^L$ and the fact that p^b and p^d are both strictly increasing in γ , it is straightforward to show the transaction is online if $\gamma < \underline{\gamma}_s^L$ and offline if $\gamma > \bar{\gamma}_s^L$. It remains to show $\pi^b(p^b) - \pi^d(p^d)$ strictly decreases in γ on $[\underline{\gamma}_s^L, \bar{\gamma}_s^L]$. Differentiating in γ , we have

$$\begin{aligned} \frac{d}{d\gamma}(\pi^b(p^b) - \pi^d(p^d)) &= \left(\frac{\partial \pi^b}{\partial p} \cdot \frac{dp^b}{d\gamma} + \frac{\partial \pi^b}{\partial \gamma} \right) \Big|_{p=p^b} - \left(\frac{\partial \pi^d}{\partial p} \cdot \frac{dp^d}{d\gamma} + \frac{\partial \pi^d}{\partial \gamma} \right) \Big|_{p=p^d} \\ &= \frac{\partial \pi^b}{\partial \gamma} \Big|_{p=p^b} - \frac{\partial \pi^d}{\partial \gamma} \Big|_{p=p^d} \\ &= -\eta_s p^b \left(1 - \frac{p^b}{q_L} \right) + \frac{\eta_s}{2} p^d \left(1 - \frac{p^d}{q_L} \right) \\ &\leq \left(-\eta_s p^b + \frac{\eta_s}{2} p^d \right) \left(1 - \frac{p^d}{q_L} \right), \end{aligned}$$

where the final inequality follows because $p^d \geq p^b$ when $\gamma \in [\underline{\gamma}_s^L, \bar{\gamma}_s^L]$, which follows from the definitions of $\underline{\gamma}_s^L$ and $\bar{\gamma}_s^L$. It remains to show $-\eta_s p^b + \frac{\eta_s}{2} p^d > 0$, or equivalently, $2p^b > p^d$. Note we must have $p^d < q_L$ because the type- L seller has zero demand for all prices above q_L , and further $2p^b > q_L$ by inspection of the expression for p^b . It follows that $2p^b > q_L > p^d$, as desired.

Step 4. We now show $\gamma_s^H \leq \gamma_s^L$ for all $\phi \geq 0$. By Lemma A.10, we have $\gamma_s^H \leq \bar{\gamma}_s^H$ and $\underline{\gamma}_s^L \leq \gamma_s^L$. Therefore, it suffices to show $\bar{\gamma}_s^H \leq \underline{\gamma}_s^L$. Using the expressions in Lemma A.10, $\bar{\gamma}_s^H \leq \underline{\gamma}_s^L$ holds if and only if $p^a(\bar{\gamma}_s^H)|_{q=q_H} \geq p^d(\underline{\gamma}_s^L)|_{q=q_L}$. Next, note

$$p^a(\bar{\gamma}_s^H)|_{q=q_H} \geq \frac{q_H}{2} \geq 2c \geq 2(1-\lambda)c \geq 2q_L \geq p^d(\underline{\gamma}_s^L)|_{q=q_L}.$$

The first inequality follows by inspecting the expression for p^a (Lemma A.3), the second from Assumption 1, the third because $\lambda \geq 0$, and the fourth inequality follows from Assumption 1. To see that the final inequality holds, note the type- L seller has zero demand for all prices above q_L , which implies $p^d(\gamma) \leq q_L$ for all $\gamma \geq 0$. It follows that $\gamma_s^H \leq \gamma_s^L$. Lastly, the result that $\gamma_s^i = \underline{\gamma}_s^i = \bar{\gamma}_s^i = 1 - \omega_s$ for $i \in \{L, H\}$ when $\phi = 0$ follows by definition of $\underline{\gamma}_s^i$ and $\bar{\gamma}_s^i$. $\square$

Lemma A.11 (Platform's revenue function). *Let p^a , p^b , and p^c be as defined in Lemma A.3, and define*

$$\begin{aligned} r^a(\gamma) &:= \gamma p^a \left(1 - \frac{p^a}{q_H} \right), \\ r^b(\gamma) &:= \gamma \eta_s p^b \left(1 - \frac{p^b}{q_L} \right), \\ r^c(\gamma) &:= \gamma \eta_r p^c \left(1 - \frac{p^c}{q_H} \right). \end{aligned}$$

Then the platform's commission revenue is given by $R(\gamma)$, where

$$R(\gamma) := \begin{cases} \mu r^a(\gamma) + (1-\mu)r^b(\gamma)^+ & \text{if } \gamma \in [0, \gamma_s^H], \\ \mu r^c(\gamma) + (1-\mu)r^b(\gamma)^+ & \text{if } \gamma \in (\gamma_s^H, \gamma_s^L], \\ \mu r^c(\gamma) & \text{if } \gamma \in (\gamma_s^L, \gamma^m], \end{cases}$$

and $x^+ = \max\{0, x\}$.

Proof. To see that the piecewise function $R(\gamma)$ is the platform's commission revenue, note μ and $1 - \mu$ are the shares of type- H and type- L sellers, respectively. By Lemma A.5, the type- H seller accepts both $\sigma = s$ and $\sigma = r$ buyers and the type- L seller rejects the $\sigma = r$ buyer. Then combining Lemmas A.3 and A.10, the type- H sellers' contribution to platform revenue is $\mu r^a(\gamma)^+$ if $\gamma \leq \gamma_s^H$ and $\mu r^c(\gamma)^+$ if $\gamma > \gamma_s^H$; similarly, the type- L sellers' contribution is $(1 - \mu) r^b(\gamma)^+$ if $\gamma \leq \gamma_s^L$ and 0 if $\gamma > \gamma_s^L$. Further, it is straightforward to verify algebraically that $r^a(\gamma) \geq 0$ and $r^c(\gamma) \geq 0$ for all $\alpha \in [\frac{1}{2}, 1]$ and $\gamma \geq 0$; the superscript $(\cdot)^+$ is suppressed accordingly. The function $R(\gamma)$ follows. $\square$

Lemma A.12 (Revenue function properties). *Let $r^a(\gamma)$, $r^b(\gamma)$, and $r^c(\gamma)$ be as defined in Lemma A.11. The following statements hold for all $\gamma \in [0, \frac{1}{2}]$ and $\alpha \in [\frac{1}{2}, 1]$.*

- (i) $r^a(\gamma)$ is strictly concave and increasing in γ , and is independent of α and ϕ .
- (ii) $r^b(\gamma)$ is strictly concave in γ , increases in α wherever $r^b(\gamma) > 0$, and is independent of ϕ .
- (iii) $r^c(\gamma)$ is strictly concave in γ for all ϕ and is strictly increasing in γ for $\phi = 0$.

Proof. (i). Using the expressions for $r^a(\gamma)$ and p^a ,

$$r^a(\gamma) = \gamma p^a \left(1 - \frac{p^a}{q_H} \right) = \frac{\gamma}{4} \left(q_H - \frac{c^2(1-\lambda)^2}{q_H(1-\gamma)^2} \right).$$

Differentiating in γ , we have

$$\frac{\partial r^a}{\partial \gamma} = \frac{1}{4} \left(q_H - \frac{c^2(1-\lambda)^2}{q_H(1-\gamma)^2} \right) - \frac{\gamma}{4} \left(\frac{2c^2(1-\lambda)^2}{q_H(1-\gamma)^3} \right). \quad (10)$$

By inspection, the first term on the right hand side of (10) strictly decreases in γ and the second term strictly increases in γ . Therefore, $\frac{d}{d\gamma} r^a$ strictly decreases in γ , which implies $r^a(\gamma)$ is strictly concave in γ . Further, using the fact that $q_H \geq 4c$ (Assumption 1) it can be shown that $\lim_{\gamma \rightarrow \frac{1}{2}} \frac{d}{d\gamma} r^a > 0$. It follows that $r^a(\gamma)$ strictly increases in γ for $\gamma \in [0, \frac{1}{2}]$.

(ii). Using the expressions for p^b and $r^b(\gamma)$ (Lemmas A.3 and A.11), we have

$$r^b(\gamma) = \eta_s \gamma p^b \left(1 - \frac{p^b}{q_L} \right) = \frac{\eta_s \gamma}{4} \left(q_L - \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2} \right).$$

Note for any $\gamma \in [0, \gamma^m]$, using the expressions for η_s and ω_s (Lemma A.1) we have

$$\begin{aligned} \frac{\partial r^b}{\partial \alpha} &= \frac{\partial r^b}{\partial \eta_s} \frac{\partial \eta_s}{\partial \alpha} + \frac{\partial r^b}{\partial \omega_s} \frac{\partial \omega_s}{\partial \alpha} \\ &= \frac{\gamma}{4} \left(q_L - \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2} \right) \frac{\partial \eta_s}{\partial \alpha} + \left(\frac{2c^2(1-\omega_s)}{q_L(1-\gamma)^2} \right) \frac{\partial \omega_s}{\partial \alpha} \\ &= \frac{\gamma}{4} \left(\frac{r^b}{\eta_s} \right) (2\lambda - 1) + \left(\frac{2c^2(1-\omega_s)}{q_L(1-\gamma)^2} \right) \frac{(1-\lambda)\lambda}{\eta_s^2} \\ &> 0, \end{aligned}$$

where the strict inequality follows from $\lambda \in [\frac{1}{2}, 1]$. Thus $r^b(\gamma)$ strictly increases in α if $r^b(\gamma) > 0$. Next,

differentiating $r^b(\gamma)$ in γ yields

$$\frac{\partial r^b}{\partial \gamma} = \frac{\eta_s}{4} \left(q_L - \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2} \right) - \frac{\eta_s \gamma}{4} \left(\frac{2c^2(1-\omega_s)^2}{q_L(1-\gamma)^3} \right) = \frac{\eta_s}{4} \underbrace{\left(q_L - \left(1 + \frac{2\gamma}{1-\gamma} \right) \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2} \right)}_{g(\gamma)}, \quad (11)$$

where for convenience $g(\gamma)$ is defined as shown in (11). Note by inspection that $g(\gamma)$ strictly increases in γ , which implies $\frac{d}{d\gamma} r^b$ strictly decreases in γ . Hence $r^b(\gamma)$ is strictly concave in γ .

(iii). Using the expressions for $r^c(\gamma)$ and p^c , for $q = q_H$ we have

$$r^c(\gamma) = \eta_r \gamma p^c \left(1 - \frac{p^c}{q_H} \right) = \eta_r \gamma q_H \left(\frac{1}{4} - \left(\frac{2(1-\lambda)c + \eta_s \phi}{4q_H \zeta} \right)^2 \right),$$

where $\zeta = \eta_r(1-\gamma) + \frac{1}{2}\eta_s(1-\gamma+\omega_s)$. Differentiating in γ and using the fact that $\frac{d}{d\gamma} \zeta = -\eta_r - \frac{1}{2}\eta_s = \frac{1}{2}\eta_s - 1$, we have

$$\begin{aligned} \frac{\partial r^c}{\partial \gamma} &= \eta_r q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H \zeta} + \frac{\eta_s \phi}{4q_H \zeta} \right)^2 \right) - \frac{2\eta_r q_H}{\zeta^3} \gamma \left(\frac{(1-\lambda)c}{2q_H} + \frac{\eta_s \phi}{4q_H} \right)^2 \left(1 - \frac{\eta_s}{2} \right) \\ &= \eta_r q_H \left(\frac{1}{4} - \frac{h^2}{q_H^2} \left(\frac{1}{\zeta^2} + \frac{2\gamma}{\zeta^3} \left(1 - \frac{\eta_s}{2} \right) \right) \right), \end{aligned} \quad (12)$$

where $h = \frac{1}{4}(2c(1-\lambda) + \eta_s \phi)$. Next, note (12) strictly decreases in γ because $\frac{\partial}{\partial \gamma} \zeta = \frac{1}{2}\eta_s - 1 < 0$ and

$$\frac{\partial}{\partial \gamma} \left(\frac{\gamma}{\zeta^3} \right) = \frac{8(2+4\gamma-\eta_s(1+2\gamma-\omega_s))}{(2(1-\gamma)-\eta_s(1-\gamma-\omega_s))^4} > 0,$$

where the strict inequality follows because $\gamma \in [0, \frac{1}{2}]$, $\eta_s \in [0, 1]$ and $\omega_s \in [0, 1]$. It follows that $r^c(\gamma)$ is strictly concave. Finally, consider the case when $\phi = 0$:

$$\frac{\partial r^c}{\partial \gamma} = \frac{\eta_r q_H}{4} \left(1 - \frac{c^2(1-\lambda)^2}{q_H^2} \left(\frac{1}{\zeta^2} + \frac{2\gamma}{\zeta^3} \left(1 - \frac{\eta_s}{2} \right) \right) \right).$$

Since $\gamma \leq \frac{1}{2}$, we have $\zeta \geq 1 - \gamma \geq \frac{1}{2}$. Also note $\omega_s \geq \omega_s \geq \frac{1}{2}$. Utilizing this bound on ζ along with $q_H \geq 4c$, $\lambda \geq \frac{1}{2}$, and $\eta_s \geq \frac{1}{2}$, we can write

$$\frac{\partial r^c}{\partial \gamma} \geq \frac{\eta_r q_H}{4} \left(1 - \frac{c^2(1-\lambda)^2}{16c^2} \left(\frac{1}{(\frac{1}{2})^2} + \frac{2(\frac{1}{2})}{(\frac{1}{2})^3} \left(1 - \frac{1}{4} \right) \right) \right) > 0.$$

We conclude that $r^c(\gamma)$ strictly increases in $[0, \gamma^m]$ when $\phi = 0$. □

Lemma A.13. *The inequality $\frac{8}{15}r^a(\gamma) > r^c(\gamma)$ holds for all $\gamma \in [0, \gamma^m]$ and $\phi \geq 0$.*

Proof. We first show the following two inequalities hold:

$$\zeta \leq 1 - \gamma + \frac{\lambda \gamma}{2} \quad (13a)$$

$$1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma + \frac{\lambda \gamma}{2})} \right)^2 \leq \frac{16}{15} \left(1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma)} \right)^2 \right) \quad (13b)$$

To see that (13a) holds, note

$$\zeta = \eta_r(1 - \gamma) + \frac{\eta_s}{2}(1 - \gamma + \omega_s) \leq \eta_r(1 - \gamma) + \frac{\eta_s}{2}(1 - \gamma + 1) = 1 - \gamma + \frac{\eta_s \gamma}{2} \leq 1 - \gamma + \frac{\lambda \gamma}{2},$$

which follows because $\omega_s \leq 1$, $\eta_r \leq 1$, and $\eta_s \leq \lambda$. Next, for (13b) we have

$$\frac{1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma+\frac{\lambda\gamma}{2})} \right)^2}{1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma)} \right)^2} \leq \frac{1 - \left(\frac{1-\lambda}{2+\lambda} \right)^2}{1 - \left(\frac{1-\lambda}{2} \right)^2} \leq \frac{16}{15}.$$

The first inequality above follows by noting the ratio in the left hand side is decreasing in q_H and increasing in γ , and because $q_H \geq 4c$ and $\gamma \leq \frac{1}{2}$ by Assumptions 1 and 2. The second inequality follows because the intermediate expression decreases in λ and because $\lambda \geq \frac{1}{2}$. We can now prove the lemma statement. Note

$$\begin{aligned} r^c(\gamma) &= \eta_r \gamma q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H \zeta} + \frac{\eta_s \phi}{4q_H \zeta} \right)^2 \right) \\ &\leq \eta_r \gamma q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H \zeta} \right)^2 \right) \\ &\leq \eta_r \gamma \frac{q_H}{4} \left(1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma+\frac{\lambda\gamma}{2})} \right)^2 \right) \\ &\leq \frac{16}{15} \eta_r \gamma \frac{q_H}{4} \left(1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma)} \right)^2 \right) \\ &= \frac{16}{15} \eta_r r^a(\gamma) \\ &< \frac{8}{15} r^a(\gamma). \end{aligned}$$

The first line follows by definition of $r^c(\gamma)$, the second line follows because $r^c(\gamma)$ strictly decreases in ϕ , the third line follows using the upper bound on ζ from (13a), the fourth line follows using the inequality (13b), the fifth line follows from the definition of $r^a(\gamma)$, and the final line follows because $\eta_r \leq \frac{1}{2}$ since $\lambda \geq \frac{1}{2}$. $\square$

B Proofs for Section 3: Optimal Commission and Platform Revenue

B.1 Proof of Proposition 1

We first present two useful lemmas used in the proof of Proposition 1 and elsewhere: Lemma B.1 presents comparative statics with respect to ϕ for different candidate solutions for the optimal commission rate γ^* , and Lemma B.2 provides a characterization of the optimal commission rate γ^* . For use in the remainder of the appendix, define $\gamma^x := \operatorname{argmax}_{\gamma \in [0,1]} r^x(\gamma)$ and $\gamma^{xy} := \operatorname{argmax}_{\gamma \in [0, \gamma^m]} \{\mu r^x(\gamma) + (1-\mu)r^y(\gamma)\}$, where $x, y \in \{a, b, c\}$ and the $r^x(\gamma)$ functions are as defined in Lemma A.11. For convenience, we describe these quantities informally below:

- $r^a(\gamma)$ is the platform's revenue from a type- H seller when the seller transacts only online, and γ^a is its

unconstrained maximizer,

- $r^b(\gamma)$ is the platform's revenue from a type- L seller when they transact online with the $\sigma = s$ buyer (and reject $\sigma = r$ buyers), and γ^b is its unconstrained maximizer,
- $r^c(\gamma)$ is the platform's revenue from a type- H seller when the seller transacts online and offline with $\sigma = r$ and $\sigma = s$ buyers, respectively, and γ^c is its unconstrained maximizer,
- γ^{ab} – which is the only quantity of the form γ^{xy} used in the following proof – is the optimal commission rate when both type- H and type- L sellers transact online.

Lemma B.1. *The following statements hold. (i) γ^a and γ^b are both independent of ϕ , (ii) γ^c strictly decreases in ϕ , and (iii) γ_s^H and γ_s^L strictly increase in ϕ .*

Recall that γ_s^H and γ_s^L denote the smallest commission rates at which the type- H and type- L sellers transact offline with $\sigma = s$ buyers, respectively (Lemma A.10). Intuitively, an increase in the switching cost leads to an increase in the commission rates at which sellers disintermediate, which is captured by statement (iii) in the lemma above.

Proof. (i). The result follows by noting that $r^a(\gamma)$ and $r^b(\gamma)$ are independent of ϕ (Lemma A.12), and thus so are their maximizers γ^a and γ^b . This is because for fixed γ , the switching cost does not affect revenue when all transactions occur online.

(ii). Applying the implicit function theorem, we have

$$\frac{d\gamma^c}{d\phi} = - \left(\frac{\partial^2 r^c}{\partial \gamma \partial \phi} \right) \left(\frac{\partial^2 r^c}{\partial \gamma^2} \right)^{-1} \Big|_{\gamma=\gamma^c}.$$

Note $\frac{\partial^2}{\partial \gamma^2} r^c < 0$ at $\gamma = \gamma^c$ because γ^c is the maximizer of r^c . Therefore, $\frac{d}{d\phi} \gamma^c$ has the same sign as $\frac{\partial^2}{\partial \gamma \partial \phi} r^c$. Using the expressions for p^c and $r^c(\gamma)$ from Lemmas A.3 and A.11, we have

$$r^c(\gamma) = \eta_r \gamma p^c \left(1 - \frac{p^c}{q_H} \right) = \eta_r \gamma q_H \left(\frac{1}{4} - \left(\frac{2(1-\lambda)c + \eta_s \phi}{4q_H \zeta} \right)^2 \right),$$

where $\zeta = \eta_r(1-\gamma) + \frac{1}{2}\eta_s(1-\gamma + \omega_s)$. Differentiating in γ and using the fact that $\frac{d}{d\gamma} \zeta = \frac{1}{2}\eta_s - 1$, we have

$$\begin{aligned} \frac{\partial r^c}{\partial \gamma} &= \eta_r q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H \zeta} + \frac{\eta_s \phi}{4q_H \zeta} \right)^2 \right) - \frac{2\eta_r q_H}{\zeta^3} \gamma \left(\frac{(1-\lambda)c}{2q_H} + \frac{\eta_s \phi}{4q_H} \right)^2 \left(1 - \frac{\eta_s}{2} \right) \\ &= \eta_r q_H \left(\frac{1}{4} - \frac{h^2}{q_H^2} \left(\frac{1}{\zeta^2} + \frac{2\gamma}{\zeta^3} \left(1 - \frac{\eta_s}{2} \right) \right) \right), \end{aligned}$$

where $h = \frac{1}{4}(2c(1-\lambda) + \eta_s \phi)$. Because h increases in ϕ , we conclude $\frac{\partial^2}{\partial \gamma \partial \phi} r^c < 0$ and thus $\frac{d}{d\phi} \gamma^c < 0$.

(iii). We first show $\frac{d}{d\phi} \gamma_s^H > 0$, followed by showing $\frac{d}{d\phi} \gamma_s^L > 0$. Note by the proof of Lemma A.10, γ_s^H is the unique solution to $\pi^a(p^a) - \pi^c(p^c) = 0$. Note $\pi^a(p^a)$ depends on γ and $\pi^c(p^c)$ depends on both γ and ϕ . For convenience, we define the function $\pi^-(\gamma, \phi) := \pi^a(p^a) - \pi^c(p^c)$. It follows that $\pi^-(\gamma_s^H, \phi) = 0$. Taking the total derivative of this equation with respect to ϕ , we have

$$\frac{d\pi^-}{ds} \Big|_{\gamma=\gamma_s^H} = \left(\frac{\partial \pi^-}{\partial \gamma} \frac{d\gamma_s^H}{d\phi} + \frac{\partial \pi^-}{\partial \phi} \right) \Big|_{\gamma=\gamma_s^H} = 0.$$

Because γ_s^H is the unique solution to $\pi^-(\gamma, \phi) = 0$ by Lemma A.10, we must have $\frac{\partial}{\partial \gamma} \pi^- \neq 0$ at $\gamma = \gamma_s^H$. We can therefore re-arrange for $\frac{d}{d\phi} \gamma_s^H$ to obtain

$$\frac{d\gamma_s^H}{d\phi} = - \left(\frac{\partial \pi^-}{\partial \phi} \right) \left(\frac{\partial \pi^-}{\partial \gamma} \right)^{-1} \Big|_{\gamma=\gamma_s^H}.$$

Next, we show $\frac{d}{d\phi} \gamma_s^H > 0$ by showing $\frac{\partial}{\partial \gamma} \pi^- < 0$ and $\frac{\partial}{\partial \phi} \pi^- > 0$ at $\gamma = \gamma_s^H$. First, the proof of Lemma A.10 shows that $\pi^-(\gamma, \phi)$ strictly decreases in γ on the interval $[\underline{\gamma}_s^H, \bar{\gamma}_s^H]$, and that $\gamma_s^H \in [\underline{\gamma}_s^H, \bar{\gamma}_s^H]$. It follows that $\frac{\partial}{\partial \gamma} \pi^-(\gamma, \phi) < 0$ at $\gamma = \gamma_s^H$. Next, for all $\gamma \in [0, \gamma^m]$ we have

$$\begin{aligned} \frac{\partial \pi^-}{\partial \phi} &= - \frac{\partial}{\partial \phi} \pi^c(p^c) \\ &= - \frac{\partial}{\partial \phi} \left\{ \frac{1}{2} \left(1 - \frac{p^c}{q_H} \right) (p^c(2(1-\gamma) - \eta_s(1-\gamma-\omega_s)) - 2c(1-\lambda) - \eta_s \phi) \right\} \\ &= \frac{\eta_s}{2} \left(1 - \frac{p^c}{q_H} \right) \\ &> 0. \end{aligned}$$

The first line follows by definition of $\pi^-(\gamma, \phi)$ and because $\frac{\partial}{\partial \phi} \pi^a(p^a) = 0$, the second by plugging in the expression for $\pi^c(p^c)$, and the third from noting p^c is the maximizer of $\pi^c(p)$ and thus applying the envelope theorem to compute the partial derivative. The strict inequality follows because p^c maximizes $\pi^c(p)$ and the seller earns zero profit for all prices above q_H , which implies $p^c < q_H$. Because $\frac{\partial}{\partial \gamma} \pi^- < 0$ and $\frac{\partial}{\partial \phi} \pi^- > 0$ at $\gamma = \gamma_s^H$, we conclude $\frac{d}{d\phi} \gamma_s^H > 0$, as desired. Next, we show $\frac{d}{d\phi} \gamma_s^L > 0$ using a similar argument to that above for $\frac{d}{d\phi} \gamma_s^H > 0$. Note by the proof of Lemma A.10, γ_s^L is the unique solution to $\pi^b(p^b) - \pi^d(p^d) = 0$. Let $\bar{\pi}(\gamma, \phi) = \pi^b(p^b) - \pi^d(p^d)$, and note $\bar{\pi}(\gamma_s^L, \phi) = 0$ by definition of γ_s^L . Taking the total derivative with respect to ϕ yields

$$\frac{d\bar{\pi}}{d\phi} \Big|_{\gamma=\gamma_s^L} = \left(\frac{\partial \bar{\pi}}{\partial \gamma} \frac{d\gamma_s^L}{d\phi} + \frac{\partial \bar{\pi}}{\partial \phi} \right) \Big|_{\gamma=\gamma_s^L} = 0.$$

Because γ_s^L is the unique solution to $\bar{\pi}(\gamma, \phi) = 0$ by Lemma A.10, we must have $\frac{\partial}{\partial \gamma} \bar{\pi} \neq 0$ at $\gamma = \gamma_s^L$. We can therefore re-arrange for $\frac{d}{d\phi} \gamma_s^L$ to obtain

$$\frac{d\gamma_s^L}{d\phi} = - \left(\frac{\partial \bar{\pi}}{\partial \phi} \right) \left(\frac{\partial \bar{\pi}}{\partial \gamma} \right)^{-1} \Big|_{\gamma=\gamma_s^L}.$$

Next, we show $\frac{d}{d\phi} \gamma_s^L > 0$ by showing $\frac{\partial}{\partial \gamma} \bar{\pi} < 0$ and $\frac{\partial}{\partial \phi} \bar{\pi} > 0$ at $\gamma = \gamma_s^L$. First, the proof of Lemma A.10 shows that $\bar{\pi}(\gamma, \phi)$ strictly decreases in γ on the interval $[\underline{\gamma}_s^L, \bar{\gamma}_s^L]$, and that $\gamma_s^L \in [\underline{\gamma}_s^L, \bar{\gamma}_s^L]$. It follows that $\frac{\partial}{\partial \gamma} \bar{\pi}(\gamma, \phi) < 0$ at $\gamma = \gamma_s^L$. Next, by parallel argument to the proof for $\frac{d}{d\phi} \gamma_s^H$ above, we have

$$\begin{aligned} \frac{\partial \bar{\pi}}{\partial \phi} &= - \frac{d}{d\phi} \pi^d(p^d) \\ &= - \frac{d}{d\phi} \left\{ \frac{1}{2} \eta_s \left(1 - \frac{p^d}{q_L} \right) (p^d(1-\gamma+\omega_s) - 2c(1-\omega_s) - \phi) \right\} \\ &= \frac{\eta_s}{2} \left(1 - \frac{p^d}{q_L} \right) \\ &> 0. \end{aligned}$$

Because $\frac{\partial}{\partial \gamma} \bar{\pi} < 0$ and $\frac{\partial}{\partial \phi} \bar{\pi} > 0$ at $\gamma = \gamma_s^L$, we conclude $\frac{d}{d\phi} \gamma_s^L > 0$. □

Lemma B.2 (Optimal commission rate). *The following statements hold.*

- (i) *There exists $\underline{\alpha} \in (\frac{1}{2}, 1]$ such that $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$ if $\alpha \leq \underline{\alpha}$ and $\phi \geq 0$.*
- (ii) *There exists $\bar{\alpha} \in [\underline{\alpha}, 1)$ such that $\gamma^* = \min\{\gamma^c, \gamma^m\}$ if $\alpha \in [\bar{\alpha}, 1]$ and $\phi = 0$.*

Unpacking the above lemma, part (i) states that when information quality α is low, the platform chooses a commission rate no greater than γ_s^H , and so all transactions occur on-platform. At high values of α under no switching cost (part (ii)), the only transactions that occur on-platform are between the type- H seller and $\sigma = r$ buyer.

Proof. (i). Note for any $\gamma > 0$ and $\alpha = \frac{1}{2}$, the type- L seller's profit is

$$\pi^b(p^b) = (1 - \gamma)q_L \left(\frac{1}{2} - \frac{(1 - \omega_s)c}{2q_L(1 - \gamma)} \right)^2 \leq (1 - \gamma)q_L \left(\frac{1}{2} - \frac{(1 - \omega_s)c}{2q_L} \right)^2 = (1 - \gamma)q_L \left(\frac{1}{2} - \frac{(1 - \lambda)c}{2q_L} \right)^2 \leq 0,$$

which follows because at $\gamma > 0$, $\omega_s = \lambda$ at $\alpha = \frac{1}{2}$, and $q_L \leq (1 - \lambda)c$ by Assumption 1. Since $\pi^b(p^b) \leq 0$ implies that the type- L seller rejects all buyers, we conclude the type- L seller's contribution to platform revenue at $\alpha = \frac{1}{2}$ is $r^b(\gamma)^+ = 0$. It follows by continuity of $\pi^b(p^b)$ in α that there exists $\tilde{\alpha}$ such that $r^b(\gamma)^+ = 0$ for all $\alpha \leq \tilde{\alpha}$. Thus, for $\alpha \leq \tilde{\alpha}$, the platform's revenue is given by (Lemma A.11)

$$R(\gamma) = \begin{cases} \mu r^a(\gamma) & \text{if } \gamma \in [0, \gamma_s^H], \\ \mu r^c(\gamma) & \text{if } \gamma \in (\gamma_s^H, \gamma^m]. \end{cases}$$

Next, using the expressions for ω_s and ω_r (Lemma A.1), we have $\omega_s = \omega_r = \lambda$ for $\alpha = \frac{1}{2}$, which implies $\omega_s = \omega_r$, and thus $\gamma_s^H = \gamma_r^H$. It follows from Lemma A.7, that $\gamma_s^H = \gamma_r^H > \gamma^m$ at $\alpha = \frac{1}{2}$. Further, because $r^a(\gamma)$ strictly increases in γ (Lemma A.12), we conclude $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$ at $\alpha = \frac{1}{2}$. Finally, the existence of the threshold $\underline{\alpha} \leq \tilde{\alpha}$ follows because γ_s^H is continuous in α and $r^a(\gamma) > r^c(\gamma)$ for $\gamma > 0$ by Lemma A.13.

(ii). By Lemma A.11, the platform's revenue function is

$$R(\gamma) = \begin{cases} \mu r^a(\gamma) + (1 - \mu)r^b(\gamma)^+ & \text{if } \gamma \in [0, \gamma_s^H], \\ \mu r^c(\gamma) + (1 - \mu)r^b(\gamma)^+ & \text{if } \gamma \in (\gamma_s^H, \gamma_s^L], \\ \mu r^c(\gamma) & \text{if } \gamma \in (\gamma_s^L, \gamma^m]. \end{cases}$$

Note $r^c(\gamma)$ is strictly concave in γ by Lemma A.12. Therefore, to show $\gamma^* = \min\{\gamma^c, \gamma^m\}$ it suffices to show that the following three inequalities hold:

$$\begin{aligned} \min\{\gamma^c, \gamma^m\} &> \gamma_s^L, \\ \mu r^c(\gamma^m) &> \max_{\gamma \in [0, \gamma_s^H]} \mu r^a(\gamma) + (1 - \mu)r^b(\gamma)^+, \end{aligned} \tag{14a}$$

$$\mu r^c(\gamma^m) > \max_{\gamma \in (\gamma_s^H, \gamma_s^L]} \mu r^c(\gamma) + (1 - \mu)r^b(\gamma)^+. \tag{14b}$$

We show $\gamma^* = \min\{\gamma^c, \gamma^m\}$ holds at $\alpha = 1$. Note $\phi = 0$ and $\alpha = 1$ implies $\omega_s = 1$ and thus $\gamma_s^H = \gamma_s^L = 0$ by Lemma A.10. Therefore, in this setting the right hand sides of (14a) and (14b) are both zero. It remains to show $r^c(\gamma^m) > 0$. Note

$$\zeta = \eta_r(1 - \gamma^m) + \frac{\eta_s}{2}(1 - \gamma^m + \omega_s) \geq \frac{1}{2}(\eta_r + \eta_s)(1 - \gamma^m) = \frac{1 - \gamma^m}{2} \geq \frac{1}{4}.$$

Because $\zeta \geq \frac{1}{4}$ and $q_H \geq 4c$ by Assumption 1, we have $q_H \zeta \geq c$. Using this inequality, we can write

$$r^c(\gamma^m) = \eta_r \gamma^m q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H \zeta} \right)^2 \right) \geq \eta_r \gamma^m q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)}{2} \right)^2 \right) > 0,$$

where the final inequality follows because $\lambda < 1$. Therefore, $\gamma^* = \min\{\gamma^c, \gamma^m\}$ if $\phi = 0$ and $\alpha = 1$. Finally, the existence of the threshold $\bar{\alpha} < 1$ follows because γ_s^H and γ_s^L and the functions $r^a(\gamma)$, $r^b(\gamma)$ and $r^c(\gamma)$ are all continuous in α . $\square$

Proposition 1. *Let $\gamma^*(\phi)$ be the platform's optimal commission rate under switching cost ϕ . There exist thresholds $\underline{\alpha} \in (\frac{1}{2}, 1]$ and $\bar{\alpha} \in [\underline{\alpha}, 1)$ such that the following statements hold.*

- (i) *Suppose information quality is low, i.e., $\alpha \leq \underline{\alpha}$. Then the optimal commission rate $\gamma^*(\phi)$ weakly increases in the switching cost ϕ for all $\phi \geq 0$.*
- (ii) *Suppose information quality is high, i.e., $\alpha > \bar{\alpha}$. Then there exists $\bar{\phi} > 0$ such that for each $\phi \geq \bar{\phi}$, the optimal commission rate is higher in the absence of switching costs, $\gamma^*(0) \geq \gamma^*(\phi)$, where the inequality is strict if $\gamma^*(\phi) < \gamma^m$. Further, there exists $\underline{\phi} \in (0, \bar{\phi}]$ such that $\gamma^*(\phi)$ strictly decreases in ϕ on $\phi \in [0, \underline{\phi}]$ wherever $\gamma^*(\phi) < \gamma^m$.*

Proof. This proof makes use of Lemmas B.1 and B.2. (i). By Lemma B.2 there exists $\underline{\alpha} \in [\frac{1}{2}, 1]$ such that if $\alpha \leq \underline{\alpha}$, then $\gamma^* = \min\{\gamma^a, \gamma_s^H\}$ for all $\phi \geq 0$. That is, the optimal commission rate γ^* ensures that all transactions occur on-platform, e.g., see role of γ_s^H in Figure 1. By Lemma B.1, γ^a is independent of ϕ and γ_s^H strictly increases in ϕ . It follows that $\gamma^* = \min\{\gamma^a, \gamma_s^H\}$ weakly increases in ϕ for all $\phi \geq 0$. In summary, when the information quality is low, all transactions occur on-platform. Further, any increase in the switching cost only strengthens the platform's pricing power, as sellers are less inclined to go off-platform, even at higher commission rates.

(ii). The proof proceeds in two steps. First we show $\gamma^*(0) \geq \gamma^*(\phi)$ holds for sufficiently large ϕ and α . Second, we address the comparative statics result.

Step 1. Note by Lemma A.11, the platform's revenue function is

$$R(\gamma) = \begin{cases} \mu r^a(\gamma) + (1-\mu)r^b(\gamma)^+ & \text{if } \gamma \in [0, \gamma_s^H], \\ \mu r^c(\gamma) + (1-\mu)r^b(\gamma)^+ & \text{if } \gamma \in (\gamma_s^H, \gamma_s^L], \\ \mu r^c(\gamma) & \text{if } \gamma \in (\gamma_s^L, \gamma^m]. \end{cases}$$

Because $\omega_s = 1$ at $\alpha = 1$ (Lemma A.1) and $\gamma_s^H = \gamma_s^L = 1 - \omega_s$ at $\phi = 0$ (Lemma A.10), we have $\gamma_s^H = \gamma_s^L = 0$ at $\phi = 0$ and $\alpha = 1$. When $\alpha = 1$, transactions with the $\sigma = s$ buyer will always occur offline, as the commission thresholds for disintermediation are zero. By continuity of γ_s^H and γ_s^L in α , it follows that there exists $\tilde{\alpha} \in [\frac{1}{2}, 1)$ such that $\gamma^* = \min\{\gamma^c, \gamma^m\}$ if $\phi = 0$ and $\alpha \geq \tilde{\alpha}$. Note that $\gamma^* = \min\{\gamma^c, \gamma^m\}$ implies that the only transactions that occur online are between the type- H seller and $\sigma = r$ buyer. Similarly, because γ_s^H strictly increases in ϕ (Lemma B.1), when the switching cost is sufficiently high, the platform can set its commission rate to guarantee that all transactions occur online. Formally, there exists $\bar{\phi} > 0$ such that $\gamma^* = \min\{\gamma^{ab}, \gamma^m\}$ if $\phi \geq \bar{\phi}$. Because γ^{ab} does not depend on ϕ , it remains to show there exists $\bar{\alpha} \in [\tilde{\alpha}, 1)$ such that $\gamma^{ab} < \gamma^c$ if $\alpha \geq \bar{\alpha}$ and $\phi = 0$. In other words, the platform's commission rate at $\phi = 0$ (γ^c) is larger than its commission rate when switching costs are large (γ^{ab}).

To show this, we first define an auxiliary function $\ell^x(\gamma)$ for each $x \in \{a, b, c\}$. To define $\ell^a(\gamma)$, we differentiate $r^a(\gamma)$ in γ to obtain

$$\frac{\partial r^a}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left\{ \gamma p^a \left(1 - \frac{p^a}{q_H} \right) \right\} = \frac{\partial}{\partial \gamma} \left\{ \frac{\gamma}{4} \left(q_H - \frac{(\rho c(1-\lambda))^2}{q_H(1-\gamma)^2} \right) \right\} = \frac{1}{4q_H} \underbrace{\left(q_H^2 - \frac{c^2(1+\gamma)(1-\lambda)^2}{(1-\gamma)^3} \right)}_{\ell^a(\gamma)}.$$

Similarly, for $\ell^b(\gamma)$ we have

$$\frac{\partial r^b}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left\{ \gamma \eta_s p^b \left(1 - \frac{p^b}{q_H} \right) \right\} = \frac{\partial}{\partial \gamma} \left\{ \frac{\gamma \eta_s}{4} \left(q_L - \frac{(\rho c(1-\omega_s))^2}{q_L(1-\gamma)^2} \right) \right\} = \frac{\eta_s}{4} \underbrace{\left(q_L - \left(1 + \frac{2\gamma}{1-\gamma} \right) \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2} \right)}_{\ell^b(\gamma)}.$$

For $\ell^c(\gamma)$,

$$\begin{aligned} \frac{\partial r^c}{\partial \gamma} &= \frac{\partial}{\partial \gamma} \left\{ \gamma \eta_r p^c \left(1 - \frac{p^c}{q_H} \right) \right\} \\ &= \frac{\partial}{\partial \gamma} \left\{ \gamma \eta_r q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H\zeta} \right)^2 \right) \right\} \\ &= \gamma q_H \underbrace{\left(\frac{\partial \eta_r}{\partial \alpha} \gamma q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H\zeta} \right)^2 \right) + \frac{2\eta_r}{\zeta} \left(\frac{(1-\lambda)c}{2q_H\zeta} \right)^2 \frac{\partial \zeta}{\partial \alpha} \right)}_{\ell^c(\gamma)}, \end{aligned}$$

Note because $r^x(\gamma)$ is strictly concave in γ for each $x \in \{a, b, c\}$ (Lemma A.12), $\ell^x(\gamma)$ strictly decreases in γ and $\ell^x(\gamma^x) = 0$ for each $x \in \{a, b, c\}$ by definition. Using these properties, it is straightforward to verify that $\gamma^b \leq \gamma^a$ for all $\alpha \in [\frac{1}{2}, 1]$. Further, because $r^a(\gamma)$ and $r^b(\gamma)$ are both strictly concave in γ , we must have $\gamma^b \leq \gamma^{ab} \leq \gamma^a$ for all $\alpha \in [\frac{1}{2}, 1]$. It is then sufficient to show there exists $\bar{\alpha} \in [\bar{\alpha}, 1)$ such that $\gamma^a < \gamma^c$ for $\alpha \geq \bar{\alpha}$. Using $\frac{\partial}{\partial \alpha} \eta_r = 1 - 2\lambda$ and $\frac{\partial}{\partial \alpha} \zeta = \frac{1}{2}((1-\lambda) - \gamma(2\lambda - 1))$, it can be shown algebraically that

$$\lim_{\alpha \rightarrow 1} \{\ell^c(\gamma) - \ell^a(\gamma)\} = 4c^2(1-\lambda)^2(2-\gamma(2-\lambda)) + q_H^2((2-\gamma(2-\lambda))^3 - 1) + \frac{c^2(1+\gamma)(1-\lambda)^2}{(1-\gamma)^3} > 0,$$

where the strict inequality follows because $\gamma \in (0, \frac{1}{2}]$ and $\lambda \in [\frac{1}{2}, 1]$. By continuity of $\ell^c(\gamma)$ and $\ell^a(\gamma)$ in α , it follows that there exists $\bar{\alpha} \in [\bar{\alpha}, 1)$ such that $\ell^a(\gamma) < \ell^c(\gamma)$ for all $\gamma \in [0, \gamma^m]$ if $\alpha \geq \bar{\alpha}$. Therefore, for all $\alpha \geq \bar{\alpha}$ we have $\ell^a(\gamma^a) = 0 < \ell^c(\gamma^a)$, which implies $\gamma^a < \gamma^c$. Because $\gamma^{ab} \leq \gamma^a$, we conclude $\gamma^{ab} < \gamma^c$ if $\alpha \geq \bar{\alpha}$ and $\phi = 0$. Because $\gamma^* = \min\{\gamma^{ab}, \gamma^m\}$ for all $\phi \geq \bar{\phi}$ and $\gamma^* = \min\{\gamma^c, \gamma^m\}$ for $\phi = 0$ as established at the beginning of the proof, we conclude $\gamma^*(0) \geq \gamma^*(\phi)$ for all $\phi \geq \bar{\phi}$ and $\alpha \geq \bar{\alpha}$. Finally, to see that the inequality is strict wherever $\gamma^*(\phi) < \gamma^m$, note $\gamma^*(\phi) < \gamma^m$ implies $\gamma^*(\phi) = \gamma^{ab} < \min\{\gamma^c, \gamma^m\} = \gamma^*(0)$.

Step 2. From part (ii), we have $\gamma^* = \min\{\gamma^c, \gamma^m\}$ for $\phi = 0$ and $\alpha \geq \bar{\alpha}$. Further, since the thresholds γ_s^H and γ_s^L and the revenue functions $r^a(\gamma)$, $r^b(\gamma)$, and $r^c(\gamma)$ are each continuous in ϕ , it follows that there exists $\underline{\phi} > 0$ such that $\gamma^* = \min\{\gamma^c, \gamma^m\}$ for all $\phi \leq \underline{\phi}$ and $\alpha \geq \bar{\alpha}$. The result follows because γ^c strictly decreases in ϕ (Lemma B.1). $\square$

B.2 Proof of Proposition 2

Proposition 2. *Let $R(\gamma^*)$ be the platform's revenue under the optimal commission rate γ^* . There exist thresholds $\underline{\alpha} \in (\frac{1}{2}, 1]$ and $\bar{\alpha} \in [\underline{\alpha}, 1)$ such that the following statements hold.*

- (i) *Suppose information quality is low, $\alpha \leq \underline{\alpha}$. Then the platform's optimal revenue $R(\gamma^*)$ weakly increases in the switching cost ϕ on $\phi \in [0, \infty)$.*
- (ii) *Suppose information quality is high, $\alpha \geq \bar{\alpha}$. Then there exists $\underline{\phi}$ such that the platform's optimal revenue $R(\gamma^*)$ strictly decreases in the switching cost ϕ on $\phi \in [0, \underline{\phi}]$.*

Proof. (i). By Lemma B.2, there exists $\underline{\alpha}$ such that if $\alpha \leq \underline{\alpha}$, then $\gamma^* = \min\{\gamma^a, \gamma_s^H, \gamma^m\}$ for all $\phi \geq 0$ and $r^b(\gamma^*)^+ = 0$. That is, the low information quality ensures that type- L sellers do not transact at all and it is optimal for the platform to choose a commission rate where type- H sellers fully transact online as the disintermediation threshold γ_s^H is sufficiently large (e.g., see Figure 1). Pick $\alpha \leq \underline{\alpha}$. It is straightforward to show that because γ^a , γ_s^H , $r^a(\gamma)$ and $r^c(\gamma)$ are all continuous in ϕ , so is $R(\gamma^*)$. It remains to show $R(\gamma^*)$ weakly increases in ϕ in three separate cases: $\gamma^* = \gamma^a$, $\gamma^* = \gamma_s^H$ and $\gamma^* = \gamma^m$.

Case I: $\gamma^* = \gamma^a$. Following the proof of Lemma B.2, we have $r^b(\gamma)^+ = 0$ for all $\gamma \geq 0$ if $\alpha \leq \underline{\alpha}$, which implies $R(\gamma) = \mu r^a(\gamma)$. Then

$$\left. \frac{dR}{d\phi} \right|_{\gamma=\gamma^a} = \mu \left(\frac{\partial r^a}{\partial \gamma} \frac{d\gamma^*}{d\phi} + \frac{\partial r^a}{\partial \phi} \right) \Big|_{\gamma=\gamma^a} = \mu \frac{\partial r^a}{\partial \phi} \Big|_{\gamma=\gamma^a} = 0. \quad (15)$$

The first equality follows from taking the total derivative with respect to ϕ , the second equality follows from the envelope theorem because γ^a is the unconstrained maximizer of r^a , and the third equality follows because r^a is independent of ϕ (Lemma A.12). Hence $R(\gamma^*)$ is independent of ϕ when $\gamma^* = \gamma^a$.

Case II: $\gamma^* = \gamma^m$. In this case, we again obtain (15), except the second equality holds because $\frac{d}{d\phi} \gamma^* = 0$ for $\gamma^* = \gamma^m$ instead of by the envelope theorem. Hence $R(\gamma^*)$ is independent of ϕ when $\gamma^* = \gamma^m$.

Case III: $\gamma^* = \gamma_s^H$. Because $\frac{\partial}{\partial \phi} r^a = 0$ (Lemma A.12), we have

$$\left. \frac{dR}{d\phi} \right|_{\gamma=\gamma_s^H} = \mu \left(\frac{\partial r^a}{\partial \gamma} \frac{d\gamma^*}{d\phi} + \frac{\partial r^a}{\partial \phi} \right) \Big|_{\gamma=\gamma_s^H} = \mu \frac{\partial r^a}{\partial \gamma} \frac{d\gamma^*}{d\phi} \Big|_{\gamma=\gamma_s^H} > 0.$$

To see why the strict inequality holds, note γ_s^H strictly increases in ϕ (Lemma B.1) and $\gamma^* = \gamma_s^H$ implies we must have $\frac{\partial}{\partial \gamma} r^a > 0$ at $\gamma = \gamma_s^H$. We conclude that $R(\gamma^*)$ strictly increases in ϕ if $\gamma^* = \gamma_s^H$. Statement (i) thus follows.

(ii). By Lemma B.2 and the continuity of the thresholds γ_s^H and γ_s^L in ϕ , there exists $\underline{\phi} > 0$ such that $\gamma^* = \min\{\gamma^c, \gamma^m\}$ for $\alpha \geq \bar{\alpha}$ and $\phi \leq \underline{\phi}$. Pick $\alpha \geq \bar{\alpha}$. We show $R(\gamma^*)$ strictly decreases in ϕ at each $\phi \in [0, \underline{\phi}]$ by considering $\gamma^* = \gamma^c$ and $\gamma = \gamma^m$ as separate cases.

Case I: $\gamma^* = \gamma^c$. In this case, we have

$$\left. \frac{dR}{d\phi} \right|_{\gamma=\gamma^c} = \mu \left(\frac{\partial r^c}{\partial \gamma} \frac{d\gamma^*}{d\phi} + \frac{\partial r^c}{\partial \phi} \right) \Big|_{\gamma=\gamma^c} = \mu \frac{\partial r^c}{\partial \phi} \Big|_{\gamma=\gamma^c},$$

where the first equality follows because $R(\gamma^*) = \mu r^c(\gamma^*)$ for $\gamma^* = \gamma^c$, and the second equality follows by the envelope theorem because γ^c is the maximizer of $r^c(\gamma)$. Next, using the expressions for $r^c(\gamma)$ and p^c from

Lemma A.3, we have

$$\begin{aligned}
\frac{\partial r^c}{\partial \phi} &= \frac{\partial}{\partial \phi} \left\{ \eta_r \gamma p^c \left(1 - \frac{p^c}{q_H} \right) \right\} \\
&= \frac{\partial}{\partial \phi} \left\{ \eta_r \gamma q_H \left(\frac{1}{4} - \left(\frac{2(1-\lambda)c + \eta_s \phi}{4q_H \zeta} \right)^2 \right) \right\} \\
&= -\frac{\eta_s \eta_r \gamma (2(1-\lambda)c + \eta_s \phi)}{8q_H \zeta^2} \\
&< 0.
\end{aligned} \tag{16}$$

It follows that $R(\gamma^*)$ strictly decreases in ϕ if $\gamma^* = \gamma^c$.

Case II: $\gamma^* = \gamma^m$. Similar to Case I, we have

$$\left. \frac{dR}{d\phi} \right|_{\gamma=\gamma^*} = \mu \left(\frac{\partial r^c}{\partial \gamma} \frac{d\gamma^m}{d\phi} + \frac{\partial r^c}{\partial \phi} \right) \Big|_{\gamma=\gamma^m} = \mu \frac{\partial r^c}{\partial \phi} \Big|_{\gamma=\gamma^m} < 0,$$

where the second equality follows because $\frac{d\gamma^m}{d\phi} = 0$ and the strict inequality follows from (16). Therefore, $R(\gamma^*)$ strictly decreases in ϕ if $\gamma^* = \gamma^m$. $\square$

C Proofs for Section 4: Optimal Information Quality

C.1 Proof of Lemma 1

Lemma 1. *There exist thresholds $\underline{\phi} > 0$ and $\bar{\phi} \geq \underline{\phi}$ such that the following statements hold.*

- (i) *Suppose the switching cost is high, $\phi > \bar{\phi}$. Then the platform's optimal revenue $R(\gamma^*)$ weakly increases in information quality α on $\alpha \in [\frac{1}{2}, 1]$.*
- (ii) *Suppose the switching cost is low, $\phi \leq \underline{\phi}$. Then there exists $\underline{\alpha} \in (\frac{1}{2}, 1]$, $\bar{\alpha} \in [\underline{\alpha}, 1)$ and $\bar{\lambda} \in [\frac{1}{2}, 1)$ such that the platform's optimal revenue $R(\gamma^*)$ weakly increases in α on $\alpha \in [\frac{1}{2}, \underline{\alpha}]$ for all $\lambda \in [\frac{1}{2}, 1]$ and strictly decreases in α on $\alpha \in [\bar{\alpha}, 1]$ if $\lambda \geq \bar{\lambda}$.*

Proof. (i). By Lemma B.1, γ_s^H and γ_s^L are both strictly increasing in ϕ for each $\alpha \in [\frac{1}{2}, 1]$. It follows that there exists $\bar{\phi}$ such that $\gamma_s^H \geq \gamma^m$ and $\gamma_s^L \geq \gamma^m$ for all $\alpha \in [\frac{1}{2}, 1]$. Therefore, by Lemma A.11, for each $\phi \geq \bar{\phi}$ the platform's revenue is given by $R(\gamma) = \mu r^a(\gamma) + (1-\mu)r^b(\gamma)^+$ for all $\alpha \in [\frac{1}{2}, 1]$ and $\gamma \in [0, \gamma^m]$. Let γ^* be the maximizer of $R(\gamma)$ on $\gamma \in [0, \gamma^m]$. Because $r^b(\gamma)$ increases in α for each $\gamma \in [0, \gamma^m]$ such that $r^b(\gamma) > 0$ (Lemma A.12), there exists $\tilde{\alpha} \in [\frac{1}{2}, 1]$ such that $r^b(\gamma^*) > 0$ if and only if $\alpha > \tilde{\alpha}$. Note $r^a(\gamma)$ is independent of α . Therefore, for each $\alpha \leq \tilde{\alpha}$, $R(\gamma^*)$ is also independent of α . Next, for each $\alpha > \tilde{\alpha}$, we consider two further cases: $\gamma^* < \gamma^m$ and $\gamma^* = \gamma^m$. If $\gamma^* < \gamma^m$, then

$$\left. \frac{dR}{d\alpha} \right|_{\gamma=\gamma^*} = \left(\frac{\partial R}{\partial \gamma} \frac{d\gamma}{d\alpha} + \frac{\partial R}{\partial \alpha} \right) \Big|_{\gamma=\gamma^*} = \frac{\partial R}{\partial \alpha} \Big|_{\gamma=\gamma^*} = (1-\mu) \frac{\partial r^b}{\partial \alpha} \Big|_{\gamma=\gamma^*} > 0$$

where the second equality follows from the envelope theorem, the third equality follows because $\frac{\partial}{\partial \alpha} r^a = 0$, and the strictly inequality follows because $\frac{\partial}{\partial \alpha} r^b > 0$, as established above. If $\gamma^* = \gamma^m$, then $\frac{\partial}{\partial \alpha} \gamma = 0$ at $\gamma = \gamma^m$, and we again obtain $\frac{d}{d\alpha} R > 0$ at $\gamma = \gamma^*$.

(ii). The proof proceeds in three steps. First, we show that for $\gamma \in [0, \gamma^m]$ and $\phi = 0$, r^c strictly decreases in α for all $\lambda \geq \bar{\lambda} = 0.52$. Second, we address the upper threshold $\bar{\alpha}$. Third, we address the lower threshold $\underline{\alpha}$.

Step 1. Using $\gamma \leq \frac{1}{2}$, we have the following lower bound on ζ :

$$\zeta = (1 - \eta_s)(1 - \gamma) + \frac{\eta_s}{2}(1 - \gamma + \omega_s) \geq \frac{1 - \eta_s}{2} + \frac{\eta_s}{2} \left(\frac{1}{2} + \omega_s \right) = \frac{1}{4}(1 + \alpha + \lambda) \geq \frac{3 + 2\lambda}{8}. \quad (17)$$

Suppose $\phi = 0$. Then

$$\begin{aligned} \frac{\partial r^c}{\partial \alpha} &= \frac{\partial}{\partial \alpha} \left\{ \eta_r \gamma q_H \left(\frac{1}{4} - \left(\frac{(1 - \lambda)c}{2q_H \zeta} \right)^2 \right) \right\} \\ &= \frac{\partial \eta_r}{\partial \alpha} \gamma q_H \left(\frac{1}{4} - \left(\frac{(1 - \lambda)c}{2q_H \zeta} \right)^2 \right) + \frac{2\eta_r \gamma q_H}{\zeta} \left(\frac{(1 - \lambda)c}{2q_H \zeta} \right)^2 \frac{\partial \zeta}{\partial \alpha} \\ &\leq 4c\gamma \left((1 - 2\lambda) \left(\frac{1}{4} - \left(\frac{1 - \lambda}{8\zeta} \right)^2 \right) + \frac{2\eta_r}{\zeta} \left(\frac{1 - \lambda}{8\zeta} \right)^2 \frac{1 - \lambda}{2} \right) \\ &\leq 4c\gamma \left((1 - 2\lambda) \left(\frac{1}{4} - \left(\frac{1 - \lambda}{3 + 2\lambda} \right)^2 \right) + \frac{4}{3 + 2\lambda} \left(\frac{1 - \lambda}{3 + 2\lambda} \right)^2 (1 - \lambda) \right). \end{aligned} \quad (18)$$

The third line follows because $\frac{\partial}{\partial \alpha} \eta_r = 1 - 2\lambda$, $\frac{\partial}{\partial \alpha} \zeta = \frac{1}{2}((1 - \lambda) - \gamma(2\lambda - 1)) \leq \frac{1}{2}(1 - \lambda)$ and $q_H \geq 4c$, and the fourth line follows from the lower bound on ζ from (17). Next, let $\bar{\lambda} = 0.52$. Plugging in $\lambda = 0.52$, we obtain the bound

$$4c\gamma \left((1 - 2\lambda) \left(\frac{1}{4} - \left(\frac{1 - \lambda}{3 + 2\lambda} \right)^2 \right) + \frac{4}{3 + 2\lambda} \left(\frac{1 - \lambda}{3 + 2\lambda} \right)^2 (1 - \lambda) \right) \leq -\frac{c\gamma}{100} < 0.$$

Next, note that the upper bound on $\frac{\partial}{\partial \alpha} r^c$ in (18) strictly decreases in λ on $\lambda \in [\frac{1}{2}, 1]$, and is strictly negative for $\lambda = 0.52$. It follows that for any $\gamma \in [0, \gamma^m]$ and $\phi = 0$, $r^c(\gamma)$ strictly decreases in α for all $\lambda \geq \bar{\lambda} = 0.52$.

Step 2. It follows from Lemma B.2 and the continuity of γ_s^H and γ_s^L that there exists $\underline{\phi} > 0$ and $\bar{\alpha} < 1$ such that for each $\phi \leq \underline{\phi}$, $\gamma^* = \min\{\gamma^c, \gamma^m\}$ for all $\alpha \in [\bar{\alpha}, 1]$. Further, following the proof of Lemma B.2, we also have $\gamma^c > \gamma_s^L$ for $\phi \leq \underline{\phi}$ and $\alpha \in [\bar{\alpha}, 1]$. Now let $\lambda \geq \bar{\lambda}$ and consider two cases: $\gamma^* < \gamma^m$ and $\gamma^* = \gamma^m$. If $\gamma^* < \gamma^m$, then $\gamma^* = \gamma^c$, and we have

$$\left. \frac{dR}{d\alpha} \right|_{\gamma=\gamma^c} = \mu \left. \frac{dr^c}{d\alpha} \right|_{\gamma=\gamma^c} = \mu \left. \frac{\partial r^c}{\partial \alpha} \right|_{\gamma=\gamma^c} < 0. \quad (19)$$

The first equality follows because $\gamma^c > \gamma_s^L$ implies $R(\gamma^c) = \mu r^c(\gamma^c)$ (Lemma A.11), the second equality follows from the envelope theorem because γ^c is a maximizer of $r^c(\gamma)$, and the strict inequality follows from Step 1. It follows that if $\phi \leq \underline{\phi}$ and $\lambda \geq \bar{\lambda}$, then $R(\gamma^*)$ strictly decreases in α on $\alpha \in [\bar{\alpha}, 1]$. In the case where $\gamma^* = \gamma^m$, we have $\frac{d}{d\alpha} \gamma^* = 0$, from which (19) again follows.

Step 3. For the lower threshold $\underline{\alpha}$, first suppose $\phi = 0$ and $\alpha = \frac{1}{2}$. Note that $\alpha = \frac{1}{2}$ implies $\omega_s = \omega_r$, and thus $\gamma_s^H = \gamma_r^H$. By Lemma A.10 we have $\gamma_r^H = 1 - \omega_r > \gamma^m$, which implies $\gamma_s^H > \gamma^m$. Further, because γ_s^H strictly increases in ϕ by Lemma B.1 and is continuous in α , there exists $\underline{\alpha} \in [\frac{1}{2}, \bar{\alpha}]$ such that $\gamma_s^H > \gamma^m$ holds for all $\phi \leq \underline{\phi}$ and $\alpha \leq \bar{\alpha}$. It follows from Lemma A.11 that the platform's revenue is given by $R(\gamma) = \mu r^a(\gamma) + (1 - \mu)r^b(\gamma)^+$. Let γ^* be the maximizer of $R(\gamma)$. Note $r^a(\gamma)$ is independent of α and $r^b(\gamma)$ strictly increases in α (Lemma A.12). The result that $R(\gamma^*)$ weakly increases in α then follows by an

identical argument to the proof of statement (i). $\square$

C.2 Proof of Proposition 3

We first present two supporting results that are used to prove Proposition 3: Lemma C.1 presents comparative statics with respect to α for different candidate solutions for the optimal commission rate γ^* , and Lemma C.2 proves a useful property that holds at the optimal information quality and commission rate (α^*, γ^*) .

Lemma C.1. *The following statements hold. (i) γ^a is independent of α for all ϕ , (ii) γ^b strictly increases in α for all ϕ , (iii) γ^c strictly increases in α if $\phi < \bar{\phi}$ for some $\bar{\phi} > 0$, (iv) γ_s^H strictly decreases in α , and (v) γ_s^L strictly decreases in α if $\phi < \bar{\phi}$ for some $\bar{\phi} > 0$.*

Proof. (i). Note $r^a(\gamma)$ is independent of α (Lemma A.12), which implies the maximizer γ^a is also independent of α .

(ii). Note $r^b(\gamma)$ does not depend on ϕ , which implies $\frac{d}{d\alpha}\gamma^b$ has the same sign for all ϕ . By the implicit function theorem, we have

$$\frac{d\gamma^b}{d\alpha} = - \left(\frac{\partial^2 r^b}{\partial \gamma \partial \alpha} \right) \left(\frac{\partial^2 r^b}{\partial \gamma^2} \right)^{-1} \Big|_{\gamma=\gamma^b}.$$

By Lemma A.12, $r^b(\gamma)$ is strictly concave in γ , which implies $\frac{\partial^2}{\partial \gamma^2} r^b < 0$ at $\gamma = \gamma^b$. It follows that $\frac{d}{d\alpha}\gamma^b$ has the same sign as $\frac{\partial^2}{\partial \gamma \partial \alpha} r^b$ at $\gamma = \gamma^b$. Next, using the expressions in Lemmas A.3 and A.11 and plugging p^b into r^b , we have

$$\begin{aligned} \frac{\partial r^b}{\partial \gamma} &= \frac{\partial}{\partial \gamma} \left\{ \frac{\eta_s \gamma}{4} \left(q_L - \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2} \right) \right\} \\ &= \frac{\eta_s}{4} \left(q_L - \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2} \right) - \frac{\eta_s \gamma}{4} \left(\frac{2c^2(1-\omega_s)^2}{q_L(1-\gamma)^3} \right) \\ &= \frac{\eta_s}{4} \left(q_L - \left(1 + \frac{2\gamma}{1-\gamma} \right) \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2} \right). \end{aligned} \quad (20)$$

Note η_s and ω_s both strictly increase in α by Lemma A.1. Thus, by inspecting (20) it can be verified that $\frac{\partial}{\partial \gamma} r^b$ also strictly increases in α . It follows that $\frac{\partial^2}{\partial \gamma \partial \alpha} r^b > 0$, and thus γ^b strictly increases in α .

(iii). By the implicit function theorem, we have

$$\frac{d\gamma^c}{d\alpha} = - \left(\frac{\partial^2 r^c}{\partial \gamma \partial \alpha} \right) \left(\frac{\partial^2 r^c}{\partial \gamma^2} \right)^{-1} \Big|_{\gamma=\gamma^c}. \quad (21)$$

By Lemma A.12, r^c is strictly concave in γ , which implies $\frac{\partial^2}{\partial \gamma^2} r^c < 0$ at $\gamma = \gamma^c$. It follows that $\frac{d}{d\alpha}\gamma^c$ has the same sign as $\frac{\partial^2}{\partial \gamma \partial \alpha} r^c$ at $\gamma = \gamma^c$. Next, note

$$r^c = \eta_r \gamma q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H \zeta} + \frac{\eta_s \phi}{4q_H \zeta} \right)^2 \right), \quad (22)$$

where $\zeta = \eta_r(1-\gamma) + \eta_s \frac{1-\gamma+\omega_s}{2}$. Differentiating in γ , we have

$$\frac{\partial r^c}{\partial \gamma} = \eta_r q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H \zeta} + \frac{\eta_s \phi}{4q_H \zeta} \right)^2 \right) - \frac{2\eta_r q_H \gamma}{\zeta^3} \left(\frac{(1-\lambda)c}{2q_H} + \frac{\eta_s \phi}{4q_H} \right)^2 \left(1 - \frac{\eta_s}{2} \right)$$

$$= \underbrace{\eta_r q_H \left(\frac{1}{4} - \frac{h^2}{q_H^2} \left(\frac{1}{\zeta^2} + 2 \frac{\gamma}{\zeta^3} \left(1 - \frac{\eta_s}{2} \right) \right) \right)}_{g(\alpha, \gamma)}$$

where $h = \frac{1}{4}(2c(1 - \lambda) + \eta_s \phi)$ and $g(\alpha, \gamma)$ is defined as above for convenience. Differentiating again in α and evaluating at $\gamma = \gamma^c$ yields

$$\left. \frac{\partial^2 r^c}{\partial \gamma \partial \alpha} \right|_{\gamma=\gamma^c} = \left(\eta_r \frac{\partial g}{\partial \alpha} + \frac{\partial \eta_r}{\partial \alpha} g \right) \Big|_{\gamma=\gamma^c} = \left(\eta_r \frac{\partial g}{\partial \alpha} \right) \Big|_{\gamma=\gamma^c}.$$

The second equality above follows because γ^c is the maximizer of $r^c(\gamma)$, which implies $\frac{\partial}{\partial \gamma} r^c = 0$ at $\gamma = \gamma^c$. By (22), this implies $g(\alpha, \gamma^c) = 0$. It remains to show $\frac{\partial}{\partial \alpha} g(\alpha, \gamma^c) > 0$. Note by inspection that $g(\alpha, \gamma)$ is increasing in ζ and η_s for each γ ; therefore, it suffices to show $\frac{\partial}{\partial \alpha} \zeta > 0$ and $\frac{\partial}{\partial \alpha} \eta_s > 0$. Using the expressions for η_s and ω_s (Lemma A.1), it can be shown algebraically that

$$\frac{\partial \zeta}{\partial \alpha} = \frac{\partial}{\partial \alpha} \left\{ (1 - \gamma)(1 - \eta_s) + \frac{\eta_s(1 - \gamma + \omega_s)}{2} \right\} = \frac{1}{2}(\gamma(2\lambda - 1) + (1 - \lambda)) \geq 0,$$

where the final inequality follows because $\lambda \geq \frac{1}{2}$. By Lemma A.1, we also have $\frac{\partial}{\partial \alpha} \eta_s = 2\lambda - 1 > 0$ when $\lambda \geq \frac{1}{2}$. Because $\frac{\partial}{\partial \alpha} \zeta > 0$ and $\frac{\partial}{\partial \alpha} \eta_s > 0$ for $\phi = 0$ and $\lambda \geq \frac{1}{2}$, we conclude $g(\alpha, \gamma^c)$ strictly increases in α . This establishes that $\frac{d}{d\alpha} \gamma^c < 0$ for $\phi = 0$. The existence of the threshold $\bar{\phi} > 0$ follows because $r^c(\gamma)$ is continuous in ϕ , which by (21) implies $\frac{d}{d\alpha} \gamma^c$ is continuous in ϕ .

(*iv*). By Lemma A.10, γ_s^H is the unique solution to $\pi^a(p^a) - \pi^c(p^c) = 0$. For convenience, define the function $\pi^-(\alpha, \gamma) := \pi^a(p^a) - \pi^c(p^c)$. By definition, for each $\alpha \in [\frac{1}{2}, 1]$ we have $\pi^-(\alpha, \gamma_s^H) = 0$. It is straightforward to verify that $\pi^a(p^a)$ and $\pi^c(p^c)$ are both differentiable in α and γ . Therefore, we can differentiate $\pi^-(\alpha, \gamma)$ with respect to α to obtain

$$\left. \frac{d\pi^-}{d\alpha} \right|_{\gamma=\gamma_s^H} = \left(\frac{\partial \pi^-}{\partial \gamma} \frac{d\gamma_s^H}{d\alpha} + \frac{\partial \pi^-}{\partial \alpha} \right) \Big|_{\gamma=\gamma_s^H} = 0.$$

Because $\pi^-(\alpha, \gamma) = 0$ strictly decreases in γ on the interval $[\underline{\gamma}_s^H, \bar{\gamma}_s^H]$ by the proof of Lemma A.10, we have $\frac{\partial}{\partial \gamma} \pi^- < 0$ at $\gamma = \gamma_s^H$. We can therefore re-arrange for $\frac{d}{d\alpha} \gamma_s^H$ to obtain

$$\frac{d}{d\alpha} \gamma_s^H = - \left(\frac{\partial \pi^-}{\partial \alpha} \right) \left(\frac{\partial \pi^-}{\partial \gamma} \right)^{-1} \Big|_{\gamma=\gamma_s^H}.$$

Because $\frac{\partial}{\partial \gamma} \pi^- < 0$, $\frac{d}{d\alpha} \gamma_s^H < 0$ holds if $\frac{\partial}{\partial \alpha} \pi^- < 0$; we show the latter inequality holds. Note

$$\frac{\partial \pi^-}{\partial \alpha} = \frac{d\pi^a(p^a)}{d\alpha} - \frac{d\pi^c(p^c)}{d\alpha} = - \frac{d\pi^c(p^c)}{d\alpha},$$

where the second equality follows because π^a and p^a are both independent of α , and the right hand side is the total derivative of $\pi^c(p^c)$ with respect to α . Further, we have

$$\left. \frac{d\pi^c}{d\alpha} \right|_{(\gamma, p)=(\gamma_s^H, p^c)} = \left(\frac{\partial \pi^c}{\partial p} \frac{\partial p^c}{\partial \alpha} + \frac{\partial \pi^c}{\partial \alpha} \right) \Big|_{(\gamma, p)=(\gamma_s^H, p^c)} = \frac{\partial \pi^c}{\partial \alpha} \Big|_{(\gamma, p)=(\gamma_s^H, p^c)},$$

because $\frac{\partial}{\partial p} \pi^c = 0$ at $p = p^c$ by the envelope theorem. Therefore, it remains to show $\frac{\partial}{\partial \alpha} \pi^c > 0$ at $(\gamma, p) =$

(γ_s^H, p^c) . Writing out this partial derivative using the expression for $\pi^c(p)$, we have

$$\begin{aligned}\frac{\partial \pi^c}{\partial \alpha} &= \frac{\partial}{\partial \alpha} \left\{ \eta_s (\omega_s b_s(p) - \phi - (1 - \gamma)p) \left(1 - \frac{p}{q_H} \right) \right\} \\ &= \frac{\partial \eta_s}{\partial \alpha} (\omega_s b_s(p) - \phi - (1 - \gamma)p) \left(1 - \frac{p}{q_H} \right) + \eta_s \frac{\partial \omega_s b_s(p)}{\partial \alpha} \left(1 - \frac{p}{q_H} \right).\end{aligned}\quad (23)$$

Next, we show the expression in (23) is strictly positive at $(\gamma, p) = (\gamma_s^H, p^c)$. Note we must have $\left(1 - \frac{p^c}{q_H} \right) > 0$ because the type- H seller has positive demand at $p = p^c$. Note also that γ_s^H is the commission rate at which the seller is indifferent between transacting online and offline; using this fact it is straightforward to show that $(\omega_s b_s(p) - \phi - (1 - \gamma)p) \geq 0$ and $b_s(p) \geq 0$ at $(\gamma, p) = (\gamma_s^H, p^c)$. Because $\frac{\partial}{\partial \alpha} \eta_s = 2\lambda - 1 > 0$, we conclude the first term in (23) is positive. For the second term, note

$$\frac{\partial \omega_s b_s(p)}{\partial \alpha} = \frac{1}{2} \frac{\partial \omega_s}{\partial \alpha} = \frac{(1 - \lambda)\lambda}{2(1 - \lambda + (\alpha(2\lambda - 1))^2)} > 0,$$

where the first equality follows using the expression for $b_s(p)$ (Lemma A.2) and the strict inequality follows because $\lambda \in [\frac{1}{2}, 1]$. Therefore, $\frac{\partial}{\partial \alpha} \pi^c(p) > 0$ at $(\gamma, p) = (\gamma_s^H, p^c)$, as desired. We conclude γ_s^H strictly decreases in α .

(v). Let $\phi = 0$. It follows from Lemma A.10 that

$$\gamma_s^L = 1 - \omega_s = \frac{(1 - \alpha)(1 - \lambda)}{\alpha(1 - \lambda) + (1 - \alpha)\lambda}.$$

Differentiating in α ,

$$\frac{d\gamma_s^L}{d\alpha} = -\frac{(1 - \lambda)\lambda}{(\alpha(1 - \lambda) + (1 - \alpha)\lambda)^2} < 0.$$

Therefore, γ_s^L strictly decreases in α on $\alpha \in [\frac{1}{2}, 1]$ when $\phi = 0$. Finally, to see that there exists a threshold $\bar{\phi} > 0$ such that $\frac{d}{d\alpha} \gamma_s^L < 0$ for $\phi \leq \bar{\phi}$, note that $\frac{d}{d\alpha} \gamma_s^L$ can be shown to be continuous in ϕ using the fact that γ_s^L is the solution to $\pi^b(p^b) - \pi^d(p^d) = 0$, where $\pi^b(p^b)$ does not depend on ϕ and $\pi^d(p^d)$ is continuous in ϕ . $\square$

Lemma C.2 (Optimal information quality and commission rate). *Suppose $\gamma^* < \gamma^m$ holds at the optimal information quality and commission rate (α^*, γ^*) . Then $\gamma^* \in [\gamma_s^H, \gamma_s^L]$.*

Proof. We show neither $\gamma^* < \gamma_s^H$ nor $\gamma^* > \gamma_s^L$ can hold if $\gamma^* < \gamma^m$. By way of contradiction, suppose $\gamma^* < \gamma_s^H$ holds at (α^*, γ^*) . Because $\gamma_s^H = 0$ at $\alpha = 1$, $\gamma^* < \gamma_s^H$ implies $\alpha^* < 1$. Further, because $r^a(\gamma)$ strictly increases in γ on $\gamma \in [0, \gamma^m]$ (Lemma A.12), we must have $r^b(\gamma^*) > 0$; otherwise, we obtain a contradiction to $\gamma^* < \gamma_s^H$. Because γ_s^H is continuous and strictly decreasing in α (Lemma C.1), there exists $\tilde{\alpha} > \alpha^*$ such that $\gamma_s^H = \gamma^*$ at $\alpha = \tilde{\alpha}$. To make dependence on α explicit, we slightly abuse notation and write $R(\alpha, \gamma)$ to denote the platform's revenue. Then we can write

$$R(\alpha^*, \gamma^*) = \mu r^a(\alpha^*, \gamma^*) + (1 - \mu) r^b(\alpha^*, \gamma^*)^+ < \mu r^a(\tilde{\alpha}, \gamma^*) + (1 - \mu) r^b(\tilde{\alpha}, \gamma^*)^+ = R(\tilde{\alpha}, \gamma^*), \quad (24)$$

where the strict inequality follows because $r^a(\gamma)$ and $r^b(\gamma)$ are independent of and increasing in α , respectively (Lemma A.12), and the two equalities follow by noting $\gamma^* \leq \gamma_s^H$ at both α^* and $\tilde{\alpha}$, and applying the definition of $R(\gamma)$ from Lemma A.11. Note (24) contradicts the optimality of α^* , which implies $\gamma^* < \gamma_s^H$ cannot hold.

Next, suppose $\gamma^* > \gamma_s^L$. Then

$$R(\alpha^*, \gamma^*) = \mu r^c(\alpha^*, \gamma^*) < \mu r^a(\alpha^*, \gamma^*) = \mu r^a\left(\frac{1}{2}, \gamma^*\right) = R\left(\frac{1}{2}, \gamma^*\right). \quad (25)$$

The first equality follows because $\gamma^* > \gamma_s^L$, the strictly inequality follows by Lemma A.13, the second equality follows because $r^a(\gamma)$ is independent of α , and the final equality follows because $\gamma_s^H \geq \gamma^m$ for $\alpha = \frac{1}{2}$. Note (25) contradicts the optimality of α^* , and so $\gamma^* > \gamma_s^L$ cannot hold. The result follows. $\square$

Proposition 3. *Let α^* be the platform's revenue-maximizing information quality when jointly optimized with the commission rate. There exist thresholds $\bar{\mu} \in [0, 1]$, $\bar{\phi} > 0$, $\underline{\alpha} \in (\frac{1}{2}, 1)$ and $\bar{\alpha} \in (\underline{\alpha}, 1)$ such that the following statements hold.*

- (i) *A no-information policy is optimal $\alpha^* = \frac{1}{2}$ if the share of type-H sellers is large $\mu > \bar{\mu}$ and there is no switching cost $\phi = 0$.*
- (ii) *A partial-information policy is optimal $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$ if the share of type-H sellers is small $\mu \leq \bar{\mu}$ and there is no switching cost $\phi = 0$. Further, α^* strictly decreases in μ for all $\mu \in [0, \bar{\mu}]$.*
- (iii) *A full-information policy is optimal $\alpha^* = 1$ for all $\mu \in [0, 1]$ if the switching cost is high $\phi \geq \bar{\phi}$.*

Proof. The proof proceeds in five steps. First, we define two thresholds, α^b and $\tilde{\alpha}$, which are used in the remainder of the proof. Second, we show that if $\phi = 0$, then there exists $\bar{\mu} \in [0, 1]$ such that the optimal information quality satisfies $\alpha^* \leq \min\{\alpha^b, \tilde{\alpha}\}$ if $\mu \geq \bar{\mu}$ and $\alpha^* \geq \max\{\alpha^b, \tilde{\alpha}\}$ if $\mu \leq \bar{\mu}$, which is used to prove statement (i). Third, we define the thresholds $\underline{\alpha}$ and $\bar{\alpha}$ and show $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$ if $\mu \leq \bar{\mu}$ and $\phi = 0$. Fourth, we show α^* is decreasing in μ when $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$, which combined with the third step proves statement (ii). Fifth, we prove statement (iii). Note that by Lemma C.2, the platform's optimal policy (α^*, γ^*) satisfies $\alpha^* \in [\frac{1}{2}, 1]$ and $\gamma^* \in [\gamma_s^H, \gamma_s^L] \cup \gamma^m$; therefore, we restrict attention to those sets throughout the proof.

Step 1. Note $\alpha = \frac{1}{2}$ implies $\omega_s = \omega_r$ and thus $\gamma_r^H = \gamma_s^H$. Because $\gamma_r^H > \gamma^m$ by Lemma A.7, and γ_s^H strictly decreases in α (Lemma C.1), it follows there exists a unique threshold $\tilde{\alpha} \in (\frac{1}{2}, 1)$ such that $\gamma_s^H \geq \gamma^m$ if and only if $\alpha \leq \tilde{\alpha}$. Next, let $\gamma^*(\alpha)$ be the optimal commission rate for fixed α . We show there exists a unique threshold $\alpha^b \in (\frac{1}{2}, 1)$ such that $r^b(\gamma^*) \geq 0$ if and only if $\alpha \geq \alpha^b$. To see this, note

$$r^b(\gamma) = \frac{\eta_s \gamma}{4} \underbrace{\left(q_L - \frac{c^2(1 - \omega_s)^2}{q_L(1 - \gamma)^2} \right)}_{h(\gamma)},$$

where for convenience we define $h(\gamma)$ to be the expression inside the parentheses. Next, differentiating h in α yields

$$\left. \frac{dh}{d\alpha} \right|_{\gamma=\gamma^*} = \left(\frac{\partial h}{\partial \gamma} \frac{d\gamma^*}{d\alpha} + \frac{\partial h}{\partial \alpha} \right) \Big|_{\gamma=\gamma^*}.$$

By inspection of h , we have $\frac{\partial}{\partial \gamma} h < 0$ and $\frac{\partial}{\partial \alpha} h > 0$ for any $\gamma \in [0, \gamma^m]$ and $\alpha \in [\frac{1}{2}, 1]$. Further, because $\phi = 0$, we have $\gamma_s^H = \gamma_s^L$, which combined with Lemma C.2 implies $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$. Because γ_s^H decreases in α , we must have $\frac{d}{d\alpha} \gamma^* \leq 0$. It follows that $\frac{d}{d\alpha} h > 0$ at $\gamma = \gamma^*(\alpha)$ for all $\alpha \in [\frac{1}{2}, 1]$. Because $r^b(\gamma) \geq 0$ if and only if $h(\gamma) \geq 0$, we conclude there exists a unique threshold $\alpha^b \in [\frac{1}{2}, 1]$ such that $r^b(\gamma^*(\alpha)) \geq 0$ if and only if $\alpha \geq \alpha^b$. Lastly, it can be verified algebraically that $r^b(\gamma) < 0$ at $\alpha = \frac{1}{2}$ and $r^b(\gamma) > 0$ at $\alpha = 1$ for any $\gamma \in [0, \gamma^m]$, which implies $\alpha^b \in (\frac{1}{2}, 1)$.

Step 2. We now show there exists $\bar{\mu} \in [0, 1]$ such that $\alpha^* \leq \min\{\alpha^b, \tilde{\alpha}\}$ if $\mu > \bar{\mu}$ and $\alpha^* \geq \max\{\alpha^b, \tilde{\alpha}\}$ if $\mu \leq \bar{\mu}$. There are two cases to consider: $\alpha^b \geq \tilde{\alpha}$ and $\alpha^b < \tilde{\alpha}$.

Case I: $\alpha^b \geq \tilde{\alpha}$. In this case, the platform's revenue as a function of α can be written as

$$R(\gamma^*) = \begin{cases} \mu r^a(\gamma^m), & \text{for } \alpha \in [\frac{1}{2}, \tilde{\alpha}), \\ \mu r^a(\gamma_s^H), & \text{for } \alpha \in [\tilde{\alpha}, \alpha^b), \\ \mu r^a(\gamma_s^H) + (1 - \mu)r^b(\gamma_s^H)^+, & \text{for } \alpha \in [\alpha^b, 1]. \end{cases}$$

Note $r^a(\gamma)$ is independent of α and strictly increasing in γ on $\gamma \in [0, \gamma^m]$ (Lemma A.12) and γ_s^H is strictly decreasing in α (Lemma C.1). Combining these with the revenue expression above imply either $\alpha^* \leq \tilde{\alpha}$ or $\alpha^* \geq \alpha^b$ must hold for all $\mu \in [0, 1]$. Because $r^a(\gamma^m)$ does not depend on α , it follows that $\alpha^* \leq \tilde{\alpha}$ if and only if the following inequality holds

$$\max_{\alpha \geq \alpha^b} \{ \mu(r^a(\gamma_s^H) - r^a(\gamma^m)) + (1 - \mu)r^b(\gamma_s^H)^+ \} \leq 0. \quad (26)$$

It remains to show (26) holds if and only if $\mu > \bar{\mu}$ for some $\mu \in (0, 1)$. Note if $\mu = 0$, then (26) cannot hold because $\max_{\alpha \geq \alpha^b} r^b(\gamma_s^H) > 0$. If $\mu = 1$, then (26) holds strictly because $\gamma_s^H < \gamma^m$ on $\alpha \geq \alpha^b$ and r^a is strictly increasing in γ . Lastly, note the argument in (26) is strictly decreasing in μ for every value of α because $r^a(\gamma_s^H) \leq r^a(\gamma^m)$ by Lemma A.12, which implies the left hand side of (26) is also strictly decreasing in μ . Because (26) does not hold at $\mu = 0$, holds strictly at $\mu = 1$, and the left hand side is strictly decreasing in μ , we conclude there exists a unique $\bar{\mu} \in (0, 1)$ such that (26) holds if and only if $\mu > \bar{\mu}$. The result follows because $\tilde{\alpha} = \min\{\alpha^b, \tilde{\alpha}\}$ and $\alpha^b = \max\{\alpha^b, \tilde{\alpha}\}$ in this case.

Case II: $\alpha^b < \tilde{\alpha}$. In this case, the platform's revenue is

$$R(\gamma^*) = \begin{cases} \mu r^a(\gamma^m), & \text{for } \alpha \in [\frac{1}{2}, \alpha^b), \\ \mu r^a(\gamma^m) + (1 - \mu)r^b(\gamma^m)^+, & \text{for } \alpha \in [\alpha^b, \tilde{\alpha}), \\ \mu r^a(\gamma_s^H) + (1 - \mu)r^b(\gamma_s^H)^+, & \text{for } \alpha \in [\tilde{\alpha}, 1]. \end{cases}$$

Note that because r^b strictly increases in α (Lemma A.12), either $\alpha^* \leq \alpha^b$ or $\alpha^* \geq \tilde{\alpha}$ must hold for all $\mu \in [0, 1]$. Similar to Case I, because $r^a(\gamma^m)$ does not depend on α , it follows that $\alpha^* \leq \alpha^b$ holds if and only if

$$\max_{\alpha \geq \alpha'} \{ \mu(r^a(\gamma_s^H) - r^a(\gamma^m)) + (1 - \mu)r^b(\gamma_s^H)^+ \} \leq 0. \quad (27)$$

Because $r^a(\gamma)$ strictly increases in γ on $\gamma \in [0, \gamma^m]$ and does not depend on α (Lemma A.12), and because $\gamma_s^H \leq \gamma^m$ if $\alpha \geq \tilde{\alpha}$, it is straightforward to verify that (27) holds only if $\mu = 1$. The result follows by setting $\bar{\mu} = 1$ and noting $\alpha^b = \min\{\alpha^b, \tilde{\alpha}\}$ and $\tilde{\alpha} = \max\{\alpha^b, \tilde{\alpha}\}$.

Step 3. We now show there exists $\underline{\alpha} \in [\frac{1}{2}, 1)$ and $\bar{\alpha} \in (\underline{\alpha}, 1)$ such that $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$ if $\mu \leq \bar{\mu}$. Define $\underline{\alpha} = \max\{\alpha^b, \tilde{\alpha}\}$. Note the result that $\alpha^* \geq \underline{\alpha}$ if $\mu \leq \bar{\mu}$ follows immediately from Step 2. It remains to show the existence of $\bar{\alpha} < 1$ such that $\alpha^* \leq \bar{\alpha}$ for $\mu \leq \bar{\mu}$. We do so by showing $\alpha^* = 1$ cannot hold for any $\mu \in [0, 1]$. Note that if $\mu = 0$ and $\alpha = 1$, then trivially we have $\gamma_s^L = 0$ and thus $R(\alpha, \gamma) = 0$ for all $\gamma \in [0, \gamma^m]$, which implies $\alpha^* < 1$. Now let $\mu > 0$. Suppose by way of contradiction that $\alpha^* = 1$ and let γ^* be the corresponding optimal commission rate. Further, define $\alpha' = \frac{1}{2}$ and let γ' be the optimal commission rate under α' . Then

we have

$$R(\alpha', \gamma') \geq \mu r^a(\alpha', \gamma') = \max_{\gamma \leq \gamma^m} \mu r^a(\alpha', \gamma) = \max_{\gamma \leq \gamma^m} \mu r^a(\alpha^*, \gamma) > \max_{\gamma \leq \gamma^m} \mu r^c(\alpha^*, \gamma) = R(\alpha^*, \gamma^*).$$

The relations above follow because $\gamma_s^H \geq \gamma^m$ at $\alpha = \frac{1}{2}$, which implies $R(\gamma) \geq \mu r^a(\gamma)$ for all $\gamma \in [0, \gamma^m]$, by definition of γ' , because $r^a(\gamma)$ is independent of α , because $r^a(\gamma) > r^c(\gamma)$ for all $\gamma \leq \gamma^m$ (Lemma A.13), and because $\alpha = 1$ and $\phi = 0$ imply $\gamma_s^H = \gamma_s^L = 0$. Note $R(\alpha', \gamma') > R(\alpha^*, \gamma^*)$ contradicts the optimality of (α^*, γ^*) . Therefore, $\alpha^* = 1$ cannot hold at $\phi = 0$ for any $\mu > 0$. It follows that there exists $\bar{\alpha} \in [\underline{\alpha}, 1)$ such that $\alpha^* \leq \bar{\alpha}$ for all $\mu \leq \bar{\mu}$.

Step 4. We now show that α^* strictly decreases in μ if $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$. From the revenue expressions in Step 2 and the definition of $\underline{\alpha} = \max\{\alpha^b, \tilde{\alpha}\}$, $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$ implies the platform's optimal revenue is

$$R(\gamma_s^H) = \mu r^a(\gamma_s^H) + (1 - \mu) r^b(\gamma_s^H)^+.$$

It can be shown that $R(\gamma_s^H)$ is continuous and differentiable in α using the definitions of $r^a(\gamma)$ and $r^b(\gamma)$ (Lemma A.11) and γ_s^H (see proof of Lemma C.1(iv)). Because α^* is a local maximizer of $R(\gamma_s^H)$, we have

$$\left. \frac{dR}{d\alpha} \right|_{(\alpha, \gamma) = (\alpha^*, \gamma_s^H)} = \left(\mu \frac{dr^a}{d\alpha} + (1 - \mu) \frac{dr^b}{d\alpha} \right) \Big|_{(\alpha, \gamma) = (\alpha^*, \gamma_s^H)} = 0. \quad (28)$$

Then by the implicit function theorem,

$$\frac{d\alpha^*}{d\mu} = - \left(\frac{d^2 R}{d\alpha d\mu} \right) \left(\frac{d^2 R}{d\alpha^2} \right)^{-1} \Big|_{(\alpha, \gamma) = (\alpha^*, \gamma_s^H)}.$$

Because α^* maximizes $R(\gamma_s^H)$ and $\alpha^* \in (\frac{1}{2}, 1)$, we have $\frac{d^2}{d\alpha^2} R < 0$ at $(\alpha, \gamma) = (\alpha^*, \gamma_s^H)$. Therefore, $\frac{d}{d\mu} \alpha^*$ has the same sign as $\frac{d^2}{d\alpha d\mu} R$. Further, because $r^a(\gamma)$, $r^b(\gamma)$, and γ_s^H do not depend on μ , we have

$$\frac{d^2 R}{d\alpha d\mu} = \frac{dr^a}{d\alpha} - \frac{dr^b}{d\alpha}.$$

It remains to show $\frac{d}{d\alpha} r^a - \frac{d}{d\alpha} r^b < 0$ at (α^*, γ_s^H) . First, because $\frac{\partial}{\partial \alpha} r^a = 0$ (Lemma A.12), we have

$$\left. \frac{dr^a}{d\alpha} \right|_{\gamma = \gamma_s^H} = \left(\frac{\partial r^a}{\partial \gamma} \frac{d\gamma_s^H}{d\alpha} \right) \Big|_{\gamma = \gamma_s^H} < 0,$$

where the strict inequality follows because $\frac{d}{d\alpha} \gamma_s^H < 0$ (Lemma C.1) and $\frac{\partial}{\partial \gamma} r^a > 0$ for all $\gamma \in [0, \gamma^m]$ (Lemma A.12). Because $\frac{d}{d\alpha} r^a < 0$, it follows from (28) that $\frac{d}{d\alpha} r^b > 0$ at (α^*, γ_s^H) . Finally, re-arranging (28) yields

$$\left(\mu \left(\frac{dr^a}{d\alpha} - \frac{dr^b}{d\alpha} \right) + \frac{dr^b}{d\alpha} \right) \Big|_{(\alpha, \gamma) = (\alpha^*, \gamma_s^H)} = 0.$$

Because $\frac{d}{d\alpha} r^b > 0$, it follows that $\mu \left(\frac{d}{d\alpha} r^a - \frac{d}{d\alpha} r^b \right) < 0$, and thus $\frac{d}{d\alpha} r^a - \frac{d}{d\alpha} r^b < 0$ at (α^*, γ_s^H) , as desired. We conclude α^* strictly decreases in μ if $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$.

Step 5. We now show statement (iii). Note γ_s^H strictly increases in ϕ for all $\alpha \in [\frac{1}{2}, 1]$ (Lemma B.1), strictly decreases in α (Lemma C.1), and $\gamma_s^L \geq \gamma_s^H$ (Lemma A.11). It follows that there exists $\bar{\phi} > 0$ such that $\gamma_s^H \geq \gamma^m$ for all $\alpha \in [\frac{1}{2}, 1]$ if $\phi \geq \bar{\phi}$. Thus, for any $\phi \geq \bar{\phi}$, the platform's optimal revenue for fixed $\alpha \in [\frac{1}{2}, 1]$

is given by

$$R(\gamma^*) = \max_{\gamma \leq \gamma^m} \{ \mu r^a(\gamma) + (1 - \mu) r^b(\gamma)^+ \}.$$

Because $r^b(\gamma)$ strictly increases in α and $r^a(\gamma)$ is independent of α (Lemma A.12), it follows that $R(\gamma^*)$ is strictly increasing in α for $\phi \geq \bar{\phi}$. The result follows. $\square$

C.3 Proof of Corollary 1

Lemma C.3. *If $\mu > \frac{2}{9}$, then the inequality $\mu r^a(\gamma) > \mu r^c(\gamma) + (1 - \mu) r^b(\gamma)^+$ holds for all $\alpha \in [\frac{1}{2}, 1]$, $\gamma \in (0, \gamma^m]$ and $\phi \geq 0$.*

Proof. Before proving the lemma, we show the following bound holds for any $\gamma \in (0, \gamma^m]$:

$$\frac{r^b(\gamma)}{r^a(\gamma)} \leq \frac{2}{15}. \quad (29)$$

To see that (29) holds, note

$$\begin{aligned} r^b(\gamma) &= \frac{q_L \eta_s \gamma}{4} \left(1 - \frac{c^2(1 - \omega_s)^2}{q_L^2(1 - \gamma)^2} \right) \\ &\leq \frac{q_L \eta_s \gamma}{4} \\ &\leq \frac{16}{15} \frac{q_L \eta_s \gamma}{4} \left(1 - \frac{c^2(1 - \lambda)^2}{q_H^2(1 - \gamma)^2} \right) \\ &\leq \frac{16}{15} \cdot \frac{1}{8} \frac{q_H \eta_s \gamma}{4} \left(1 - \frac{c^2(1 - \lambda)^2}{q_H^2(1 - \gamma)^2} \right) \\ &\leq \frac{2}{15} r^a(\gamma). \end{aligned}$$

The third line above follows because $\left(1 - \frac{c^2(1 - \lambda)^2}{q_H^2(1 - \gamma)^2} \right)$ is minimized at $\lambda = \frac{1}{2}$, $q_H = 4c$, and $\gamma = \frac{1}{2}$, which corresponds to a minimal value of $\frac{15}{16}$. The fourth line follows because $8q_L \leq q_H$ by Assumption 1. We can now write

$$\mu r^c(\gamma) + (1 - \mu) r^b(\gamma)^+ \leq \frac{8}{15} \mu r^a(\gamma) + (1 - \mu) \frac{2}{15} r^a(\gamma) < \frac{8}{15} \mu r^a(\gamma) + \frac{7}{2} \mu \cdot \frac{2}{15} r^a(\gamma) = \mu r^a(\gamma),$$

where the first inequality follows by combining Lemma A.13 and (29), and the second inequality follows because $\mu > \frac{2}{9}$ implies $(1 - \mu) \leq \frac{7\mu}{2}$. $\square$

Corollary 1. *There exist thresholds $\bar{\mu} \in [0, 1)$, $\underline{\phi} \geq 0$, and $\bar{\phi} > \underline{\phi}$ such that partial-information is optimal $\alpha^* \in (\frac{1}{2}, 1)$ if the switching cost is moderate $\phi \in [\underline{\phi}, \bar{\phi}]$ and the share of type-H sellers is large $\mu > \bar{\mu}$.*

Proof. The proof proceeds in two steps. First, we show that under the optimal policy (α^*, γ^*) , $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$ holds for all $\phi \geq 0$ if $\mu > \frac{2}{9}$. Second, we prove the main result. With a slight abuse of notation, we write the platform's revenue as $R(\alpha, \gamma)$ to make dependence on α explicit.

Step 1. We focus on the most general case where $\gamma^m > \gamma_s^L$ holds at $\alpha = \alpha^*$; the cases where $\gamma^m \leq \gamma_s^H$ and $\gamma^m \in (\gamma_s^H, \gamma_s^L)$ follow by parallel argument and are omitted. First, note $\gamma^* \notin (0, \gamma_s^H)$ follows directly from

Lemma C.2. It remains to show $\gamma^* \notin (\gamma_s^H, \gamma_s^L]$ and $\gamma^* \notin (\gamma_s^L, \gamma^m]$. Note any $\gamma \in (\gamma_s^H, \gamma_s^L]$ and $\mu > \frac{2}{9}$,

$$R(\alpha^*, \gamma) = \mu r^c(\alpha^*, \gamma) + (1 - \mu) \mu r^b(\alpha^*, \gamma) < \mu r^a(\alpha^*, \gamma) \leq \mu r^a(\frac{1}{2}, \gamma^m) = R(\frac{1}{2}, \gamma^m),$$

where the first inequality follows from Lemma C.3 and the second follows because $r^a(\gamma)$ is independent of α and increases in γ (Lemma A.12). Hence, $\gamma^* \notin (\gamma_s^H, \gamma_s^L]$ for $\mu > \frac{2}{9}$. Similarly, by again using Lemma C.3, for any $\gamma > \gamma_s^L$ and $\mu > \frac{2}{9}$ we have

$$R(\alpha^*, \gamma) = \mu r^c(\alpha^*, \gamma) < \mu r^a(\alpha^*, \gamma) \leq R(\frac{1}{2}, \gamma^m),$$

which implies $\gamma^* \leq \gamma_s^L$. We have thus shown $\gamma^* = \gamma_s^H$ if $\mu > \frac{2}{9}$ and $\gamma_s^L < \gamma^m$. Because $\gamma_s^H \leq \gamma_s^L$ (Lemma A.11), we conclude $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$ if $\mu > \frac{2}{9}$.

Step 2. To begin, define α_0 to be the solution to $1 - \omega_s = \frac{q_L \lambda}{c}$, and note $\alpha_0 < 1$ since ω_s strictly increases in α and $\omega_s = 1$ at $\alpha = 1$ (Lemma A.1). Further, define $\hat{\alpha} = \alpha_0 + \epsilon < 1$ for any $\epsilon \in (0, 1 - \alpha_0)$ and define $\hat{\phi} \geq 0$ to be the smallest switching cost such that $\gamma_s^H \geq \gamma^m$ for all $\phi \geq \hat{\phi}$ at $\alpha = \hat{\alpha}$. Note $\hat{\phi}$ exists because γ_s^H strictly increases in ϕ for all $\alpha \in [\frac{1}{2}, 1]$ (Lemma B.1); let $\phi = \hat{\phi}$ be fixed in the remainder. To prove the corollary statement, it suffices to show there exists $\bar{\mu} < 1$ such that $R(\hat{\alpha}, \gamma^*(\hat{\alpha})) > R(\frac{1}{2}, \gamma^*(\frac{1}{2}))$ and $R(\hat{\alpha}, \gamma^*(\hat{\alpha})) > R(1, \gamma^*(1))$ both hold for all $\mu \geq \bar{\mu}$ at $\phi = \hat{\phi}$, i.e., the platform's revenue at $\alpha = \hat{\alpha}$ is strictly higher than the revenue at both $\alpha = \frac{1}{2}$ and $\alpha = 1$, under the corresponding optimal commission rate γ^* in each instance.

Case I: Comparison with $\alpha = \frac{1}{2}$. First, at $\alpha = \alpha_0$ we have

$$r^b(\alpha_0, \gamma^m) = \frac{q_L \eta_s \gamma}{4} \left(1 - \left(\frac{(1 - \omega_s)c}{q_L(1 - \gamma^m)} \right)^2 \right) = \frac{q_L \eta_s \gamma}{4} \left(1 - \left(\frac{\lambda}{1 - \gamma^m} \right)^2 \right) \geq 0,$$

where the second equality follows by construction of α_0 and the inequality follows because $\gamma^m \leq 1 - \lambda$ by Assumption 2. For all $\mu > \frac{2}{9}$ we can now write

$$R(\hat{\alpha}, \gamma^*(\hat{\alpha})) \geq R(\hat{\alpha}, \gamma^m) = \mu r^a(\hat{\alpha}, \gamma^m) + (1 - \mu) r^b(\hat{\alpha}, \gamma^m)^+ > \mu r^a(\hat{\alpha}, \gamma^m) = R(\frac{1}{2}, \gamma^m) = R(\frac{1}{2}, \gamma^*(\frac{1}{2})) \quad (30)$$

To see that the strict inequality in (30) holds, note that $r^b(\alpha_0, \gamma^m) \geq 0$ as established above, $\hat{\alpha} > \alpha_0$, and $r^b(\alpha, \gamma^m)$ increases in α because ω_s and γ_s both increase in α (Lemma A.1). The second equality in (30) holds because $r^a(\gamma)$ is independent of α , and the final equality follows because $\gamma_s^H \geq \gamma^m$ at $\alpha = \frac{1}{2}$ and $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$ by Step 1. We conclude $R(\hat{\alpha}, \gamma^*(\hat{\alpha})) > R(\frac{1}{2}, \gamma^*(\frac{1}{2}))$ for $\mu > \frac{2}{9}$.

Case II: Comparison with $\alpha = 1$. First, note $\gamma_s^H < \gamma^m$ at $(\alpha, \phi) = (1, \hat{\phi})$. To see this, note $\gamma_s^H(\hat{\alpha}) = \gamma^m$ at $(\alpha, \phi) = (\hat{\alpha}, \hat{\phi})$ by definition of $\hat{\phi}$, γ_s^H is strictly decreasing in α (Lemma C.1), and $\hat{\alpha} < 1$. Next, let $\mu = 1$. We can now write

$$R(1, \gamma^*(1)) = \mu r^a(1, \gamma_s^H) + (1 - \mu) r^b(1, \gamma_s^H)^+ < \mu r^a(\hat{\alpha}, \gamma^m) \leq R(\hat{\alpha}, \gamma^m) \leq R(\hat{\alpha}, \gamma^*(\hat{\alpha})). \quad (31)$$

The first equality in (31) follows because $\gamma_s^H < \gamma^m$ implies $\gamma^* = \gamma_s^H$ by Step 1. To see that the strict inequality holds at $\mu = 1$, note $r^a(\gamma)$ is strictly increasing in γ and independent of α (Lemmas A.12 and C.1), and $\gamma_s^H < \gamma^m$ as established above. We have thus shown $R(1, \gamma^*(1)) < R(\hat{\alpha}, \gamma^*(\hat{\alpha}))$ holds for $\mu = 1$. It follows by continuity of $R(\alpha, \gamma)$ in μ that there exists $\bar{\mu} \in [\frac{2}{9}, 1)$ such that $R(1, \gamma^*(1)) < R(\hat{\alpha}, \gamma^*(\hat{\alpha}))$ for all $\mu \geq \bar{\mu}$. Because $\mu \geq \frac{2}{9}$, it follows that $R(\hat{\alpha}, \gamma^*(\hat{\alpha})) > R(\frac{1}{2}, \gamma^*(\frac{1}{2}))$ and $R(\hat{\alpha}, \gamma^*(\hat{\alpha})) > R(1, \gamma^*(1))$ for all $\mu \geq \bar{\mu}$ for $\phi = \hat{\phi}$, as desired. Finally, the thresholds $\underline{\phi}$ and $\bar{\phi}$ can be shown to exist using the continuity of $R(\alpha, \gamma)$ in ϕ . $\square$

D Proofs for Section 5.1: Platform-Access Fees

In section D.1, we first present several supporting results (Lemmas D.1–D.5) that are needed for the proof of Proposition 4. Throughout this section, we use $R_C(\alpha, \gamma)$ and $R_A(\alpha, \psi)$ to denote the platform's revenue under the commission and access fee mechanisms, respectively. For conciseness, define $R_C^* := R_C(\alpha^*, \gamma^*)$ and $R_A^* := R_A(\alpha^*, \psi^*)$, where (α^*, γ^*) and (α^*, ψ^*) are the optimal policies under commission and access fees, respectively.

D.1 Preliminary Results for Proposition 4

Lemma D.1 (Characterization of access fees). *Let Π_0^i be a type- i seller's on-platform profit under a commission rate of $\gamma = 0$.*

(i) *For each $\alpha \in [\frac{1}{2}, 1]$,*

$$\begin{aligned}\Pi_0^H &:= \frac{q_H}{4} \left(1 - \frac{(1-\lambda)c}{q_H} \right)^2, \\ \Pi_0^L &:= \frac{\eta_s q_L}{4} \left(1 - \frac{(1-\omega_s)c}{q_L} \right)^2.\end{aligned}$$

(ii) *Π_0^H is independent of α , Π_0^L strictly increases in α , and $0 < \Pi_0^L < \Pi_0^H$ for all $\alpha \in [\frac{1}{2}, 1]$.*

(iii) *For each $\psi > 0$, the platform's revenue under access fees $R_A(\psi)$ is weakly increasing in α on $\alpha \in [\frac{1}{2}, 1]$.*

(iv) *The platform's optimal access fee satisfies $\psi^* \in \{\Pi_0^H, \Pi_0^L\}$, with corresponding optimal revenue $R_A^* = \max\{\mu\Pi_0^H, \Pi_0^L\}$.*

Proof. (i). By Lemma A.3, the profit for the type- H and type- L sellers are given by $\pi^a(p^a)$ and $\pi^b(p^b)$, respectively. Thus, the on-platform earnings under access fees are given by setting $\gamma = 0$ in $\pi^a(p^a)$ and $\pi^b(p^b)$, which yields the expressions in statement (i).

(ii). By inspection, Π_0^H is independent of α . Because $\frac{\partial}{\partial \alpha} \eta_s > 0$ and $\frac{\partial}{\partial \alpha} \omega_s > 0$ (Lemma A.1), we have $\frac{d}{d\alpha} \Pi_0^L > 0$. Finally, $0 < \Pi_0^L < \Pi_0^H$ follows from the expressions in part (i) and because $q_H \geq 8q_L$ by Assumption 1.

(iii). Define $\bar{\Pi}_0^L := \lim_{\alpha \rightarrow 1} \Pi_0^L$, and note $\bar{\Pi}_0^L < \Pi_0^H$ by part (ii). We consider three cases: $\psi \leq \bar{\Pi}_0^L$, $\psi \in (\bar{\Pi}_0^L, \Pi_0^H]$, and $\psi > \Pi_0^H$. First, if $\psi \leq \bar{\Pi}_0^L$, because Π_0^L strictly increases in α , for each $\psi \leq \bar{\Pi}_0^L$ there exists $\alpha_L \in [\frac{1}{2}, 1]$ such the type- L seller joins the platform if and only if $\alpha \geq \alpha_L$. Further, the type- H seller joins for all $\alpha \in [\frac{1}{2}, 1]$. Therefore, if $\psi \leq \bar{\Pi}_0^L$, the platform's revenue under access fees is

$$R_A(\psi) = \begin{cases} \mu\psi, & \text{if } \alpha < \alpha_L, \\ \psi, & \text{if } \alpha \geq \alpha_L. \end{cases}$$

Because $\mu \leq 1$, $R_A(\psi)$ weakly increases in α . Next, if $\psi \in (\bar{\Pi}_0^L, \Pi_0^H]$, then the type- L seller does not join for any $\alpha \in [\frac{1}{2}, 1]$, which implies $R_A(\psi) = \mu\psi$ for all $\alpha \in [\frac{1}{2}, 1]$, and thus $R_A(\psi)$ is independent of α . Finally, if $\psi > \Pi_0^H$, then neither seller type joins, which implies $R_A(\psi) = 0$ for all $\alpha \in [\frac{1}{2}, 1]$. Therefore, in all three cases the platform's revenue is weakly increasing in α .

(iv). By part (i), if $\psi \leq \Pi_0^L$ then both seller types join, which generates a revenue of ψ . If $\psi \in [\Pi_0^L, \Pi_0^H]$, then only the type- H seller joins the platform, which generates revenue $\mu\psi$. It follows that $\psi^* \in \{\Pi_0^H, \Pi_0^L\}$ and thus $R_A^* = \max\{\mu\Pi_0^H, \Pi_0^L\}$. $\square$

Lemma D.2. *If $\phi \geq 0$, then the following inequalities hold for all $\alpha \in [\frac{1}{2}, 1]$:*

$$\frac{r^a(\gamma)}{\Pi_0^H} \leq \frac{9\gamma}{7} \quad \gamma \in [0, \gamma^m], \quad (33a)$$

$$\frac{r^b(\gamma)}{\Pi_0^L} \leq \frac{1}{2-\gamma} \quad \gamma \in [0, \gamma^m]. \quad (33b)$$

Further, if $\phi = 0$ and $\gamma \leq 1 - \omega_s$, then the following inequality holds for all $\alpha \in [\frac{1}{2}, 1]$:

$$\frac{r^b(\gamma)}{\Pi_0^L} \leq \begin{cases} \frac{1}{2-\gamma} & \gamma \in [0, 2 - \sqrt{3}], \\ \gamma \frac{(1+\gamma)(1-3\gamma)}{(1-\gamma)^2(1-2\gamma)^2} & \gamma \in [2 - \sqrt{3}, \gamma^m]. \end{cases} \quad (34)$$

Proof. We first show that (33a) holds. Note

$$\frac{1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma)}\right)^2}{\left(1 - \frac{(1-\lambda)c}{q_H}\right)^2} \leq \frac{1 - \left(\frac{(1-\lambda)c}{q_H}\right)^2}{\left(1 - \frac{(1-\lambda)c}{q_H}\right)^2} \leq \frac{9}{7}, \quad (35)$$

where the second inequality above follows because $\lambda \geq \frac{1}{2}$ and $q_H \geq 4c$. Then we can write

$$r^a(\gamma) = \frac{q_H\gamma}{4} \left(1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma)}\right)^2\right) \leq \frac{q_H\gamma}{4} \cdot \frac{9}{7} \left(1 - \frac{(1-\lambda)c}{q_H}\right)^2 = \frac{9\gamma}{7} \Pi_0^H,$$

where the first and second equalities follow by definition of $r^a(\gamma)$ and Π_0^H , and the inequality follows from (35). Next, we show (33b) and (34). In the case where $r^b(\gamma) \leq 0$, (33b) and (34) follow trivially; we assume $r^b(\gamma) > 0$ for the remainder of the proof. Let $z = \frac{(1-\omega_s)c}{q_L}$. We first show $z \leq 1 - \gamma$ for all $\phi \geq 0$ and $z \geq 2\gamma$ if $\phi = 0$. To see $z \leq 1 - \gamma$ for $\phi \geq 0$, note $r^b(\gamma) > 0$ implies $q_L(1 - \gamma) > (1 - \omega_s)c$ (Lemma A.3), which by definition of z implies $z \leq 1 - \gamma$. To see that $z \geq 2\gamma$ when $\phi = 0$, note

$$z = \frac{(1 - \omega_s)c}{q_L} \geq \frac{(1 - \omega_s)c}{(1 - \lambda)c} \geq 2(1 - \omega_s) \geq 2\gamma,$$

which follows because $q_L \leq (1 - \lambda)c$, $\lambda \geq \frac{1}{2}$, and $\gamma \leq \gamma_s^L = 1 - \omega_s \leq 1 - \omega_s$ at $\phi = 0$. Thus, $z \leq 1 - \gamma$ for $\phi \geq 0$ and $z \geq 2\gamma$ for $\phi = 0$. We now show (34) assuming $\phi = 0$; (33b) follows by a similar argument. First, using the definition of z , we have

$$\frac{r^b(\gamma)}{\Pi_0^L} = \frac{\gamma q_L^2}{(q_L - (1 - \omega_s)c)^2} \left(1 - \left(\frac{(1 - \omega_s)c}{q_L(1 - \gamma)}\right)^2\right) = \frac{\gamma}{(1 - z)^2} \left(1 - \frac{z^2}{(1 - \gamma)^2}\right). \quad (36)$$

Differentiating in z yields

$$\frac{\partial}{\partial z} \left(\frac{r^b(\gamma)}{\Pi_0^L} \right) = \frac{\gamma}{(1 - z)^4} \left((1 - z)^2 \left(-2 \frac{z}{(1 - \gamma)^2} \right) + 2 \left(1 - \frac{z^2}{(1 - \gamma)^2} \right) (1 - z) \right) = \frac{2\gamma}{(1 - z)^3} \left(1 - \frac{z}{(1 - \gamma)^2} \right),$$

which implies the ratio (36) strictly increases in z on $z \in [0, (1 - \gamma)^2]$ and has a single maximizer at

$z = (1 - \gamma)^2 < 1 - \gamma$. Because $z \in [2\gamma, 1 - \gamma]$ when $\phi = 0$, we have two cases to consider: If $2\gamma < (1 - \gamma)^2$, the maximizer of the ratio (36) on the interval $z \in [2\gamma, 1 - \gamma]$ is $z = (1 - \gamma)^2$; if $2\gamma \geq (1 - \gamma)^2$, the maximizer is $z = 2\gamma$. Note $2\gamma \geq (1 - \gamma)^2$ if and only if $\gamma \geq 2 - \sqrt{3}$. Plugging $z = 2\gamma$ and $z = (1 - \gamma)^2$ into (36) yields (34), as desired. Lastly, in the case where $\phi \geq 0$, (33b) follows by a similar argument where only $z \leq 1 - \gamma$ is assumed to hold. $\square$

Lemma D.3. *The platform's optimal commission revenue is strictly smaller than the optimal access fee revenue, $R_C^* < R_A^*$, if at least one the following conditions holds at the optimal commission rate γ^* :*

- (i) *The type-H seller transacts offline with the $\sigma = s$ buyer, $\gamma^* > \gamma_s^H$.*
- (ii) *The type-L seller does not transact with any buyer, $r^b(\gamma^*) \leq 0$.*

Proof. (i). Note $\gamma_s^L \geq \gamma_s^H$ by Lemma A.10. We therefore consider two cases: $\gamma^* > \gamma_s^L$ and $\gamma^* \leq \gamma_s^L$. If $\gamma^* > \gamma_s^L$, then the platform's revenue under commission fees is given by $R_C(\gamma^*) = \mu r^c(\gamma^*)$ (Lemma A.11). Then

$$R_C^* = \mu r^c(\gamma^*) < \mu r^a(\gamma^*) < \mu \Pi_0^H \leq R_A^*,$$

where the equality follows by Lemma A.11 because $\gamma > \gamma_s^L$, and the next three inequalities follow from Lemmas A.13, D.2, and D.1, in order. Now suppose $\gamma^* \in (\gamma_s^H, \gamma_s^L]$. Then there are two further cases to consider: $\gamma^* \leq \gamma_r^H$ and $\gamma^* > \gamma_r^H$.

Case I: $\gamma^* \leq \gamma_r^H$. Note by Lemma A.11 the platform's optimal commission revenue in this case is given by $R_C^* = \mu r^c(\gamma^*) + (1 - \mu)r^b(\gamma^*)^+$. We show $R_C^* \leq R_A^*$ by first establishing upper bounds on $r^b(\gamma^*)$ and $r^c(\gamma^*)$. First, for $r^b(\gamma^*)$ we have

$$r^b(\gamma^*) = \frac{q_L \eta_s \gamma^*}{4} \left(1 - \frac{c^2(1 - \omega_s)^2}{q_L^2(1 - \gamma)^2} \right) \leq \frac{q_L \eta_s \gamma^*}{4} \leq \frac{q_L \eta_s \gamma^m}{4} = \frac{q_L \eta_s}{8}.$$

For $r^c(\gamma^*)$,

$$\begin{aligned} r^c(\gamma^*) &= \eta_r \gamma^* q_H \left(\frac{1}{4} - \left(\frac{(1 - \lambda)c}{2q_H \zeta} + \frac{\eta_s \phi}{4q_H \zeta} \right)^2 \right) \\ &\leq \frac{\eta_r \gamma^* q_H}{4} \left(1 - \frac{c^2(1 - \lambda)^2}{q_H^2 \zeta^2} \right) \\ &\leq \frac{\eta_r \gamma^* q_H}{4} \left(1 - \frac{(1 - \lambda)c}{q_H \zeta} \right)^2 \cdot \frac{9}{7} \\ &\leq \frac{9}{7} \cdot \frac{q_H}{16} \left(1 - \frac{(1 - \lambda)c}{q_H \zeta} \right)^2, \end{aligned}$$

where the second line follows by setting $\phi = 0$, the third line from (35), and the final line follows because $\gamma^* \leq \gamma^m = \frac{1}{2}$ and $\eta_r \leq \frac{1}{2}$. Combining the bounds above, we can then write

$$\begin{aligned} \mu r^c(\gamma^*) + (1 - \mu)r^b(\gamma^*)^+ &\leq \mu \cdot \frac{9}{7} \cdot \frac{q_H}{16} \left(1 - \frac{(1 - \lambda)c}{q_H \zeta} \right)^2 + (1 - \mu) \frac{q_L \eta_s}{8} \\ &\leq 2 \max \left\{ \mu \cdot \frac{9}{7} \cdot \frac{q_H}{16} \left(1 - \frac{(1 - \lambda)c}{q_H \zeta} \right)^2, (1 - \mu) \frac{q_L \eta_s}{8} \right\} \\ &\leq \max \left\{ \mu \cdot \frac{9}{7} \cdot \frac{q_H}{8} \left(1 - \frac{(1 - \lambda)c}{q_H \zeta} \right)^2, (1 - \mu) \frac{q_L \eta_s}{4} \right\} \end{aligned}$$

$$\begin{aligned}
&\leq \max \left\{ \mu \cdot \frac{q_H}{5} \left(1 - \frac{(1-\lambda)c}{q_H \zeta} \right)^2, (1-\mu) \frac{q_L \eta_s}{4} \right\} \\
&< \max \{ \mu \Pi_0^H, \Pi_0^L \} \\
&= R_A^*.
\end{aligned}$$

In the first inequality, we used the fact that $x + y \leq 2 \max\{x, y\}$ for any non-negative x and y . This completes the proof for the case $\gamma^* \leq \gamma_r^H$.

Case II: $\gamma^* > \gamma_r^H$. In this case, the platform's revenue is given by $R_C^* = (1-\mu)r^b(\gamma^*)^+$. Then we have

$$R_C^* = (1-\mu)r^b(\gamma^*) < (1-\mu)\Pi_0^L \leq \Pi_0^L \leq R_A^*,$$

where the first and third inequalities follow from Lemmas D.2 and D.1, respectively.

(ii). Because $r^b(\gamma^*) \leq 0$, only the type- H seller transacts online. Note that if $r^b(\gamma^*) \leq 0$ implies $\gamma^* \leq \gamma_r^H$, otherwise the platform's commission revenue is zero. Then there are two cases to consider: $\gamma^* \leq \gamma_s^H$ and $\gamma^* > \gamma_s^H$. If $\gamma^* \leq \gamma_s^H$, then by Lemma A.11 the platform's commission revenue is $R_c(\gamma^*) = \mu r^a(\gamma^*)$. Then we have

$$R_C^* = \mu r^a(\gamma^*) \leq \mu \Pi_0^H \leq R_A^*,$$

where the first and second inequalities follow from Lemma D.2 and D.1, respectively. If $\gamma^* > \gamma_s^H$, then $R_C(\gamma^*) = \mu r^c(\gamma^*)$, and

$$R_C^* = \mu r^c(\gamma^*) \leq \mu r^a(\gamma^*) \leq \mu \Pi_0^H \leq R_A^*,$$

where the first, second, and third inequalities follow from Lemmas A.13, D.2, and D.1, respectively. $\square$

Lemma D.4. Define $\hat{\mu} = \Pi_0^L / \Pi_0^H$. If the inequality $R_A^* < R_C^*$ holds for some $\mu \neq \hat{\mu}$, then it also holds for $\mu = \hat{\mu}$.

Proof. To make dependence on μ explicit, we write $R_C^*(\mu)$, $R_A^*(\mu)$ and $\gamma^*(\mu)$ to denote the platform's optimal revenue under commission fees, optimal revenue under access fees, and optimal commission rate, respectively. We also let $R_C(\gamma, \mu)$ be the platform's commission revenue for fixed γ and optimal α , where $R_C^*(\mu) = R_C(\gamma^*(\mu), \mu)$. First, suppose $R_A^*(\mu) < R_C^*(\mu)$ for some $\mu \neq \hat{\mu}$. We show $R_A^*(\hat{\mu}) < R_C^*(\hat{\mu})$ must also hold. We consider two cases: $\mu > \hat{\mu}$ and $\mu < \hat{\mu}$.

Case I: $\mu > \hat{\mu}$. Note

$$\begin{aligned}
R_C^*(\hat{\mu}) &= R_C^*(\mu) + R_C^*(\hat{\mu}) - R_C^*(\mu) \\
&\geq R_C^*(\mu) + R_C(\gamma^*(\mu), \hat{\mu}) - R_C^*(\mu) \\
&= R_C^*(\mu) + \hat{\mu} r^a(\gamma^*(\mu)) + (1-\mu)r^b(\gamma^*(\mu))^+ - R_C^*(\mu) \\
&= R_C^*(\mu) + (\hat{\mu} - \mu)r^a(\gamma^*(\mu)) + (\mu - \hat{\mu})r^b(\gamma^*(\mu))^+ \\
&> R_C(\mu) + (\hat{\mu} - \mu)\Pi_0^H \\
&= R_C(\mu) + R_A^*(\hat{\mu}) - R_A^*(\mu) \\
&> R_A^*(\hat{\mu}).
\end{aligned}$$

The second line follows by optimality of $\gamma^*(\hat{\mu})$ under $\mu = \hat{\mu}$, the third because $R_C^*(\mu) > R_A^*(\mu)$ implies

$\gamma^*(\mu) \leq \gamma_s^H$ by Lemma D.3 and thus $R_C(\gamma^*(\mu), \mu') = \mu' r^a(\gamma^*(\mu)) + (1 - \mu') r^b(\gamma^*(\mu))^+$ for any $\mu' \in [0, 1]$, the fourth by expanding $R_C^*(\mu)$, the fifth because $r^a(\gamma) < \Pi_0^H$ for any $\gamma \leq \gamma^m$ by Lemma D.2, and because $(\mu - \hat{\mu}) r^b(\gamma^*(\mu))^+ \geq 0$, the sixth because $R_A^*(\mu) = \Pi_0^H$ for all $\mu \geq \hat{\mu}$ by Lemma D.1, and the seventh because $R_A^*(\mu) < R_C^*(\mu)$ by assumption. Therefore, $R_A^*(\mu) < R_C^*(\mu)$ for $\mu > \hat{\mu}$ implies $R_A^*(\hat{\mu}) < R_C^*(\hat{\mu})$.

Case II: $\mu < \hat{\mu}$. Following a similar argument to Case I, we have

$$\begin{aligned}
R_C^*(\hat{\mu}) &= R_C^*(\mu) + R_C^*(\hat{\mu}) - R_C^*(\mu) \\
&\geq R_C^*(\mu) + R_C(\gamma^*(\mu), \hat{\mu}) - R_C^*(\mu) \\
&= R_C^*(\mu) + (\hat{\mu} - \mu) r^a(\gamma^*(\mu)) + (\mu - \hat{\mu}) r^b(\gamma^*(\mu))^+ \\
&> R_C^*(\mu) \\
&\geq R_A^*(\mu) \\
&= R_A^*(\hat{\mu}),
\end{aligned}$$

where the fourth line follows because $r^a(\gamma) > r^b(\gamma)$ for all $\gamma \in [0, \gamma^m]$. $\square$

Lemma D.5. Let $\hat{\mu} = \Pi_0^L / \Pi_0^H$. There exists $\bar{\phi} > 0$ such that for each $\phi \geq \bar{\phi}$, $R_C^* > R_A^*$ if and only if $\mu \in [\underline{\mu}, \bar{\mu}]$, where $\underline{\mu} \in (0, \hat{\mu})$ and $\bar{\mu} \in (\hat{\mu}, 1)$.

Proof. The proof proceeds in three steps. First, we show that the following two inequalities hold at $\alpha = 1$:

$$2r^b(\gamma^m) \geq \Pi_0^L, \quad (37a)$$

$$2r^a(\gamma^m) - \Pi_0^H > \Pi_0^L. \quad (37b)$$

Second, we show that there exists $\bar{\phi} > 0$ such that $R_C^* > R_A^*$ for all $\phi \geq \bar{\phi}$ at $\mu = \hat{\mu}$. Third, we show the existence of the thresholds $\underline{\mu} \in (0, \hat{\mu})$ and $\bar{\mu} \in (\hat{\mu}, 1)$ for each $\phi \geq \bar{\phi}$.

Step 1. First, consider the expressions for Π_0^L and $r^b(\gamma)$:

$$\begin{aligned}
\Pi_0^L &= \eta_s q_L \left(\frac{1}{2} - \frac{(1 - \omega_s)c}{2q_L} \right)^2, \\
r^b(\gamma) &= \frac{\eta_s \gamma}{4} \left(q_L - \frac{c^2(1 - \omega_s)^2}{q_L(1 - \gamma)^2} \right).
\end{aligned}$$

Using $\lim_{\alpha \rightarrow 1} \eta_s = \lambda$ and $\lim_{\alpha \rightarrow 1} \omega_s = 1$, we have:

$$\lim_{\alpha \rightarrow 1} \{2r^b(\gamma^m) - \Pi_0^L\} = \frac{\lambda \gamma^m q_L}{2} - \frac{\lambda q_L}{4} = 0.$$

It follows that (37a) holds if $\alpha = 1$. Next, to see that (37b) holds, note

$$\begin{aligned}
2r^a(\gamma^m) - \Pi_0^H &= \left(q_H - \frac{4c^2(1 - \lambda)^2}{q_H} \right) - \left(\frac{q_H}{4} \left(1 - \frac{(1 - \lambda)c}{q_H} \right)^2 \right) \\
&= q_H \left(1 - \frac{4c^2(1 - \lambda)^2}{q_H^2} \right) - \left(\frac{q_H}{4} \left(1 - \frac{(1 - \lambda)c}{q_H} \right)^2 \right)
\end{aligned}$$

$$\begin{aligned}
&= \frac{q_H}{4} \left(\left(1 - \frac{4(1-\lambda)^2 c^2}{q_H^2} \right) - \left(1 - \frac{(1-\lambda)c}{q_H} \right)^2 \right) \\
&= \frac{q_H}{4} \left(\frac{2(1-\lambda)c}{q_H} - \frac{5(1-\lambda)^2 c^2}{q_H^2} \right) \\
&= \frac{(1-\lambda)c}{2} \left(1 - \frac{5(1-\lambda)c}{2q_H} \right) \\
&\geq \frac{q_L}{2} \left(1 - \frac{5(1-\lambda)c}{2q_H} \right) \tag{38a}
\end{aligned}$$

$$\begin{aligned}
&> \frac{q_L \lambda}{4} \tag{38b} \\
&\geq \frac{q_L \eta_s}{4} \left(1 - \frac{(1-\omega_s)c}{q_L} \right)^2 \\
&= \Pi_0^L.
\end{aligned}$$

The first five lines follow by algebra and using $\gamma^m = \frac{1}{2}$ and $q_H \geq 4c$. The inequality (38a) follows because $q_L \leq (1-\lambda)c$ by Assumption 1 and (38b) follows algebraically using $q_H \geq 4c$. The final inequality follows because $\eta_s \leq \lambda$ and because $r^b(\gamma^m) > 0$ by (37a) for $\alpha = 1$, which implies $q_L \geq (1-\omega_s)c$ and thus $\left(1 - \frac{(1-\omega_s)c}{q_L}\right)^2 \leq 1$.

Step 2. We now show that (37a) and (37b) imply there exists $\bar{\phi} > 0$ and $\hat{\mu}$ such that $R_C^* > R_A^*$ at $\hat{\mu}$ for all $\phi \geq \bar{\phi}$. By Lemmas A.10 and A.11, there exists $\bar{\phi} > 0$ such that for all $\phi \geq \bar{\phi}$ the platform's commission revenue is given by $R_C(\gamma) = \mu r^a(\gamma) + (1-\mu)r^b(\gamma)^+$. Further, by Lemma D.1, under the access fee mechanism it is optimal to set $\alpha = 1$, with corresponding optimal revenue given by $R_A^* = \max\{\mu\Pi_0^H, \Pi_0^L\}$. Therefore, to show $R_C^* > R_A^*$, it suffices to find $\gamma \in [0, \gamma^m]$ such that at $\alpha = 1$,

$$\hat{\mu} r^a(\gamma) + (1-\hat{\mu})r^b(\gamma)^+ > \max\{\hat{\mu}\Pi_0^H, \Pi_0^L\}.$$

We show the inequality above holds for $\gamma = \gamma^m$. Note

$$\begin{aligned}
\hat{\mu} r^a(\gamma^m) + (1-\hat{\mu})r^b(\gamma^m)^+ &> \frac{\hat{\mu}}{2}\Pi_0^H (1+\hat{\mu}) + \frac{(1-\hat{\mu})}{2}\Pi_0^L \\
&= \frac{\hat{\mu}}{2}\Pi_0^H + \frac{\hat{\mu}}{2}\Pi_0^L + \frac{(1-\hat{\mu})}{2}\Pi_0^L \\
&= \frac{\Pi_0^L}{2} + \frac{\Pi_0^L}{2} \\
&\geq R_A^*.
\end{aligned}$$

The first line follows from (37a) and (37b), and the second and third lines follow by definition of $\hat{\mu}$. Therefore, $R_C^* > R_A^*$ at $\hat{\mu}$.

Step 3. We now show the existence of the thresholds $\underline{\mu} \in (0, \hat{\mu})$ and $\bar{\mu} \in (\hat{\mu}, 1)$. Fix $\phi \geq \bar{\phi}$ and $\alpha = 1$. Because $R_C^* > R_A^*$ at $\hat{\mu}$ and $R_C^* - R_A^*$ is continuous in μ , it suffices to show $R_C^* - R_A^* = 0$ has exactly one solution in μ on each of the intervals $[0, \hat{\mu})$ and $(\hat{\mu}, 1]$. First consider the interval $[0, \hat{\mu})$. Note $R_A^* = \max\{\mu\Pi_0^H, \Pi_0^L\}$ by Lemma D.1 and $\hat{\mu} = \Pi_0^L/\Pi_0^H$ by definition. It follows that $R_A^* = \Pi_0^L$ for all $\mu \in [0, \hat{\mu})$. Next, because $R_C^* = \mu r^a(\gamma^*) + (1-\mu)r^b(\gamma^*)^+$, we must also have $\gamma^* \leq \gamma_s^H$ by Lemma A.11. We can then write

$$\frac{d}{d\mu} \{R_C^* - R_A^*\} = \frac{\partial}{\partial \gamma} \{R_C^* - R_A^*\} \frac{d\gamma^*}{d\mu} + \frac{\partial}{\partial \mu} \{R_C^* - R_A^*\}$$

$$\begin{aligned}
&= \frac{\partial}{\partial \gamma} \{ \mu r^a(\gamma) + (1 - \mu) r^b(\gamma)^+ \} \frac{d\gamma^*}{d\mu} + r^a(\gamma^*) - r^b(\gamma^*) \\
&= r^a(\gamma^*) - r^b(\gamma^*) \\
&> 0,
\end{aligned}$$

where the first line follows from taking the total derivative and the second line follows because $\frac{d}{d\mu} \Pi_0^L = 0$. To see why the third line holds, consider the two cases $\gamma^* < \gamma_s^H$ and $\gamma^* = \gamma_s^H$. If $\gamma^* < \gamma_s^H$, then $\frac{\partial}{\partial \gamma} \{ \mu r^a(\gamma) + (1 - \mu) r^b(\gamma)^+ \} = 0$ at γ^* by the envelope theorem; if $\gamma^* = \gamma_s^H$, then $\frac{d}{d\mu} \gamma^* = \frac{d}{d\mu} \gamma_s^H = 0$ because γ_s^H does not depend on μ (Lemma A.10). The final line follows because $r^a(\gamma) > r^b(\gamma)$ for all $\gamma \in [0, \gamma^m]$. It follows that $R_C^* - R_A^*$ strictly increases in μ on $[0, \hat{\mu})$ wherever $R_C^* - R_A^* > 0$. Hence $R_C^* - R_A^* = 0$ has at most one solution on the interval $[0, \hat{\mu})$. Further, note $R_C^* < R_A^*$ for $\mu = 0$ as a consequence of Lemma D.3 and $R_C^* > R_A^*$ at $\hat{\mu}$ by Step 2. It follows that $R_C^* - R_A^* = 0$ has exactly one solution $\underline{\mu} \in (0, \hat{\mu})$ on the interval $[0, \hat{\mu})$. Next, we address the interval $(\hat{\mu}, 1]$. Because $R_A^* = \max\{\mu \Pi_0^H, \Pi_0^L\}$ by Lemma D.1 and $\hat{\mu} = \Pi_0^L / \Pi_0^H$ by definition, we have $R_A^* = \mu \Pi_0^H$ for all $\mu \in (\hat{\mu}, 1]$. Similar to the $\mu \in [0, \hat{\mu})$ case above, it follows that $R_C^* - R_A^* = \mu(r^a(\gamma^*) - \Pi_0^H) + (1 - \mu)r^b(\gamma^*)^+$ and $\gamma^* \leq \gamma_s^H$. Differentiating in μ yields

$$\begin{aligned}
\frac{d}{d\mu} \{R_C^* - R_A^*\} &= \frac{\partial}{\partial \gamma} \{R_C^* - R_A^*\} \frac{d\gamma^*}{d\mu} + \frac{\partial}{\partial \mu} \{R_C^* - R_A^*\} \\
&= \frac{\partial}{\partial \gamma} \{ \mu r^a(\gamma) + (1 - \mu) r^b(\gamma)^+ \} \frac{d\gamma^*}{d\mu} + r^a(\gamma^*) - r^b(\gamma^*)^+ \\
&= r^a(\gamma^*) - \Pi_0^H - r^b(\gamma^*)^+ \\
&< 0,
\end{aligned}$$

where the second line follows because $\frac{d}{d\mu} \Pi_0^H = 0$, the third line follows by a parallel argument to the $\mu \in [0, \hat{\mu})$ case above, and the final line follows because $r^a(\gamma) < \Pi_0^H$ for all $\gamma \in [0, \gamma^m]$ by Lemma D.2. It follows that $R_C^* - R_A^*$ strictly decreases in μ on $(\hat{\mu}, 1]$. Hence $R_C^* - R_A^* = 0$ has at most one solution on the interval $(\hat{\mu}, 1]$. Further, note $R_C^* < R_A^*$ at $\mu = 1$ by Lemma D.3 and $R_C^* > R_A^*$ at $\hat{\mu}$ by Step 2. It follows that $R_C^* - R_A^* = 0$ has exactly one solution $\bar{\mu} \in (\hat{\mu}, 1]$ on the interval $(\hat{\mu}, 1]$. Because $R_C^* - R_A^* = 0$ has one solution $\underline{\mu}$ on the interval $[0, \hat{\mu})$, one solution $\bar{\mu}$ on the interval $(\hat{\mu}, 1]$, and $R_C^* > R_A^*$ at $\hat{\mu}$, we conclude $R_C^* > R_A^*$ if and only if $\mu \in [\underline{\mu}, \bar{\mu}]$. $\square$

D.2 Proof of Proposition 4

Proposition 4. *Let R_A^* and R_C^* be the platform's revenue under the optimal pricing and information policies for access and commission fees, respectively. Let Π_0^i be the on-platform profit of a type- i seller under a commission rate of $\gamma = 0$. There exists $\bar{\phi} > 0$ such that the following statements hold.*

- (i) *Suppose the switching cost is low, $\phi \leq \bar{\phi}$. Then access fees generate higher revenue than commission fees $R_A^* \geq R_C^*$ for all $\mu \in [0, 1]$.*
- (ii) *Suppose the switching cost is high, $\phi > \bar{\phi}$. Then there exists $\underline{\mu} \in \left(0, \frac{\Pi_0^L}{\Pi_0^H}\right)$ and $\bar{\mu} \in \left(\frac{\Pi_0^L}{\Pi_0^H}, 1\right)$ such that access fees generate lower revenue than commission fees $R_A^* < R_C^*$ if and only if the share of type- H sellers is moderate $\mu \in [\underline{\mu}, \bar{\mu}]$.*

Proof. The proof of part (i) uses Lemmas D.2–D.4. The proof of part (ii) follows almost immediately from Lemma D.5; our focus here is to show the threshold $\bar{\phi}$ from Lemma D.5 and part (i) of the proposition statement are the same.

(i). Fix $\phi = 0$. By Lemma D.3, $R_A^* \geq R_C^*$ if either $r^b(\gamma^*) \leq 0$ or $\gamma^* > \gamma_s^H$, from which the result immediately follows. It remains to address the case where $r^b(\gamma^*) > 0$ and $\gamma^* \leq \gamma_s^H$. By Lemma D.4, if $R_A^* > R_C^*$ holds under $\mu = \hat{\mu}$, then $R_A^* > R_C^*$ also holds for any $\mu \in [0, 1]$, where $\hat{\mu} = \Pi_0^L / \Pi_0^H$. Therefore, it suffices to show $R_A^* > R_C^*$ at $\mu = \hat{\mu}$ and $\phi = 0$. We consider two cases defined by the piecewise upper bound on $\frac{r^b(\gamma)}{\Pi_0^L}$ given in Lemma D.2. First, suppose $\gamma^* \leq 2 - \sqrt{3}$. Because $r^b(\gamma^*) > 0$ and $\gamma^* \leq \gamma_s^H$, by Lemma A.11 the platform's optimal commission revenue is given by $R_C^* = \mu r^a(\gamma^*) + (1 - \mu)r^b(\gamma^*)^+$. We can then write

$$\begin{aligned}
R_C^* &= \hat{\mu} r^a(\gamma^*) + (1 - \hat{\mu}) r^b(\gamma^*)^+ \\
&\leq \gamma^* \frac{9}{7} \hat{\mu} \Pi_0^H + \frac{1}{2 - \gamma^*} (1 - \hat{\mu}) \Pi_0^L \\
&< \gamma^* \frac{9}{7} \hat{\mu} \Pi_0^H + \frac{1}{2 - \gamma^*} \Pi_0^L \\
&= \hat{\mu} \Pi_0^H \left(\gamma^* \frac{9}{7} + \frac{1}{2 - \gamma^*} \right) \\
&\leq \hat{\mu} \Pi_0^H \left((2 - \sqrt{3}) \frac{9}{7} + \frac{1}{\sqrt{3}} \right) \\
&< \hat{\mu} \Pi_0^H \\
&= R_A^*.
\end{aligned}$$

The relations above uses the bounds Lemma D.2, the definition of $\hat{\mu}$, and the observation that $\left(\gamma \frac{9}{7} + \frac{1}{2 - \gamma}\right)$ strictly increases in γ on $\gamma \in [0, 2 - \sqrt{3}]$. Next, suppose $\gamma^* \in [2 - \sqrt{3}, \gamma^m]$. Again applying the bounds from Lemma D.2, we have

$$\begin{aligned}
R_C^* &= \hat{\mu} r^a(\gamma^*) + (1 - \hat{\mu}) r^b(\gamma^*)^+ \\
&\leq \gamma^* \frac{9}{7} \hat{\mu} \Pi_0^H + \gamma \frac{(1 + \gamma^*)(1 - 3\gamma^*)}{(1 - \gamma^*)^2 (1 - 2\gamma^*)^2} \Pi_0^L \\
&= \hat{\mu} \Pi_0^H \underbrace{\left(\gamma^* \frac{9}{7} + \gamma^* \frac{(1 + \gamma^*)(1 - 3\gamma^*)}{(1 - \gamma^*)^2 (1 - 2\gamma^*)^2} \right)}_{g(\gamma^*)} \\
&< \hat{\mu} \Pi_0^H \\
&= R_A^*.
\end{aligned}$$

The fourth line follows because $g(\gamma^*) \leq 1$ in the interval $\gamma \in [2 - \sqrt{3}, \gamma^m]$, and the final line follows from Lemma D.1. It follows that $R_A^* > R_C^*$ for all $\mu \in [0, 1]$ at $\phi = 0$. Next, it is straightforward to verify that R_A^* does not depend on ϕ and R_C^* is continuous in ϕ . It follows that there exists $\bar{\phi} > 0$ such that for $\phi \leq \bar{\phi}$, $R_A^* \geq R_C^*$ for all $\mu \in [0, 1]$. Statement (i) follows. In the remainder of the proof, let $\bar{\phi}$ be the largest threshold such that $R_A^* \geq R_C^*$ holds for all $\mu \in [0, 1]$ and $\phi \leq \bar{\phi}$.

(ii). Let $\hat{\mu} = \frac{\Pi_0^L}{\Pi_0^H}$. By Lemma D.5, there exists $\underline{\phi} > 0$ and $\underline{\mu} \in (0, \hat{\mu})$ and $\bar{\mu} \in (\hat{\mu}, 1)$ such that if $\phi > \underline{\phi}$ then $R_A^* < R_C^*$ if and only if $\mu \in [\underline{\mu}, \bar{\mu}]$. Pick $\underline{\phi}$ to be the smallest such threshold. It remains to show $\underline{\phi} = \bar{\phi}$. Suppose by way of contradiction that $\underline{\phi} > \bar{\phi}$. Then there exists $\hat{\phi} \in (\bar{\phi}, \underline{\phi})$ such that $R_A^* < R_C^*$ holds at $\phi = \hat{\phi}$ for $\mu \in [\underline{\mu}, \bar{\mu}]$. Further, note that $R_A^* < R_C^*$ implies $\gamma^* \leq \gamma_s^H$ by Lemma D.3. Thus, using the expression for platform commission revenue (Lemma A.11), $R_A^* < R_C^*$ holds if

$$R_A^* < \max_{\alpha \in [\frac{1}{2}, 1], \gamma \leq \gamma_s^H} \mu r^a(\gamma) + (1 - \mu) r^b(\gamma)^+. \quad (39)$$

Next, note that because γ_s^H increases in ϕ (Lemma B.1), so does the right hand side of (39). It follows that (39) and thus $R_A^* < R_C^*$ hold for all $\phi \geq \hat{\phi}$. However, this yields a contradiction to the selection of $\underline{\phi}$ as the smallest such threshold. We conclude $\underline{\phi} = \bar{\phi}$. $\square$

E Proofs for Section 5.2: Should Platforms Ban Sellers?

E.1 Proof of Proposition 5

Before proving the main result, we first characterize the commission thresholds for disintermediation in each period, analogous to Lemma A.10.

Lemma E.1 (Commission thresholds for disintermediation). *For each $i \in \{L, H\}$, $\sigma \in \{r, s\}$, and $t \in \{1, 2\}$, there exists a unique threshold γ_σ^{it} such that a type- i seller disintermediates with a signal- σ buyer in period t if and only if $\gamma > \gamma_\sigma^{it}$. Further,*

- (i) $\gamma_\sigma^{i2} := 1 - \omega_\sigma$ for $\sigma \in \{r, s\}$, and
- (ii) γ_s^{i1} weakly increases in d on $d \in [0, 1]$, where $\gamma_s^{i1} \geq \gamma_s^{i2}$ for all $d \in [0, 1]$ and $\gamma_s^{i1} = \gamma_s^{i2}$ for $d = 0$.

Proof. We focus on proving the main lemma statement for the type- H seller, and note the results for the type- L seller follow by similar arguments. For (i), we assume without loss of generality that the seller is undetected in period 1; otherwise, no transaction occurs in period 2.

(i). The result follows closely from Lemmas A.2 and A.4. Let p and b be an arbitrary online and offline price in period 2, respectively. Because the game ends after period 2, the buyer's and seller's surplus from transacting offline are simply $p - b$ and $\omega_\sigma b - (1 - \gamma)p$, respectively, which yields a Nash bargaining solution of $b_\sigma(p) = \frac{p(1-\gamma+\omega_\sigma)}{2\omega_\sigma}$. Because $\phi = 0$, it follows by parallel argument to Lemma A.4 that both $p - b_\sigma(p) > 0$ and $\omega_\sigma b_\sigma(p) - (1 - \gamma)p > 0$ hold if and only if $\gamma > 1 - \omega_s$.

(ii). The result follows by parallel argument to the proof of Lemma A.10; we verify the main steps here. Analogous to Lemma A.7, it can be shown that Assumption 2 implies $\gamma_r^{i2} > \gamma^m$, meaning the type- i seller never transacts offline with the $\sigma = r$ buyer. Therefore, we focus on the threshold γ_s^{i2} , beginning with $i = H$. In particular, we proceed in two steps: First, we show there exists $\underline{\gamma}$ and $\bar{\gamma} > \underline{\gamma}$ such that the type- H seller transacts online with the $\sigma = s$ buyer if $\gamma < \underline{\gamma}$ and offline if $\gamma > \bar{\gamma}$. Second, we show there exists a unique $\gamma_s^H \in [\underline{\gamma}, \bar{\gamma}]$ such that the transaction is offline if and only if $\gamma > \gamma_s^H$.

Step 1. Let π^{x2} denote the type- H seller's expected optimal profit in period 2 conditional on non-detection in period 1, where $x \in \{a, c\}$ as per Lemma A.3. For online price p , offline price b , and detection probability d , the seller's surplus from transacting offline in period 1 is $\omega_\sigma b - d\pi^{x2} - (1 - \gamma)p$, and the buyer's surplus is again $p - b$. Solving for the Nash bargaining price yields $b_\sigma^x(p) = \frac{p(1-\gamma+\omega_\sigma)+d\pi^{x2}}{2\omega_\sigma}$. Analogous to Lemma A.4, it follows that the buyer and seller both have strictly positive surplus from transacting offline if and only if

$$\gamma > 1 - \omega_\sigma + \frac{d\pi^{x2}}{p}. \quad (40)$$

Note 40 implies $\gamma_\sigma^{i1} = \gamma_\sigma^{i2}$ for $i \in \{L, H\}$ and $\sigma = \{r, s\}$ follows. Further, analogous to Lemma A.9, note the inequalities $\gamma \leq 1 - \omega_\sigma + \frac{d\pi^{x2}}{p^a}$ and $\gamma > 1 - \omega_\sigma + \frac{d\pi^{x2}}{p^c}$ are a necessary and sufficient condition for the transaction to be online, respectively. Next, if the type- H seller transacts online with a $\sigma = s$ buyer in period

1, their expected profit over both periods is

$$\pi^a(p) := ((1-\gamma)p - (1-\lambda)c) \left(1 - \frac{p}{q_H}\right) + \pi^{x2}.$$

If the seller transacts offline with the $\sigma = s$ buyer in period 1, their expected profit over both periods is

$$\pi^c(p) := \left(1 - \frac{p}{q_H}\right) \left(((1-\gamma)p - (1-\lambda)c) + \eta_s(\omega_s b_s(p) - d\pi^{x2} - (1-\gamma)p) \right) + \pi^{x2}.$$

The unconstrained maximizers of $\pi^a(p)$ and $\pi^c(p)$ are then given by p^a and p^c , where

$$p^a := \frac{1}{2} \left(q_H + \frac{(1-\lambda)c}{1-\gamma} \right),$$

$$p^c := \frac{1}{2} \left(q_H + \frac{(1-\lambda)c}{\zeta} + \frac{\eta_s d \pi^{x2}}{2\zeta} \right),$$

and where $\zeta = \eta_r(1-\gamma) + \eta_s \frac{1-\gamma+\omega_s}{2}$. It remains to show there exists a unique threshold γ_s^{H1} such that $\pi^a(p^a) < \pi^c(p^c)$ if and only if $\gamma > \gamma_s^{H1}$. First, define $\bar{\gamma}$ to be the largest solution to $\gamma = 1 - \omega_s + \frac{d\pi^{x2}}{p^a}$. Because p^a increases in γ and π^{x2} decreases in γ , it follows that $\gamma > \bar{\gamma}$ implies (40) holds at $p = p^a$, and thus the type- H seller transacts offline with the $\sigma = s$ buyer. Similarly, define $\underline{\gamma}$ to be the smallest solution to

$$\gamma = 1 - \omega_s + \frac{d\pi^{x2}(\gamma)}{p^c} = 1 - \omega_s + \frac{d}{\frac{1}{2} \left(\frac{q_H}{\pi^{x2}} + \frac{(1-\lambda)c}{\pi^{x2}\zeta} + \frac{\eta_s d}{2\zeta} \right)}. \quad (41)$$

Note ζ and π^{x2} both decrease in γ , which implies the right hand side of (41) decreases in γ . As a result, for any $\gamma < \underline{\gamma}$, (40) cannot hold at $p = p^c$, which implies the transaction with the $\sigma = s$ buyer occurs online. We have thus shown the transaction between the type- H seller and $\sigma = s$ buyer occurs offline if $\gamma > \bar{\gamma}$ and online if $\gamma < \underline{\gamma}$. This completes Step 1.

Step 2. First, note $p^c \geq p^a$ for all $\gamma \in [\underline{\gamma}, \bar{\gamma}]$. To see why, suppose by contradiction that $p^c < p^a$ for some $\gamma' \in [\underline{\gamma}, \bar{\gamma}]$, and define

$$f(p, \gamma) := \gamma - \left(1 - \omega_s + \frac{d\pi^{x2}}{p} \right).$$

By definition of $\underline{\gamma}$ and $\bar{\gamma}$, we have $f(\gamma', p^a) < 0$ and $f(\gamma', p^c) \geq 0$. However, $f(p, \gamma)$ increases in p for each $\gamma \in (0, \gamma^m]$, which yields a contradiction. We conclude $p^c \geq p^a$ for all $\gamma \in [\underline{\gamma}, \bar{\gamma}]$. Analogous to the proof of Lemma A.10, we show that $\pi^a(p^a) - \pi^c(p^c)$ strictly decreases in γ on $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, from which the existence of the unique threshold γ_s^{H1} follows. Differentiating $\pi^a(p^a) - \pi^c(p^c)$ in γ , we have

$$\begin{aligned} \frac{d}{d\gamma}(\pi^a(p^a) - \pi^c(p^c)) &= \left(\frac{\partial \pi^a}{\partial p} \cdot \frac{dp^a}{d\gamma} + \frac{\partial \pi^a}{\partial \gamma} \right) \Big|_{p=p^a} - \left(\frac{\partial \pi^c}{\partial p} \cdot \frac{dp^c}{d\gamma} + \frac{\partial \pi^c}{\partial \gamma} \right) \Big|_{p=p^c} \\ &= \frac{\partial \pi^a}{\partial \gamma} \Big|_{p=p^a} - \frac{\partial \pi^c}{\partial \gamma} \Big|_{p=p^c} \\ &= -p^a \left(1 - \frac{p^a}{q_H} \right) + \left(1 - \frac{\eta_s}{2} \right) p^c \left(1 - \frac{p^c}{q_H} \right) + d \left(1 - \frac{p^c}{q_H} \right) \eta_s \frac{\partial \pi^{x2}}{\partial \gamma} \\ &\leq \left(\left(1 - \frac{\eta_s}{2} \right) p^c - p^a \right) \left(1 - \frac{p^a}{q_H} \right). \end{aligned}$$

The second line above follows from the envelope theorem and the third line follows from evaluating the derivative algebraically. To see that the inequality in the fourth line holds, note π^{x2} decreases in γ and $p^c \geq p^a$

for $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, as established above. Since $\left(1 - \frac{p^a}{q}\right) > 0$ and $\eta_s \in (0, 1)$, to show $\frac{d}{d\gamma}(\pi^a(p^a) - \pi^c(p^c)) < 0$ it suffices to show that $2p^a - p^c > 0$. The preceding inequality follows immediately from the observations that $p^c \leq q$ and $2p^a > q$. This establishes the existence of the unique threshold γ_s^{H1} . Finally, to see that γ_s^{H1} increases in d , note that $\pi^a(p^a)$ is independent of d , and $\pi^c(p^c)$ can be written as

$$\pi^c(p^c) = q_H \zeta \left(\frac{1}{2} - \frac{(1-\lambda)c}{2q_H \zeta} - \frac{\eta_s d \pi^{x2}}{4q_H \zeta} \right)^2.$$

Clearly, $\pi^c(p^c)$ decreases in d wherever $\pi^c(p^c) > 0$. Because γ_s^{H1} is the solution to $\pi^a(p^a) = \pi^c(p^c)$ and $\pi^a(p^a) - \pi^c(p^c)$ decreases in γ on $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, γ_s^{H1} also decreases in d . The proof for the threshold γ_s^{L1} follows by parallel argument and is omitted. $\square$

Proposition 5. *Let $R^0(\gamma^*)$ and $R^d(\gamma^*)$ be the platform's optimal revenue under the blind eye and banning policies, respectively. Then, there exist thresholds $\underline{\alpha} \in (\frac{1}{2}, 1]$ and $\bar{\alpha} \in [\underline{\alpha}, 1)$ such that the following statements hold.*

- (i) *Suppose information quality is low $\alpha \leq \underline{\alpha}$. Then for all detection probabilities $d \in [0, 1]$, the banning policy generates weakly higher revenue than the blind eye policy, $R^0(\gamma^*) \leq R^d(\gamma^*)$.*
- (ii) *Suppose information quality is high $\alpha \geq \bar{\alpha}$. Then there exists $\bar{d} \in (0, 1)$ such that if the detection probability is low $d \in (0, \bar{d}]$, the banning policy generates strictly lower revenue than the blind eye policy, i.e., $R^0(\gamma^*) > R^d(\gamma^*)$.*

Proof. (i). First suppose $\alpha = \frac{1}{2}$ and $d = 0$. Then by Lemma E.1 and Assumption 2, we have $\gamma^m \leq \gamma_s^{it}$ for $i \in \{L, H\}$ and $t \in \{1, 2\}$. Further, because γ_s^{i1} and γ_s^{i2} are weakly increasing in and independent of d , respectively, it follows by continuity of γ_s^{i1} and γ_s^{i2} in α that there exists $\underline{\alpha} \in (0, 1]$ such that if $\alpha \leq \underline{\alpha}$, then $\gamma^* \leq \gamma_s^{it}$ for $i \in \{L, H\}$, $t \in \{1, 2\}$ and all $d \in [0, 1]$, i.e., all transactions occur online. Then by Lemma E.1, the platform's revenue is given by

$$R(\gamma^*) = 2(\mu r^a(\gamma^*) + (1 - \mu)r^b(\gamma^*))^+,$$

where $r^a(\gamma)$ and $r^b(\gamma)$ are as defined in Lemma A.11. It follows that for $\alpha \leq \underline{\alpha}$ and $d \in [0, 1]$, we have

$$\frac{dR}{dd} \Big|_{\gamma=\gamma^*} = \left(\frac{\partial R}{\partial \gamma} \frac{d\gamma^*}{dd} + \frac{\partial R}{\partial d} \right) \Big|_{\gamma=\gamma^*} = \left(\frac{\partial R}{\partial \gamma} \frac{d\gamma^*}{dd} \right) \Big|_{\gamma=\gamma^*} \geq 0. \quad (42)$$

The second equality above follows because $r^a(\gamma)$ and $r^b(\gamma)$ are both independent of d , which implies $\frac{\partial}{\partial d} R = 0$. To see that the inequality holds, note $\gamma_s^{i1} \geq \gamma_s^{i2}$ holds for all $d \in [0, 1]$ by Lemma E.1. Thus, there are two cases to consider: $\gamma^* < \min\{\gamma_s^{L2}, \gamma_s^{H2}\}$ and $\gamma^* = \min\{\gamma_s^{L2}, \gamma_s^{H2}\}$. If $\gamma^* < \min\{\gamma_s^{L2}, \gamma_s^{H2}\}$, then $\frac{\partial}{\partial \gamma} R = 0$ holds at $\gamma = \gamma^*$ by the envelope theorem, and the weak inequality in (42) holds as an equality. If $\gamma^* = \min\{\gamma_s^{L2}, \gamma_s^{H2}\}$, then $\frac{\partial}{\partial \gamma} R \geq 0$ must hold at $\gamma = \gamma^*$ because γ^* is the optimal commission rate. Further, by Lemma E.1 both γ_s^{L2} and γ_s^{H2} weakly increase in d , which implies $\frac{d}{dd} \gamma^* \geq 0$. The inequality in (42) again follows.

(ii). First suppose $\alpha = 1$ and $d = 0$. Then by Lemma E.1 we have $\gamma^* > \gamma_s^{it} = 0$ for $i \in \{L, H\}$ and $t \in \{1, 2\}$, i.e., all transactions with $\sigma = s$ buyers occur offline. It follows by continuity of γ_s^{it} in α and d that there exists $\bar{\alpha} \in [\underline{\alpha}, 1)$ and $\bar{d} > 0$ such that $\gamma^* > \gamma_s^{it}$ for $i \in \{L, H\}$ and $t \in \{1, 2\}$ if $\alpha \geq \bar{\alpha}$ and $d \leq \bar{d}$. Note that $\gamma^* > \gamma_s^{it}$ for $i \in \{L, H\}$ and $t \in \{1, 2\}$ implies the platform only extracts commissions from type- H sellers. Further,

type- H sellers are banned after period 1 with probability d only if they transact with a $\sigma = s$ buyer in period 1, and thus the probability a type- H seller is undetected following period 1 is $\eta_r + \eta_s(1 - d)$. It follows from Lemma E.1 that the platform's revenue for $\alpha \geq \bar{\alpha}$ and $d \leq \bar{d}$ is

$$R(\gamma^*) = \mu r^c(\gamma^*) + \mu(\eta_r + \eta_s(1 - d))r^c(\gamma^*),$$

where

$$r^c(\gamma) = \gamma \eta_r p^c \left(1 - \frac{p^c}{q_H} \right) = \gamma \eta_r q_H \left(\frac{1}{4} - \left(\frac{(1 - \lambda)c}{2q_H \zeta} + \frac{\eta_s d \pi^{x2}}{4q_H \zeta} \right)^2 \right). \quad (43)$$

It follows that for $\alpha \geq \bar{\alpha}$ and $d \leq \bar{d}$,

$$\left. \frac{dR}{dd} \right|_{\gamma=\gamma^*} = \left(\frac{\partial R}{\partial \gamma} \frac{d\gamma^*}{dd} + \frac{\partial R}{\partial d} \right) \Big|_{\gamma=\gamma^*} = \left. \frac{\partial R}{\partial d} \right|_{\gamma=\gamma^*} = \left(-\mu \eta_s r^c(\gamma) + \mu(1 + \eta_r + \eta_s(1 - d)) \frac{\partial r^c}{\partial \gamma} \right) \Big|_{\gamma=\gamma^*} < 0.$$

To see the second equality above, note that if $\gamma^* = \gamma^m$, then $\frac{d}{dd}\gamma^* = 0$, and if $\gamma^* < \gamma^m$ then $\frac{\partial}{\partial \gamma} R = 0$ holds at $\gamma = \gamma^*$ by the envelope theorem. The strict inequality follows because $\frac{\partial}{\partial d} r^c < 0$ by inspection of the expression for $r^c(\gamma)$ in (43). Because $R(\gamma^*)$ strictly decreases in d on $d \in [0, \bar{d}]$, statement (ii) follows. $\square$

F Repeat Interactions with Returning Buyers

Our main model examines disintermediation in a single-shot setting where sellers transact with a given buyer at most once. In practice, sellers may interact with the same buyer repeatedly, allowing them to learn the buyer's type from earlier transactions. In this section, we consider a dynamic variant of our main model, in which sellers form beliefs about buyers' types through an initial transaction, instead of through the platform signal. The purpose of this section is to establish that our main insights hold when information about buyers is transmitted in this alternative manner.

F.1 Model Setup

Formally, we consider a two-period model where each seller is matched to the same buyer in both periods, and a share $1 - \lambda$ of buyers are type- r (i.e., risky). As before, type- s buyers impose the transaction cost $c_s = 0$ on sellers. However, in contrast to our main model, the transaction cost imposed by type- r buyers is stochastic in each period, where $c_r = c > 0$ with probability $\rho \in [0, 1]$ and $c_r = 0$ with probability $1 - \rho$. As a consequence of this cost structure, when the realized transaction cost is c , the seller immediately learns the buyer is type- r ; when the realized transaction cost is 0, the seller's posterior belief that the buyer is type- r is $\frac{(1-\lambda)(1-\rho)}{(1-\lambda)(1-\rho)+\lambda}$ (see Lemma F.1). The parameter ρ thus captures sellers' *ability to learn* - as ρ increases, sellers form stronger posterior beliefs and can distinguish buyer types more accurately. To isolate the effect of the learning parameter ρ , we assume the platform's signal is uninformative ($\alpha = \frac{1}{2}$).

At the beginning of the horizon, the platform sets the commission rate γ . Each seller then commits to an online price p for both periods, and is randomly matched to one buyer. In each period, a seller's choices are to transact online, offline, or not at all. If the seller rejects the buyer in period 1, they earn zero profit and are not matched to a new buyer. This setup assumes that a seller's decision to complete a follow-up transaction with a buyer is independent of potential matches with new buyers (for example, in freelance marketplaces, sellers typically juggle multiple projects simultaneously, so the possibility of future contracts does not impact

transactions with current buyers). We adopt notation from the main model and re-define σ to denote the signal generated by the period 1 transaction, where the seller observes $\sigma = r$ if the realized transaction cost from period 1 is $c > 0$ and observes $\sigma = s$ if the realized cost is 0.

To facilitate our analysis, we make two assumptions that are analogous to Assumptions 1 and 2 from the main model. First, we again assume that the two seller types are well-separated with respect to their quality level:

Assumption 3. *The seller qualities satisfy $q_H \geq 4c$ and $q_L \in \left[\frac{2(1-\lambda)(2-\rho)\rho}{2-(1-\lambda)\rho}c, \rho c \right]$.*

The above assumption implies that the type- H seller transacts with all buyers, whereas the type- L seller always rejects the $\sigma = r$ buyer. In addition, we impose the requirement the quality level of the type- L seller is high enough to guarantee that this seller transacts on the platform in period 1 for every value of $\gamma \in [0, \gamma^m]$; this assumption precludes the less interesting case where only the type- H seller participates. We also impose an assumption that allows us to focus on the case where sellers do not disintermediate prior to forming a belief about the buyer's type (i.e., in period 1).

Assumption 4. *The maximum commission rate γ^m , share of type- H sellers λ , switching cost ϕ , and type- H seller quality q_H satisfy the inequality $\gamma^m \leq 1 - \lambda + \frac{\phi}{q_H}$.*

Assumption 4 is similar to Assumption 2 from the main model, and similarly implies that sellers do not disintermediate with $\sigma = r$ buyers.

Although we assume the learning parameter ρ to be exogenous, in practice it may be impacted by platform design decisions. For example, in the context of freelance marketplaces like Upwork, a low value of ρ may be the result of features that smooth out differences in how buyers interact with sellers, including customer support, a common structure on job postings, or AI-assisted communication (Upwork 2024a). Similarly, a high value of ρ may correspond to a less regulated environment in which sellers can more accurately screen buyers, as a consequence of risky buyers being more likely to “reveal” themselves. Note that ρ plays a similar role to α from our main model, since it captures the accuracy with which sellers can learn buyers' types.

F.2 Seller Learning and Platform Revenue

With repeated interactions, it is natural to expect the threat of disintermediation to depend on the accuracy with which sellers can infer buyers' types from an initial transaction. To the extent that platforms can influence seller learning (e.g., through features or policies that change how buyers and sellers interact), it is valuable to understand the impact of the sellers' learning parameter ρ on platform revenue.

Proposition 6. *Suppose the switching cost is $\phi = 0$. Then there exists $\bar{\mu} \in [0, 1)$ and $\bar{\rho} \in [0, 1)$ such that if the share of type- H sellers is large $\mu \geq \bar{\mu}$, the platform's optimal revenue $R(\gamma^*)$ strictly decreases in the learning parameter ρ on $\rho \in [0, \bar{\rho})$ and strictly increases in ρ on $\rho \in (\bar{\rho}, 1]$.*

All proofs for this section are in Appendix F.4. Similar to our prior results, the non-monotonic behavior in Proposition 6 can be understood by considering the effects induced by a change in the learning parameter ρ , which we briefly outline. When ρ is small ($\rho < \bar{\rho}$), sellers cannot easily identify safe (type- s) buyers, leading all transactions to occur online in equilibrium in both periods. In this case, an increase in ρ increases the threat of disintermediation, which degrades the platform's pricing power and lowers revenue. However, when ρ is large ($\rho > \bar{\rho}$), sellers disintermediate in period 2 with the zero-cost buyers (which consists of both type- r

and type- s buyers), but transact on-platform with the costly buyers (consisting only of type- r buyers). In this setting, further increases in ρ allow sellers to identify type- r buyers with increased accuracy and avoid transacting off-platform with them, which increases on-platform transaction volume and boosts revenue. In summary, although the ability to screen buyers is necessary for disintermediation, Proposition 6 suggests that in settings where sellers already transact off-platform (large ρ), further improvements to seller learning can help revenue by stemming the flow of any additional disintermediation.

F.3 Optimal Commission Rate

We conclude this section by showing that a variation of our main result from Proposition 1 holds when sellers learn through an initial transaction instead of via the platform signal.

Proposition 7. *Let $\gamma^*(\phi)$ be the optimal commission rate under switching cost ϕ . There exists $\bar{\phi} > 0$ and $\bar{\rho} \in [0, 1)$ such that for any $\rho \geq \bar{\rho}$ and $\phi \geq \bar{\phi}$, the optimal commission rate is higher in the absence of switching costs, $\gamma^*(0) \geq \gamma^*(\phi)$, where the inequality is strict if $\gamma^*(\phi) < \gamma^m$.*

Proposition 7 mirrors Proposition 1(ii) by showing that in a disintermediation-prone environment (i.e., no switching cost and high ρ), it is optimal for the platform to “double down” on the on-platform transactions - that is, choose a commission rate higher than the corresponding rate when there is no disintermediation. The intuition follows similarly to our discussion in Section 3.1.

F.4 Proofs

This section contains the proofs for Propositions 6 and 7. We first provide some supporting lemmas that are analogous to those presented in Appendix A for the main model.

F.4.1 Preliminary Results

First, Lemmas F.1 – F.5 below characterize the relevant probabilities, commission thresholds for disintermediation, the sellers’ profit functions, and the platform’s revenue function. These results are analogous to those presented in Appendix A for the main model; the proofs follow similarly and are omitted to avoid repetition.

Lemma F.1 (Signal probabilities and sellers’ beliefs). *The following statements hold for $\sigma \in \{r, s\}$.*

- (i) *The probability a seller receives the signal σ following the period 1 transaction is η_σ , where $\eta_s := \lambda + (1 - \lambda)(1 - \rho)$ and $\eta_r := (1 - \lambda)\rho$.*
- (ii) *The seller’s posterior belief that a buyer with signal σ has true type $j = s$ is ω_σ , where $\omega_s := \lambda/\eta_s$ and $\omega_r := 0$.*
- (iii) *The probability a buyer with signal σ pays the seller if transacting offline is ω_σ .*

Lemma F.2 (Disintermediation thresholds fixed p). *Let Assumption 4 hold. In period 1, all transactions occur online for all $\gamma \in [0, \gamma^m]$. In period 2, given an online price $p > 0$, both seller types transact offline with the $\sigma = s$ buyer if and only if $\gamma \geq \hat{\gamma}_s(p)$, where*

$$\hat{\gamma}_s(p) := 1 - \omega_s + \frac{\phi}{p}$$

and the offline price is given by

$$b_s(p) := \frac{p(1 - \gamma + \omega_s) + \phi}{2\omega_s}.$$

Further, neither seller transacts offline with the $\sigma = r$ buyer in period 2 for all $\gamma \in [0, \gamma^m]$ and $p > 0$.

Lemma F.3 (Sellers' profit and price cases). *Fix the commission rate γ and consider a unit mass of sellers with quality q and online price $p \leq q$. Let $\Pi(p)$ be the sellers' profit and let $\tilde{p}$ be the maximizer of $\Pi(p)$.*

(i) *If the sellers transacts online in period 2 with both $\sigma = r$ and $\sigma = s$ buyers,*

$$\begin{aligned}\Pi(p) &= \pi^a(p) := 2((1 - \gamma)p - (1 - \lambda)\rho c) \left(1 - \frac{p}{q}\right) \\ \tilde{p} &= p^a := \frac{1}{2} \left(q + \frac{\rho c(1 - \lambda)}{1 - \gamma} \right).\end{aligned}$$

(ii) *If the sellers reject $\sigma = r$ and transact online with $\sigma = s$,*

$$\begin{aligned}\Pi(p) &= \pi^b(p) := ((1 - \gamma)p - (1 - \lambda)\rho c + \eta_s((1 - \gamma)p - (1 - \omega_s)\rho c)) \left(1 - \frac{p}{q}\right) \\ \tilde{p} &= p^b := \frac{1}{2} \left(q + \frac{\rho c(1 - \lambda)(2 - \rho)}{(1 - \gamma)(2 - (1 - \lambda)\rho)} \right).\end{aligned}$$

(iii) *If the seller transact online with $\sigma = r$ and offline with $\sigma = s$,*

$$\begin{aligned}\Pi(p) &= \pi^c(p) := ((1 - \gamma)p - (1 - \lambda)\rho c + \eta_s(\omega_s b - (1 - \omega_s)\rho c - \phi) + \eta_r((1 - \gamma)p - \rho c)) \left(1 - \frac{p}{q}\right) \\ \tilde{p} &= p^c := \frac{1}{2} \left(q + \frac{4\rho c(1 - \lambda) + \phi(1 - (1 - \lambda)\rho)}{(1 - \gamma)(3 + (1 - \lambda)\rho) + \lambda} \right).\end{aligned}$$

(iv) *If the sellers reject $\sigma = r$ and transact offline with $\sigma = s$,*

$$\begin{aligned}\Pi(p) &= \pi^d(p) := ((1 - \gamma)p - (1 - \lambda)\rho c + \eta_s(\omega_s b - (1 - \omega_s)\rho c - \phi)) \left(1 - \frac{p}{q}\right) \\ \tilde{p} &= p^d := \frac{1}{2} \left(q + \frac{2\rho c(1 - \lambda)(2 - \rho) + \phi(1 - (1 - \lambda)\rho)}{(1 - \gamma)(3 - (1 - \lambda)\rho) + \lambda} \right).\end{aligned}$$

Lemma F.4 (Disintermediation threshold and platform revenue). *For each seller type $i \in \{L, H\}$, there exists a unique threshold γ_s^i such that the type- i seller transacts offline with the $\sigma = s$ buyer if and only if $\gamma > \gamma_s^i$. Further, $\gamma_s^H \leq \gamma_s^L$ for all $\phi \geq 0$, and $\gamma_s^H = \gamma_s^L = 1 - \omega_s$ if $\phi = 0$.*

Lemma F.5 (Platform's revenue function). *Let p^x for $x \in \{a, b, c, d\}$ be as defined in Lemma F.3, and define*

$$\begin{aligned}r^a(\gamma) &:= 2\gamma p^a \left(1 - \frac{p^a}{q_H}\right), \\ r^b(\gamma) &:= \gamma(1 + \lambda + (1 - \lambda)(1 - \rho))p^b \left(1 - \frac{p^b}{q_L}\right), \\ r^c(\gamma) &:= \gamma(1 + (1 - \lambda)\rho)p^c \left(1 - \frac{p^c}{q_H}\right), \\ r^d(\gamma) &:= \gamma p^d \left(1 - \frac{p^d}{q_L}\right).\end{aligned}$$

Then the platform's commission revenue is given by $R(\gamma)$, where

$$R(\gamma) := \begin{cases} \mu r^a(\gamma) + (1 - \mu)r^b(\gamma)^+ & \text{if } \gamma \in [0, \gamma_s^H], \\ \mu r^c(\gamma) + (1 - \mu)r^b(\gamma)^+ & \text{if } \gamma \in (\gamma_s^H, \gamma_s^L], \\ \mu r^c(\gamma) + (1 - \mu)r^d(\gamma)^+ & \text{if } \gamma \in (\gamma_s^L, \gamma^m], \end{cases}$$

and $x^+ = \max\{0, x\}$.

F.4.2 Proof of Proposition 6

Before presenting the proof of Proposition 6, we first present Lemmas F.6 and F.7 which describe useful properties of the platform's revenue function and the optimal commission rate, respectively. For the remainder of Appendix F.4, define $\gamma^x := \operatorname{argmax}_{\gamma \in [0, 1]} r^x(\gamma)$ and $\gamma^{xy} := \operatorname{argmax}_{\gamma \in [0, \gamma^m]} \{\mu r^x(\gamma) + (1 - \mu)r^y(\gamma)\}$, where $x, y \in \{a, b, c\}$ and the $r^x(\gamma)$ functions are defined in Lemma F.5.

Lemma F.6 (Revenue function properties). *For any $\gamma \in (0, \frac{1}{2}]$ and $\lambda \in [\frac{1}{2}, 1)$, (i) $r^a(\gamma)$ and $r^b(\gamma)$ both strictly decrease in ρ on $\rho \in [0, 1]$, and (ii) $r^c(\gamma)$ strictly increases in ρ on $\rho \in [0, 1]$.*

Proof. (i). First note for any $\gamma \in (0, \gamma^m]$,

$$\frac{\partial r^a}{\partial \rho} = \frac{\partial}{\partial \rho} \left\{ \frac{\gamma}{2} \left(q_H - \frac{(c(1 - \lambda)\rho)^2}{q_H(1 - \gamma)^2} \right) \right\} = -\frac{c^2\gamma(1 - \lambda)^2\rho}{(1 - \gamma)^2q_H} < 0.$$

For $r^b(\gamma)$, we have

$$\begin{aligned} \frac{\partial r^b}{\partial \rho} &= \frac{\partial}{\partial \rho} \left\{ \frac{\gamma}{4} \left(q_L(2 - (1 - \lambda)\rho) - \frac{(c(1 - \lambda)(2 - \rho)\rho)^2}{q_L(1 - \gamma)^2(2 - (1 - \lambda)\rho)} \right) \right\} \\ &= \frac{\gamma}{4} \left(-q_L(1 - \lambda) - \frac{c^2(1 - \lambda)^2(2 - \rho)\rho(8 + 3(1 - \lambda)\rho^2 - 2(5 - \lambda)\rho)}{q_L(1 - \gamma)^2(2 - (1 - \lambda)\rho)^2} \right) \\ &< 0, \end{aligned}$$

where the strict inequality follows because $\rho \in [0, 1]$ and $\lambda \in [\frac{1}{2}, 1]$ imply $8 + 3(1 - \lambda)\rho^2 > 2(5 - \lambda)\rho$. Thus, $r^a(\gamma)$ and $r^b(\gamma)$ both strictly decrease in ρ .

(ii). To prove the result, we first show that $\frac{\partial}{\partial \rho} r^c > 0$ at $\rho = 1$. We then show $\frac{\partial^2}{\partial \rho^2} r^c \leq 0$ for all $\rho \in [0, 1]$, which implies $\frac{\partial}{\partial \rho} r^c > 0$ for all $\rho \in [0, 1]$. To begin, note

$$\begin{aligned} \frac{\partial r^c}{\partial \rho} &= \frac{\gamma}{4} \cdot \frac{\partial}{\partial \rho} \left\{ (1 + (1 - \lambda)\rho) \left(q_H - \frac{(4c(1 - \lambda)\rho)^2}{q_H((1 - \gamma)(3 + (1 - \lambda)\rho) + \lambda)^2} \right) \right\} \\ &= \frac{\gamma}{4} \left((1 - \lambda) \left(q_H - \frac{(4c(1 - \lambda)\rho)^2}{q_H((1 - \gamma)(3 + (1 - \lambda)\rho) + \lambda)^2} \right) - (1 + (1 - \lambda)\rho) \left(\frac{32c^2(1 - \lambda)^2(3(1 - \gamma) + \lambda)\rho}{q_H((1 - \gamma)(3 + (1 - \lambda)\rho) + \lambda)^3} \right) \right) \\ &\geq \gamma c \left((1 - \lambda) \left(1 - \frac{((1 - \lambda)\rho)^2}{((1 - \gamma)(3 + (1 - \lambda)\rho) + \lambda)^2} \right) - (1 + (1 - \lambda)\rho) \left(\frac{2(1 - \lambda)^2(3(1 - \gamma) + \lambda)\rho}{((1 - \gamma)(3 + (1 - \lambda)\rho) + \lambda)^3} \right) \right), \end{aligned} \tag{44}$$

where the third line above follows because $\frac{d}{d\rho} r^c$ increases in q_H and $q_H \geq 4c$ by Assumption 3. Then

substituting $\rho = 1$,

$$\left(\frac{\partial r^c}{\partial \rho} \right) \Big|_{\rho=1} \geq \gamma^c \left((1-\lambda) \left(1 - \frac{(1-\lambda)^2}{((1-\gamma)(4-\lambda)+\lambda)^2} \right) - (2-\lambda) \left(\frac{2(1-\lambda)^2(3(1-\gamma)+\lambda)}{((1-\gamma)(4-\lambda)+\lambda)^3} \right) \right). \quad (45)$$

Next, it can be shown algebraically that (45) holds if $g(\gamma) > 0$, where

$$g(\gamma) := (1-\lambda) \left(((1-\gamma)(4-\lambda)+\lambda)^2 - (1-\lambda)^2 \right) ((1-\gamma)(4-\lambda)+\lambda) - (2-\lambda) \left(2(1-\lambda)^2(3(1-\gamma)+\lambda) \right).$$

With some effort, it can be verified that $\frac{\partial^2}{\partial \gamma^2} g > 0$ and $\lim_{\gamma \rightarrow \frac{1}{2}} \frac{\partial}{\partial \gamma} g \leq 0$, which together imply $g(\gamma)$ strictly decreases in γ on $\gamma \in [0, \gamma^m]$. It is straightforward to verify that $g(\gamma^m) > 0$ because $\lambda \in [\frac{1}{2}, 1)$ and $\gamma^m \leq \frac{1}{2}$. It follows that $g(\gamma) > 0$ for all $\gamma \in [0, \gamma^m]$. Therefore, $\frac{\partial}{\partial \rho} r^c > 0$ at $\rho = 1$. It remains to show $\frac{\partial^2}{\partial \rho^2} r^c \leq 0$. Following (44), we have

$$\begin{aligned} \frac{\partial r^c}{\partial \rho} &\geq \gamma^c \left((1-\lambda) \left(1 - \frac{((1-\lambda)\rho)^2}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^2} \right) - (1+(1-\lambda)\rho) \left(\frac{2(1-\lambda)^2(3(1-\gamma)+\lambda)\rho}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3} \right) \right) \\ &= \gamma^c (1-\lambda) \left(1 - \frac{((1-\lambda)\rho)^2}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^2} - (1+(1-\lambda)\rho) \left(\frac{2(1-\lambda)(3(1-\gamma)+\lambda)\rho}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3} \right) \right) \\ &= \gamma^c (1-\lambda) \left(1 - \frac{(1-\lambda)\rho}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3} \cdot k(\rho) \right) \\ &= \gamma^c (1-\lambda) (1 - h(\rho) \cdot k(\rho)), \end{aligned}$$

where

$$\begin{aligned} k(\rho) &:= (1-\lambda)\rho((1-\gamma)(3+(1-\lambda)\rho)+\lambda) + 2(1+(1-\lambda)\rho)(3(1-\gamma)+\lambda), \\ h(\rho) &:= \frac{(1-\lambda)\rho}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3}. \end{aligned}$$

Observe that $k(\rho)$ weakly increases in ρ for all $\gamma \in [0, \frac{1}{2}]$ and $\lambda \in [\frac{1}{2}, 1]$. It remains to show $h(\rho)$ also increases in ρ . Note

$$\frac{\partial h}{\partial \rho} = \frac{(1-\lambda)((1-\gamma)(3+(1-\lambda)\rho)+\lambda) - 3\rho(1-\gamma)(1-\lambda)}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^4} = \frac{(1-\lambda)(3(1-\gamma)+\lambda - 2\rho(1-\gamma)(1-\lambda))}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^4} \geq 0,$$

where the inequality follows because $\gamma \in [0, \frac{1}{2}]$ and $\lambda \in [\frac{1}{2}, 1]$. The result follows. $\square$

Lemma F.7 (Optimal commission rate). *Suppose that $\mu = 1$ and $\phi = 0$. Then there exists $\underline{\rho} \in [0, 1)$ and $\bar{\rho} \in (\underline{\rho}, 1)$ such that $\gamma^* > \gamma_s^H$ if $\rho > \bar{\rho}$, $\gamma^* = \gamma_s^H$ if $\rho \in [\underline{\rho}, \bar{\rho}]$, and $\gamma^* = \gamma^m$ if $\rho < \underline{\rho}$.*

Proof. Note γ_s^H is continuous and decreasing in ρ and $\lim_{\rho \rightarrow 0} \gamma_s^H = 1 - \lambda \geq \gamma^m$. It follows that there exists $\underline{\rho} \in (0, 1]$ such that $\gamma_s^H < \gamma^m$ if and only if $\rho > \underline{\rho}$. Therefore, if $\rho \leq \underline{\rho}$, then platform revenue is given by $R(\gamma) = r^a(\gamma)$ for $\gamma \leq \gamma^m$. Because $r^a(\gamma)$ strictly increases in γ (Lemma A.12), we have $\gamma^* = \gamma^m$ for $\rho \leq \underline{\rho}$, as desired. The remainder of the proof addresses the interval $(\underline{\rho}, 1]$. We focus on showing there exists $\bar{\rho} \in (\underline{\rho}, 1)$ such that $\gamma^* > \gamma_s^H$ if and only if $\rho > \bar{\rho}$; we later strengthen this result to show $\gamma^* = \gamma_s^H$ if $\rho \in (\underline{\rho}, \bar{\rho}]$. For $\mu = 1$, the platform's revenue is

$$R(\gamma) = \begin{cases} r^a(\gamma) & \text{if } \gamma \leq \gamma_s^H, \\ r^c(\gamma) & \text{if } \gamma > \gamma_s^H. \end{cases}$$

Next, for convenience define the function

$$h(\rho) := \max_{\gamma \geq \gamma_s^H} r^c(\gamma) - \max_{\gamma \leq \gamma_s^H} r^a(\gamma),$$

Note $\gamma^* > \gamma_s^H$ if and only if $h(\rho) > 0$. To see why, note $\gamma^* > \gamma_s^H$ immediately implies $h(\rho) > 0$ by definition of $R(\gamma)$. For the reverse direction, it is straightforward to show $r^a(\gamma) > r^c(\gamma)$ for all $\gamma \in (0, \gamma^m]$, meaning $r^c(\gamma)$ cannot attain its maximum over $\gamma \geq \gamma_s^H$ at $\gamma = \gamma_s^H$. Thus, $h(\rho) > 0$ implies $\gamma^* > \gamma_s^H$. It remains to show there exists $\bar{\rho} \in (\underline{\rho}, 1)$ such that $h(\rho) > 0$ if and only if $\rho \geq \bar{\rho}$. Note for fixed γ , $r^a(\gamma)$ strictly decreases in ρ and $r^c(\gamma)$ strictly increases in ρ (Lemma F.6). Further, γ_s^H strictly decreases in ρ . It follows that $\max_{\gamma \leq \gamma_s^H} r^a(\gamma)$ strictly decreases in ρ and $\max_{\gamma \geq \gamma_s^H} r^c(\gamma)$ strictly increases in ρ . Hence, $h(\rho)$ strictly increases in ρ . Further, because $\lim_{\rho \rightarrow \underline{\rho}} \gamma_s^H = \gamma^m$ and $\lim_{\rho \rightarrow 1} \gamma_s^H = 0$, we have $\lim_{\rho \rightarrow \underline{\rho}} h(\rho) = -\max_{\gamma \leq \gamma^m} r^a(\gamma) < 0$ and $\lim_{\rho \rightarrow 1} h(\rho) = \max_{\gamma \geq 0} r^c(\gamma) > 0$. It follows that there exists $\bar{\rho} \in (\underline{\rho}, 1)$ such that $h(\rho) > 0$ if and only if $\rho > \bar{\rho}$. We conclude $\gamma^* > \gamma_s^H$ if and only if $\rho > \bar{\rho}$. Lastly, note that $\rho \in (\underline{\rho}, \bar{\rho}]$ implies $\gamma^* = \arg\max_{\gamma \leq \gamma_s^H} r^a(\gamma)$. Using the expressions in Lemma F.3, it is straightforward to show $r^a(\gamma)$ strictly increases in γ . It follows that $\gamma^* = \gamma_s^H$ for $\rho \in (\underline{\rho}, \bar{\rho}]$. $\square$

Proposition 6. *Suppose the switching cost is $\phi = 0$. Then there exists $\bar{\mu} \in [0, 1)$ and $\bar{\rho} \in [0, 1)$ such that if the share of type-H sellers is large $\mu \geq \bar{\mu}$, the platform's optimal revenue $R(\gamma^*)$ strictly decreases in the learning parameter ρ on $\rho \in [0, \bar{\rho})$ and strictly increases in ρ on $\rho \in (\bar{\rho}, 1]$.*

Proof. The proof largely follows from Lemmas F.6 and F.7. Let $\mu = 1$, and let $\bar{\rho}$ be as defined in Lemma F.7. First suppose $\rho \leq \bar{\rho}$, which implies $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$ by the statement and proof of Lemma F.7; we consider $\gamma^* = \gamma_s^H$ and $\gamma^* = \gamma^m$ separately. By Lemma F.4, $\gamma^* = \gamma_s^H$ implies the platform's optimal revenue is $R(\gamma^*) = \mu r^a(\gamma^*) + (1 - \mu)r^b(\gamma^*)^+$. Then, for $\rho \in [0, \bar{\rho}]$ we have

$$\begin{aligned} \left. \frac{dR}{d\rho} \right|_{\gamma=\gamma^*} &= \left(\frac{\partial R}{\partial \gamma} \frac{d\gamma_s^H}{d\rho} + \frac{\partial R}{\partial \rho} \right) \Big|_{\gamma=\gamma_s^H} \\ &= \left(\left(\mu \frac{\partial r^a}{\partial \gamma} + (1 - \mu) \frac{\partial r^b}{\partial \gamma} \right) \frac{d\gamma_s^H}{d\rho} + \left(\mu \frac{\partial r^a}{\partial \rho} + (1 - \mu) \frac{\partial r^b}{\partial \rho} \right) \right) \Big|_{\gamma=\gamma_s^H} \\ &< \left(\mu \frac{\partial r^a}{\partial \gamma} + (1 - \mu) \frac{\partial r^b}{\partial \gamma} \right) \frac{d\gamma_s^H}{d\rho} \Big|_{\gamma=\gamma_s^H}, \end{aligned}$$

where the strict inequality follows because $\frac{\partial}{\partial \rho} r^a < 0$ and $\frac{\partial}{\partial \rho} r^b < 0$ by Lemma F.6. It remains to show

$$\left(\mu \frac{\partial r^a}{\partial \gamma} + (1 - \mu) \frac{\partial r^b}{\partial \gamma} \right) \frac{d\gamma_s^H}{d\rho} \Big|_{\gamma=\gamma_s^H} \leq 0. \quad (46)$$

Because $\phi = 0$, we have $\gamma_s^H = 1 - \omega_s$ by Lemma A.10, which implies

$$\frac{d}{d\rho} \gamma_s^H = -\frac{(1 - \lambda)\lambda}{(1 - (1 - \lambda)\rho)^2} < 0.$$

Further, because $\left(\mu \frac{\partial r^a}{\partial \gamma} + (1 - \mu) \frac{\partial r^b}{\partial \gamma} \right) \geq 0$ must hold at $\gamma = \gamma^*$, we conclude that (46) holds. Therefore, $R(\gamma^*)$ strictly decreases in ρ for $\rho \in [0, \bar{\rho})$ if $\gamma^* = \gamma_s^H$. The case where $\gamma^* = \gamma^m$ follows by a similar argument, where the condition (46) (with γ^m in place of γ_s^H) holds trivially because $\frac{d}{d\rho} \gamma^m = 0$. We conclude $R(\gamma^*)$ strictly decreases in ρ for $\rho \in [0, \bar{\rho})$. It remains to show that $R(\gamma^*)$ increases in ρ on $(\bar{\rho}, 1]$. By Lemma F.7,

$\rho > \bar{\rho}$ implies $\gamma_s^H < \gamma^*$, and thus $R(\gamma^*) = \mu r^c(\gamma^*)$. Therefore, we have

$$\left. \frac{dR}{d\rho} \right|_{\gamma=\gamma^*} = \mu \left(\frac{\partial r^c}{\partial \gamma} \frac{d\gamma^*}{d\rho} + \frac{\partial r^c}{\partial \rho} \right) \Big|_{\gamma=\gamma^*} = \mu \frac{\partial r^c}{\partial \rho} \Big|_{\gamma=\gamma^*} > 0.$$

To see why the second equality holds, consider two cases: $\gamma^* = \gamma^m$ and $\gamma^* < \gamma^m$. If $\gamma^* = \gamma^m$, then $\frac{d}{d\rho}\gamma^* = 0$; if $\gamma^* < \gamma^m$, then $\frac{\partial}{\partial \rho} r^c = 0$ at $\gamma = \gamma^*$ by the envelope theorem. Finally, the strict inequality follows because $\frac{\partial}{\partial \rho} r^c > 0$ by Lemma F.6. We have thus shown $R(\gamma^*)$ strictly increases in ρ on $\rho \in (\bar{\rho}, 1]$ for $\mu = 1$. The existence of the threshold $\bar{\mu} < 1$ in the proposition statement then follows by continuity of $\frac{d}{d\rho} R(\gamma^*)$ in μ . $\square$

F.4.3 Proof of Proposition 7

Proposition 7. *Let $\gamma^*(\phi)$ be the optimal commission rate under switching cost ϕ . There exists $\bar{\phi} > 0$ and $\bar{\rho} \in [0, 1)$ such that for any $\rho \geq \bar{\rho}$ and $\phi \geq \bar{\phi}$, the optimal commission rate is higher in the absence of switching costs, $\gamma^*(0) \geq \gamma^*(\phi)$, where the inequality is strict if $\gamma^*(\phi) < \gamma^m$.*

Proof. Note by Lemma F.5, the platform's revenue is

$$R(\gamma) = \begin{cases} \mu r^a(\gamma) + (1 - \mu) r^b(\gamma)^+ & \text{if } \gamma \in [0, \gamma_s^H], \\ \mu r^c(\gamma) + (1 - \mu) r^b(\gamma)^+ & \text{if } \gamma \in (\gamma_s^H, \gamma_s^L], \\ \mu r^c(\gamma) + (1 - \mu) r^d(\gamma)^+ & \text{if } \gamma \in (\gamma_s^L, \gamma^m]. \end{cases}$$

Further, by Lemma F.4, we have $\gamma_s^H = \gamma_s^L = 0$ at $\phi = 0$ and $\rho = 1$. By continuity of γ_s^H and γ_s^L in ρ , it follows that there exists $\hat{\rho} \in [0, 1]$ such that $\gamma^* = \min\{\gamma^{cd}, \gamma^m\}$ if $\phi = 0$ and $\rho \geq \hat{\rho}$. Similarly, because γ_s^H strictly increases in ϕ (Lemma F.4), there exists $\bar{\phi} > 0$ such that $\gamma^* = \min\{\gamma^{ab}, \gamma^m\}$ if $\phi \geq \bar{\phi}$. Because γ^{ab} does not depend on ϕ , it remains to show there exists $\bar{\rho} \geq \hat{\rho}$ such that $\gamma^{ab} < \gamma^{cd}$ if $\rho \geq \bar{\rho}$ and $\phi = 0$, which we do in four steps. First, we define four auxilliary functions, $\ell^x(\gamma)$ for $x \in \{a, b, c, d\}$, which have the useful property that $\ell^x(\gamma^x) = 0$. Second, we show there exists $\bar{\rho} \in [\hat{\rho}, 1)$ such that $\ell^a(\gamma) < \ell^c(\gamma)$ for all $\gamma \in [0, \gamma^m]$ if $\rho \geq \bar{\rho}$. Third, we show $\ell^b(\gamma) < \ell^d(\gamma)$ for all $\gamma \in [0, \gamma^m]$ and $\rho \in [0, 1]$. Fourth, we combine the first three steps to prove the proposition statement.

Step 1. Fix $\phi = 0$. We begin by defining four auxiliary functions $\ell^a(\gamma)$, $\ell^b(\gamma)$, $\ell^c(\gamma)$, and $\ell^d(\gamma)$. Note differentiating $r^a(\gamma)$ in γ yields

$$\frac{\partial r^a}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left\{ \gamma \left(\frac{q_H}{2} - \frac{(\rho c(1 - \lambda))^2}{2q_H(1 - \gamma)^2} \right) \right\} = \underbrace{\frac{q_H}{2} \left(1 - \frac{(\rho c(1 - \lambda))^2}{q_H^2} \cdot \frac{(1 + \gamma)}{(1 - \gamma)^3} \right)}_{\ell^a(\gamma)}, \quad (47)$$

where $\ell^a(\gamma)$ is defined as shown in (47). Similarly, for $r^b(\gamma)$,

$$\begin{aligned} \frac{\partial r^b}{\partial \gamma} &= \frac{\partial}{\partial \gamma} \left\{ \frac{q_L(2 - (1 - \lambda)\rho)}{4} \left(\gamma - \frac{(c(1 - \lambda)(2 - \rho)\rho)^2}{q_L^2(2 - (1 - \lambda)\rho)^2} \cdot \frac{\gamma}{(1 - \gamma)^2} \right) \right\} \\ &= \frac{q_L(2 - (1 - \lambda)\rho)}{4} \underbrace{\left(1 - \frac{(c(1 - \lambda)(2 - \rho)\rho)^2}{q_L^2(2 - (1 - \lambda)\rho)^2} \cdot \frac{(1 + \gamma)}{(1 - \gamma)^3} \right)}_{\ell^b(\gamma)}. \end{aligned} \quad (48)$$

For $r^c(\gamma)$,

$$\begin{aligned}\frac{\partial r^c}{\partial \gamma} &= \frac{\partial}{\partial \gamma} \left\{ \frac{q_H}{4} (1 + (1 - \lambda)\rho) \left(\gamma - \frac{(4c(1 - \lambda)\rho)^2}{q_H^2} \cdot \frac{\gamma}{((1 - \gamma)(3 + (1 - \lambda)\rho) + \lambda)^2} \right) \right\} \\ &= \frac{q_H}{4} (1 + (1 - \lambda)\rho) \underbrace{\left(1 - \frac{(4c(1 - \lambda)\rho)^2}{q_H^2} \cdot \frac{(1 + \gamma)(3 + (1 - \lambda)\rho) + \lambda}{((1 - \gamma)(3 + (1 - \lambda)\rho) + \lambda)^3} \right)}_{\ell^c(\gamma)}.\end{aligned}\quad (49)$$

Lastly, for $r^d(\gamma)$,

$$\begin{aligned}\frac{\partial r^d}{\partial \gamma} &= \frac{\partial}{\partial \gamma} \left\{ \frac{q_L}{4} \left(\gamma - \frac{(2c(1 - \lambda)(2 - \rho)\rho)^2}{q_L^2} \cdot \frac{\gamma}{((1 - \gamma)(3 - (1 - \lambda)\rho) + \lambda)^2} \right) \right\} \\ &= \frac{q_L}{4} \underbrace{\left(1 - \frac{(2c(1 - \lambda)(2 - \rho)\rho)^2}{q_L^2} \cdot \frac{(1 + \gamma)(3 - (1 - \lambda)\rho) + \lambda}{((1 - \gamma)(3 - (1 - \lambda)\rho) + \lambda)^3} \right)}_{\ell^d(\gamma)}.\end{aligned}\quad (50)$$

Further, note for $x \in \{a, b, c, d\}$ we have $\ell^x(\gamma^x) = 0$.

Step 2. We now show there exists $\bar{\rho} \in [\hat{\rho}, 1)$ such that $\ell^a(\gamma) < \ell^c(\gamma)$ for all $\gamma \in [0, \gamma^m]$ if $\rho \geq \bar{\rho}$. Using the expressions given in (47) and (49), note $\ell^a(\gamma) < \ell^c(\gamma)$ holds if and only if

$$\frac{(\rho c(1 - \lambda))^2}{q_H^2} \cdot \frac{1 + \gamma}{(1 - \gamma)^3} > \frac{(4c(1 - \lambda)\rho)^2}{q_H^2} \cdot \frac{(1 + \gamma)(3 + (1 - \lambda)\rho) + \lambda}{((1 - \gamma)(3 + (1 - \lambda)\rho) + \lambda)^3},$$

or equivalently, $h(\gamma) < 0$, where

$$h(\gamma) = \frac{16((1 + \gamma)(3 + (1 - \lambda)\rho) + \lambda)}{((1 - \gamma)(3 + (1 - \lambda)\rho) + \lambda)^3} - \frac{1 + \gamma}{(1 - \gamma)^3}.$$

It remains to show that for any $\gamma \in (0, \gamma^m]$, $h(\gamma) < 0$ if $\rho > \bar{\rho}$ for some $\bar{\rho} \in [\hat{\rho}, 1)$. First, observe that

$$\begin{aligned}\lim_{\rho \rightarrow 1} h(\gamma) &= 16 \frac{(1 + \gamma)(4 - \lambda) + \lambda}{((1 - \gamma)(4 - \lambda) + \lambda)^3} - \frac{1 + \gamma}{(1 - \gamma)^3} \\ &= 16 \frac{1 + \gamma}{(1 - \gamma)^3} \frac{(4 - \lambda) + \frac{\lambda}{1 + \gamma}}{((4 - \lambda) + \frac{\lambda}{1 - \gamma})^3} - \frac{1 + \gamma}{(1 - \gamma)^3} \\ &< 16 \frac{1 + \gamma}{(1 - \gamma)^3} \frac{(4 - \lambda) + \lambda}{((4 - \lambda) + \lambda)^3} - \frac{1 + \gamma}{(1 - \gamma)^3} \\ &= 16 \frac{1 + \gamma}{(1 - \gamma)^3} \frac{1}{4^2} - \frac{1 + \gamma}{(1 - \gamma)^3} \\ &= 0,\end{aligned}$$

where the strict inequality follows because $\gamma \in (0, \gamma^m]$. We also have

$$\frac{\partial h}{\partial \rho} = -\frac{32(1 - \lambda)((1 - \gamma^2)(3 + (1 - \lambda)\rho) + \lambda(1 - 2\gamma))}{((1 - \gamma)(3 + (1 - \lambda)\rho) + \lambda)^4} \leq 0,$$

where the inequality follows because $\gamma \in [0, \frac{1}{2}]$. We have thus shown that for any $\gamma \in [0, \gamma^m]$, $h(\gamma)$ strictly decreases in ρ and $\lim_{\rho \rightarrow 1} h(\gamma) < 0$. It follows that for each $\gamma \in [0, \gamma^m]$, there exists $\bar{\rho}(\gamma) \in [0, 1)$ such that $h(\gamma) < 0$ and thus $\ell^a(\gamma) < \ell^c(\gamma)$ if $\rho > \bar{\rho}(\gamma)$. The result follows by choosing $\bar{\rho}$ to be the larger of $\max_{\gamma \geq 0} \bar{\rho}(\gamma)$ and the threshold $\hat{\rho}$ defined at the beginning of the proof.

Step 3. The proof is similar to Step 2. Using the expressions from (48) and (50), it can be shown that $\ell^b(\gamma) < \ell^d(\gamma)$ holds if and only if $g(\gamma) < 0$, where

$$g(\gamma) = \frac{4((1+\gamma)(3-(1-\lambda)\rho) + \lambda)}{((1-\gamma)(3-(1-\lambda)\rho) + \lambda)^3} - \frac{1+\gamma}{(1-\gamma)^3}.$$

We show that for any $\gamma \in (0, \gamma^m]$, $g(\gamma) < 0$ for all $\rho \in [0, 1]$ and $\lambda \in [\frac{1}{2}, 1]$. We first show $g(\gamma)$ decreases in λ . To see this, note

$$\begin{aligned} \frac{\partial g}{\partial \lambda} &= \frac{\partial}{\partial \lambda} \left\{ \frac{4}{((1-\gamma)(3-(1-\lambda)\rho) + \lambda)^2} \cdot \frac{(1+\gamma)(3-(1-\lambda)\rho) + \lambda}{((1-\gamma)(3-(1-\lambda)\rho) + \lambda)} - \frac{1+\gamma}{(1-\gamma)^3} \right\} \\ &= \frac{\partial}{\partial \lambda} \left\{ \frac{4}{((1-\gamma)(3-(1-\lambda)\rho) + \lambda)^2} \right\} \cdot \frac{(1+\gamma)(3-(1-\lambda)\rho) + \lambda}{((1-\gamma)(3-(1-\lambda)\rho) + \lambda)} \\ &\quad + \frac{4}{((1-\gamma)(3-(1-\lambda)\rho) + \lambda)^2} \cdot \frac{\partial}{\partial \lambda} \left\{ \frac{(1+\gamma)(3-(1-\lambda)\rho) + \lambda}{((1-\gamma)(3-(1-\lambda)\rho) + \lambda)} \right\} \\ &< -\frac{4}{((1-\gamma)(3-(1-\lambda)\rho) + \lambda)^2} \cdot \frac{2\gamma(3-\rho)}{((1-\gamma)(3-(1-\lambda)\rho) + \lambda)^2} \\ &< 0. \end{aligned}$$

The first strict inequality above follows because $(1-\gamma)(3-(1-\lambda)\rho) + \lambda$ strictly increases in λ , which implies $\frac{\partial}{\partial \lambda} \left\{ \frac{4}{((1-\gamma)(3-(1-\lambda)\rho) + \lambda)^2} \right\} < 0$. Because $g(\gamma)$ strictly decreases in λ , plugging in $\lambda = 0$ yields the upper bound

$$g(\gamma) < \frac{4(1+\gamma)(3-\rho)}{((1-\gamma)(3-\rho))^3} - \frac{1+\gamma}{(1-\gamma)^3} = \frac{1+\gamma}{(1-\gamma)^3} \left(\frac{4}{(3-\rho)^2} - 1 \right) \leq 0$$

for $\rho \in [0, 1]$ and $\lambda \in [\frac{1}{2}, 1]$. We have thus shown $g(\gamma) < 0$, which implies $\ell^b(\gamma) < \ell^d(\gamma)$.

Step 4. We now complete the proof by showing $\gamma^{cd} > \gamma^{ab}$ holds for $\phi = 0$ and $\rho \geq \bar{\rho}$. Analogous to Lemma A.12, it can be shown that $r^x(\gamma)$ is strictly concave in γ for $x \in \{a, b, c, d\}$. It follows that $\mu r^a(\gamma) + (1-\mu)r^b(\gamma)$ and $\mu r^c(\gamma) + (1-\mu)r^d(\gamma)$ are both strictly concave in γ . Therefore, to show $\gamma^{cd} > \gamma^{ab}$, it suffices to show

$$\left(\mu \frac{\partial r^c}{\partial \gamma} + (1-\mu) \frac{\partial r^d}{\partial \gamma} \right) \Big|_{\gamma=\gamma^{ab}} > 0. \quad (51)$$

Using the expressions for $\ell^a(\gamma)$ and $\ell^b(\gamma)$, it is straightforward to verify that $\gamma^b \leq \gamma^a$. Further, because $r^a(\gamma)$ and $r^b(\gamma)$ are both strictly concave, we must have $\gamma^b \leq \gamma^{ab} \leq \gamma^a$. By Step 2, we have $\gamma^a < \gamma^c$ for $\rho \geq \bar{\rho}$, which implies $\gamma^{ab} < \gamma^c$ and thus $\frac{\partial}{\partial \gamma} r^c > 0$ at $\gamma = \gamma^{ab}$ for $\rho \geq \bar{\rho}$. Therefore, (51) follows immediately if $\frac{\partial}{\partial \gamma} r^d \geq 0$ at $\gamma = \gamma^{ab}$. It remains to show (51) also holds when $\frac{\partial}{\partial \gamma} r^d < 0$ at $\gamma = \gamma^{ab}$. By Step 2, for each $\gamma \in (0, \gamma^m]$ we have $\ell^a(\gamma) < \ell^c(\gamma)$ for $\rho \geq \bar{\rho}$. It follows that

$$\frac{\partial r^c / \partial \gamma}{\partial r^a / \partial \gamma} = \frac{1 + (1-\lambda)\rho}{2} \cdot \frac{\ell^c(\gamma)}{\ell^a(\gamma)} > \frac{1 + (1-\lambda)\rho}{2}. \quad (52)$$

Similarly, by Step 3 we have $\ell^b(\gamma) < \ell^d(\gamma)$ for any $\gamma \in (0, \gamma^m]$, which implies

$$\frac{\partial r^d / \partial \gamma}{\partial r^b / \partial \gamma} = \frac{1}{2 - (1-\lambda)\rho} \cdot \frac{\ell^d(\gamma)}{\ell^b(\gamma)} > \frac{1}{2 - (1-\lambda)\rho}. \quad (53)$$

We can now write for $\rho \geq \bar{\rho}$,

$$\begin{aligned}
\left(\mu \frac{\partial r^c}{\partial \gamma} + (1 - \mu) \frac{\partial r^d}{\partial \gamma} \right) \Big|_{\gamma = \gamma^{ab}} &> \left(\mu \frac{\partial r^a}{\partial \gamma} \cdot \frac{1 + (1 - \lambda)\rho}{2} + (1 - \mu) \frac{\partial r^b}{\partial \gamma} \cdot \frac{1}{2 - (1 - \lambda)\rho} \right) \Big|_{\gamma = \gamma^{ab}} \\
&\geq \frac{1 + (1 - \lambda)\rho}{2} \left(\mu \frac{\partial r^a}{\partial \gamma} + (1 - \mu) \frac{\partial r^b}{\partial \gamma} \right) \Big|_{\gamma = \gamma^{ab}} \\
&\geq 0,
\end{aligned}$$

where the strict inequality follows from applying (52) and (53) and noting that $\gamma^{ab} \leq \gamma^a \leq \gamma^c$ implies $\frac{\partial}{\partial \gamma} r^a \geq 0$ and $\frac{\partial}{\partial \gamma} r^c \geq 0$ at $\gamma = \gamma^{ab}$; the second inequality holds because $\frac{1 + (1 - \lambda)\rho}{2} \geq \frac{1}{2 - (1 - \lambda)\rho}$ for all $\lambda \in [\frac{1}{2}, 1]$ and $\rho \in [0, 1]$, and because $\gamma^b \leq \gamma^{ab}$ implies $\frac{\partial}{\partial \gamma} r^b \leq 0$ at $\gamma = \gamma^{ab}$; and the final inequality follows by definition of γ^{ab} . Because $\gamma^* = \min\{\gamma^{ab}, \gamma^m\}$ for all $\phi \geq \bar{\phi}$ and $\gamma^* = \min\{\gamma^{cd}, \gamma^m\}$ for $\phi = 0$ as established at the beginning of the proof, we conclude $\gamma^*(0) \geq \gamma^*(\phi)$ for all $\phi \geq \bar{\phi}$ and $\rho \geq \bar{\rho}$. Finally, to see that the inequality is strict wherever $\gamma^*(\phi) < \gamma^m$, note $\gamma^*(\phi) < \gamma^m$ implies $\gamma^*(\phi) = \gamma^{ab} < \min\{\gamma^{cd}, \gamma^m\} = \gamma^*(0)$. $\square$